



Toward Responsive DBMS: Optimal Join Algorithms, Enumeration, Factorization, Ranking, and Dynamic Programming

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Part 6 : Any- k

Slides: <https://northeastern-datalab.github.io/responsive-dbms-tutorial>

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Data Lab: <https://db.khoury.northeastern.edu>



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Outline tutorial

1: Introduction (Nikos) ~40min

2: Tree Decompositions (Mirek) ~20min

3: Acyclic Queries & Enumeration (Wolfgang) ~25min

BREAK

4: Factorization (Nikos) ~10min

5: Dynamic Programming & Semirings (Wolfgang) ~20min

6: Any- k or Ranked Enumeration (Nikos) ~35min

7. Decomposition of Comparison Predicates (Mirek) ~10min

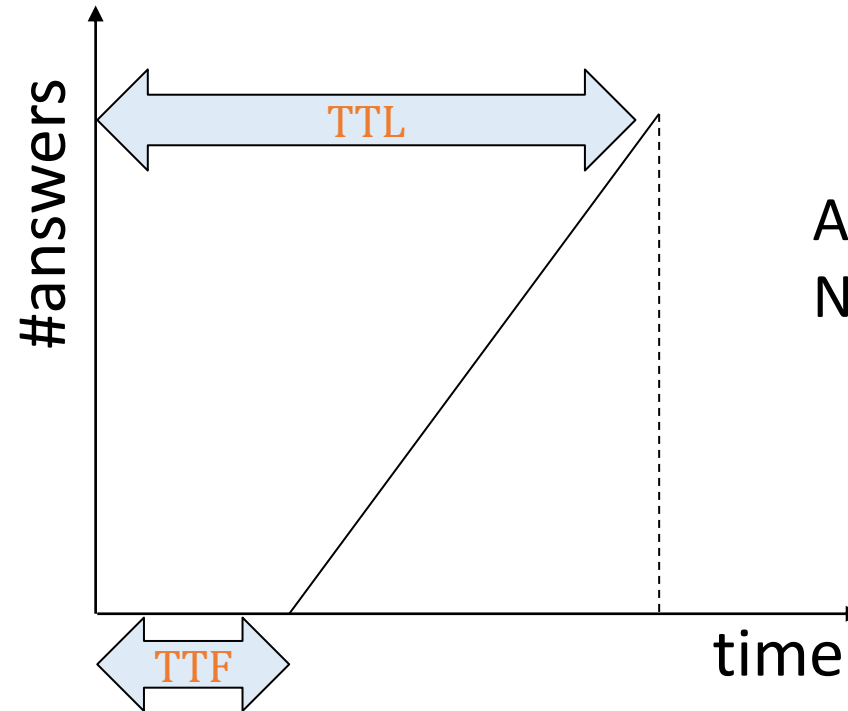
8. Conclusion (Mirek) ~10min

Any-k Algorithms

“Any-k”

Anytime algorithms + Top-k for Join Queries

Most important results first
(ranking function on output
tuples, e.g. sum of weights)



All results eventually returned
No need to set k in advance

- $TT(k)$ = Time-to- k^{th}
- TTF = Time-to-First = $TT(1)$
- TTL = Time-to-Last = $TT(|\text{out}|)$

Outline Part 6

Part 6: Any- k

- Warm-up: Incremental QuickSort
- Any- k for joins
- AnyK-Part
- AnyK-Rec
- AnyK-Part+
- Experimental Results & Summary

Incremental Sorting

- Simplest case of ranked enumeration: **incremental sorting** of n numbers
- What is the best we can hope for?
 - We need $O(n)$ to look at each number once
 - The output is returned sorted so $O(k \log k)$ for k elements
 - **$TT(k) = O(n + k \log k)$**
- One simple approach: Heapsort
 - Build a heap and pop n times
 - $TT(k) = O(n + k \log n) = O(n + k \log k)$
- An alternative approach, faster in practice: **Incremental QuickSort (IQS)**

Quicksort

Pick pivot

51 81 74 12 58 92 86 67 33 18 41 49 63 29 37

≤ 51

33 37 29 12 49 41 18 51 74 86 92 58 63 67 81

> 51

Recursion

18 29 12 33 41 49 37 51 63 67 58 74 86 92 81

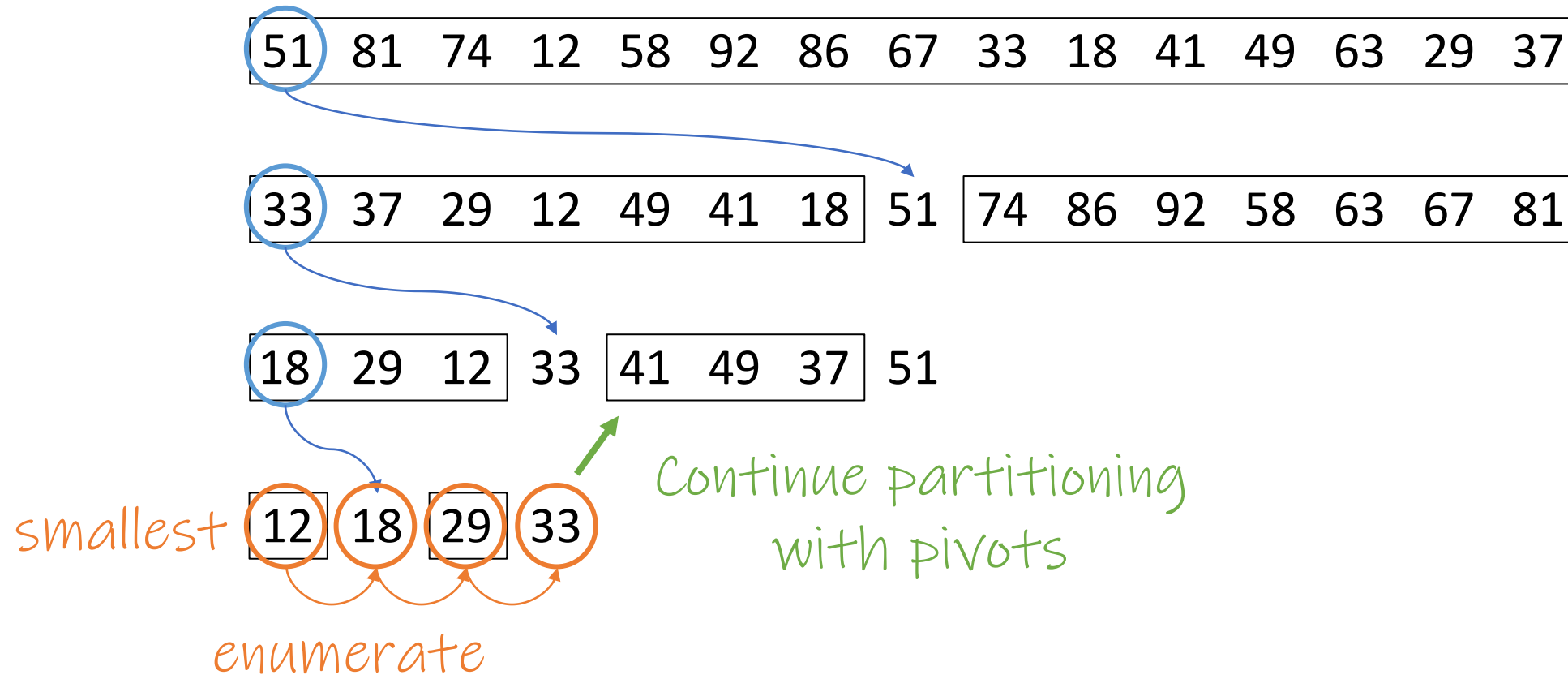
12 18 29 33 37 41 49 51 58 63 67 74 81 86 92

sorted

$O(n \log n)$ for all to be sorted
How can we get the top ones faster?

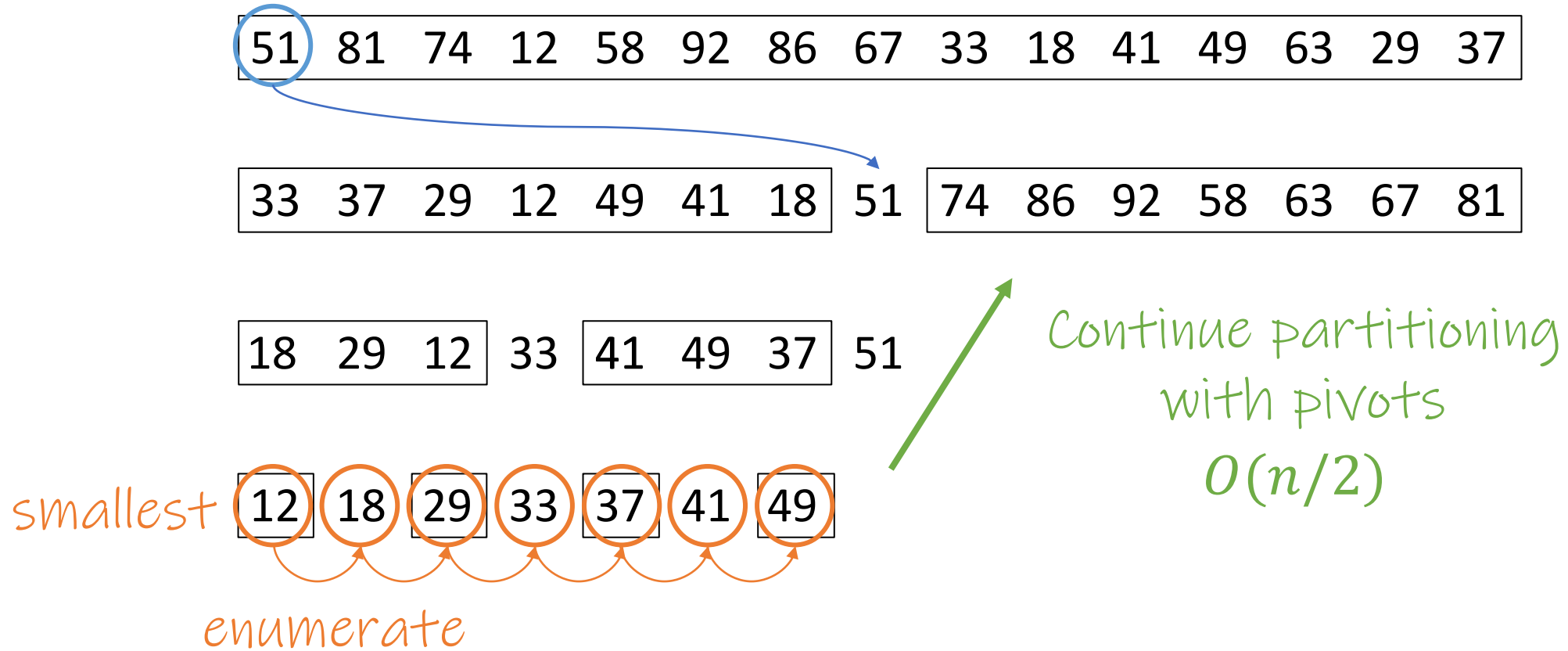
(In expectation with random pivots. Can be derandomized)

Incremental Quicksort



$$TT(k) = O(n + k \log k) \quad \text{😊}$$

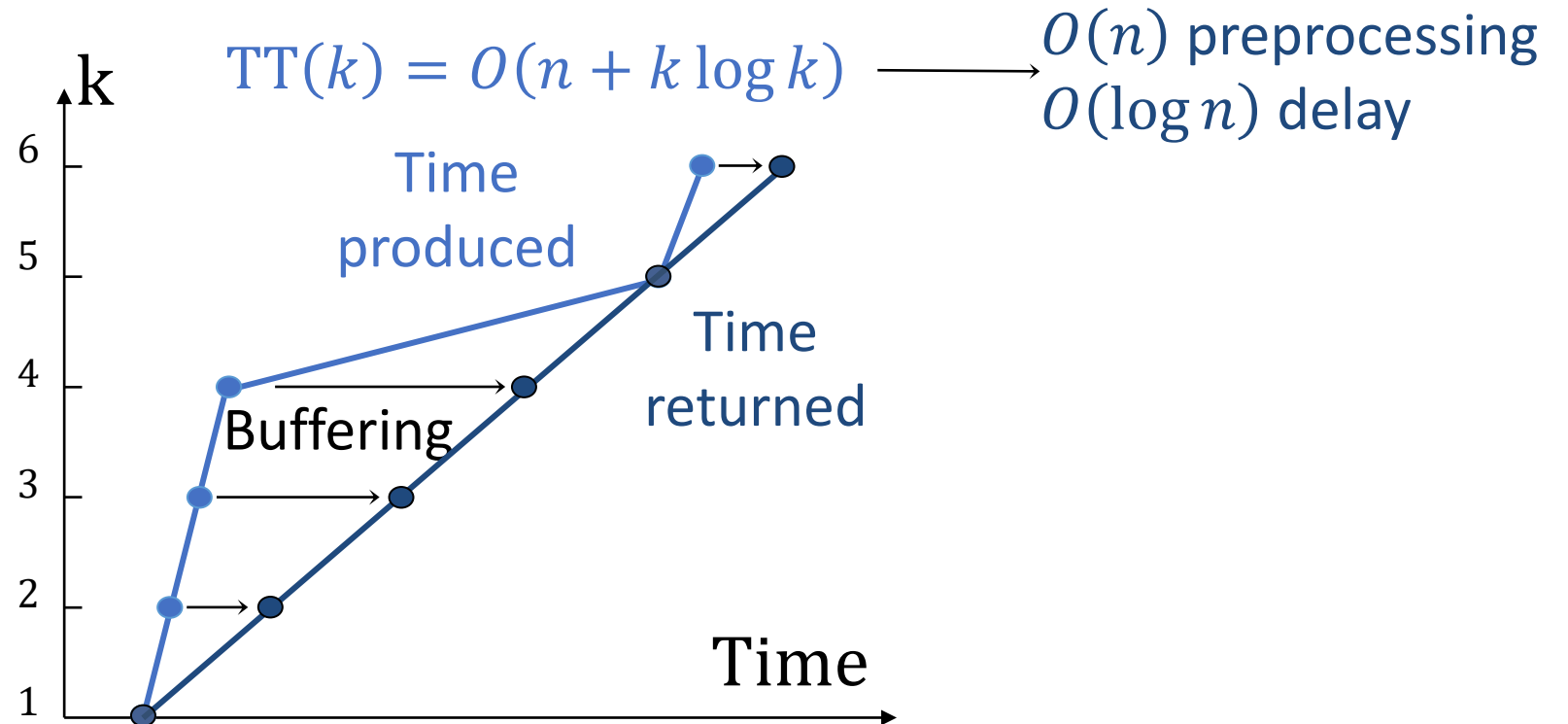
Incremental Quicksort



$$\text{TT}(k) = O(n + k \log k) \quad \text{😊}$$

Delay = $O(n)$??☹??

Enumeration measures: TT(k) vs Delay



Requiring low delay (logarithmic here) would make the algorithm slower!

$$TT(k) = O(n + k \log k) \quad \text{😊}$$

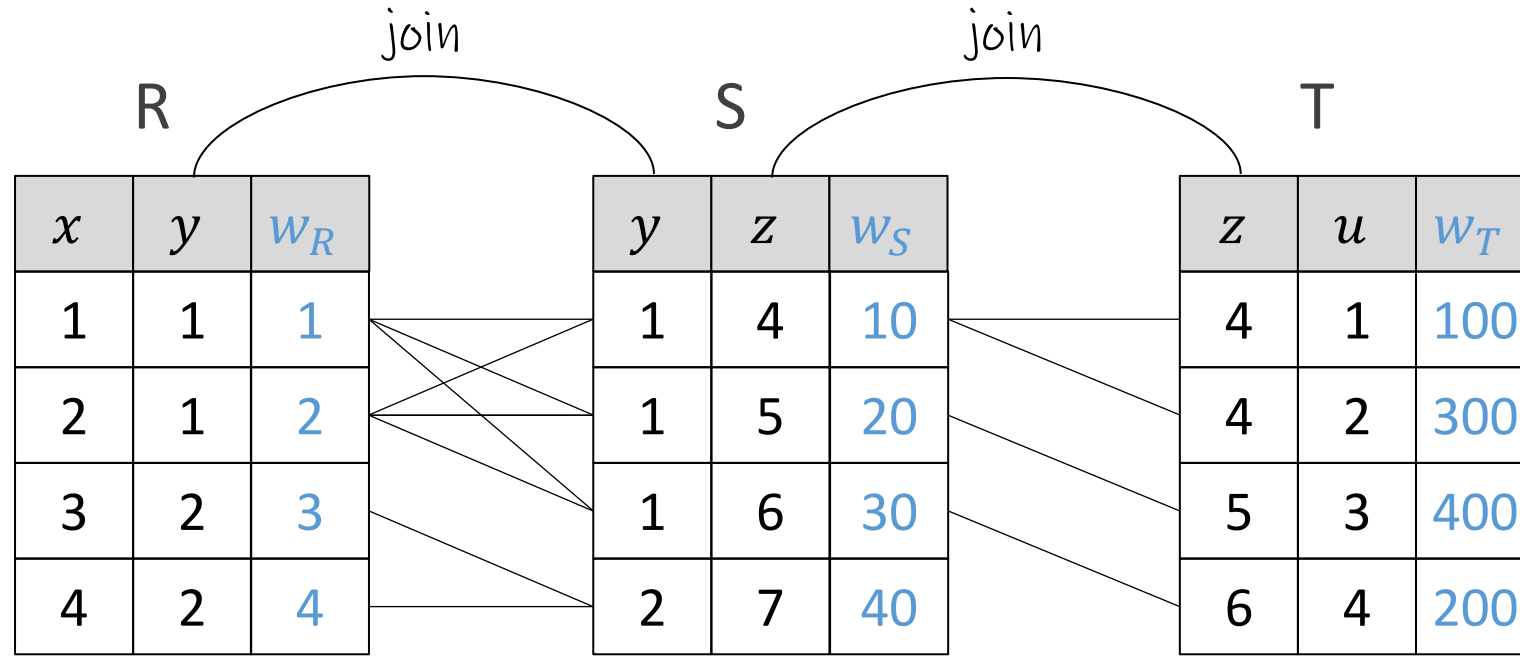
$$\text{Delay} = O(n) \quad \text{??😞??}$$

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Ranked Enumeration for Joins: Example



$R \bowtie S \bowtie T$

24 results

Weights

Rank-1

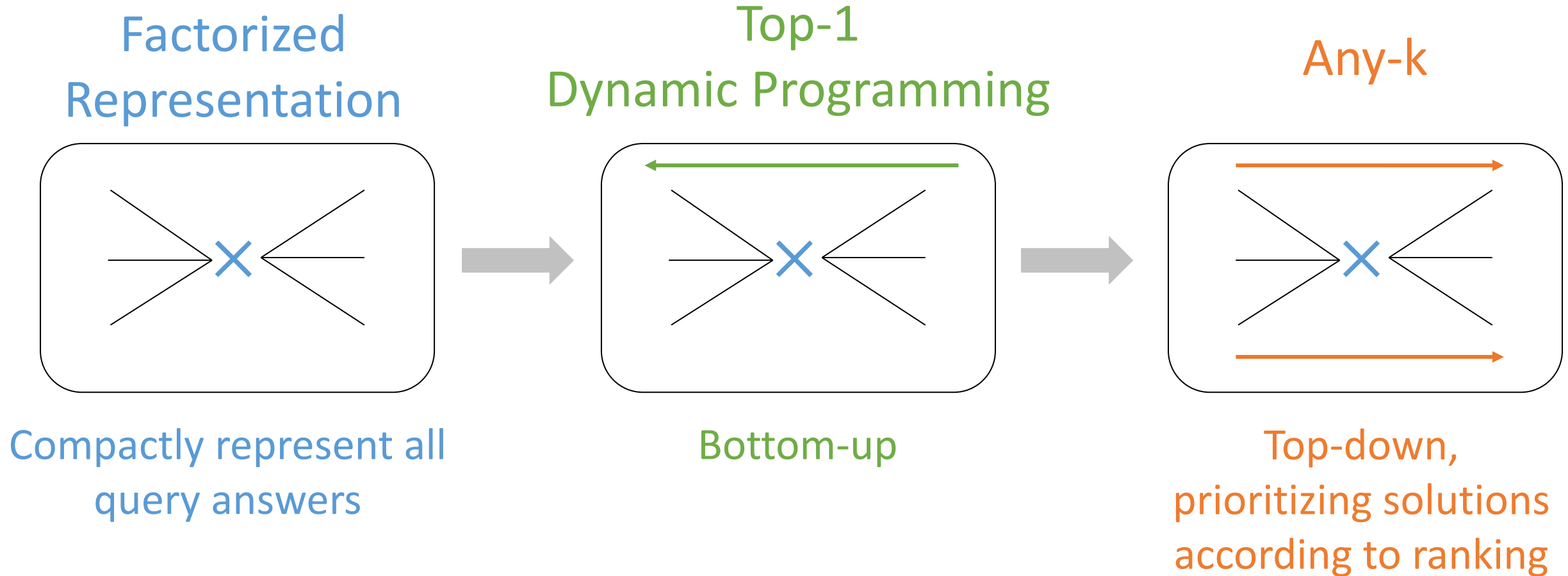
Rank-2

Rank-3

(1, 1, 4, 1, **111**) $\Rightarrow$ (2, 1, 4, 1, **112**) $\Rightarrow$ (1, 1, 6, 4, **231**) $\Rightarrow$...

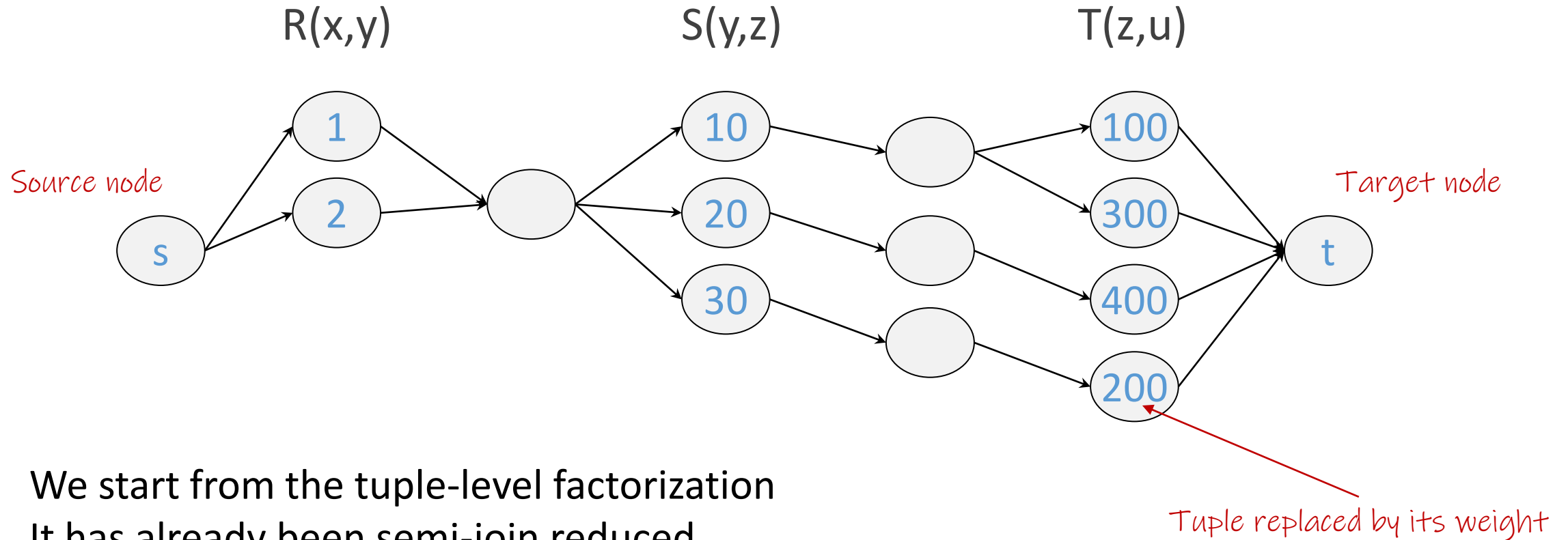
sum of weights

Overview of our Approach



Top-1: Dynamic Programming

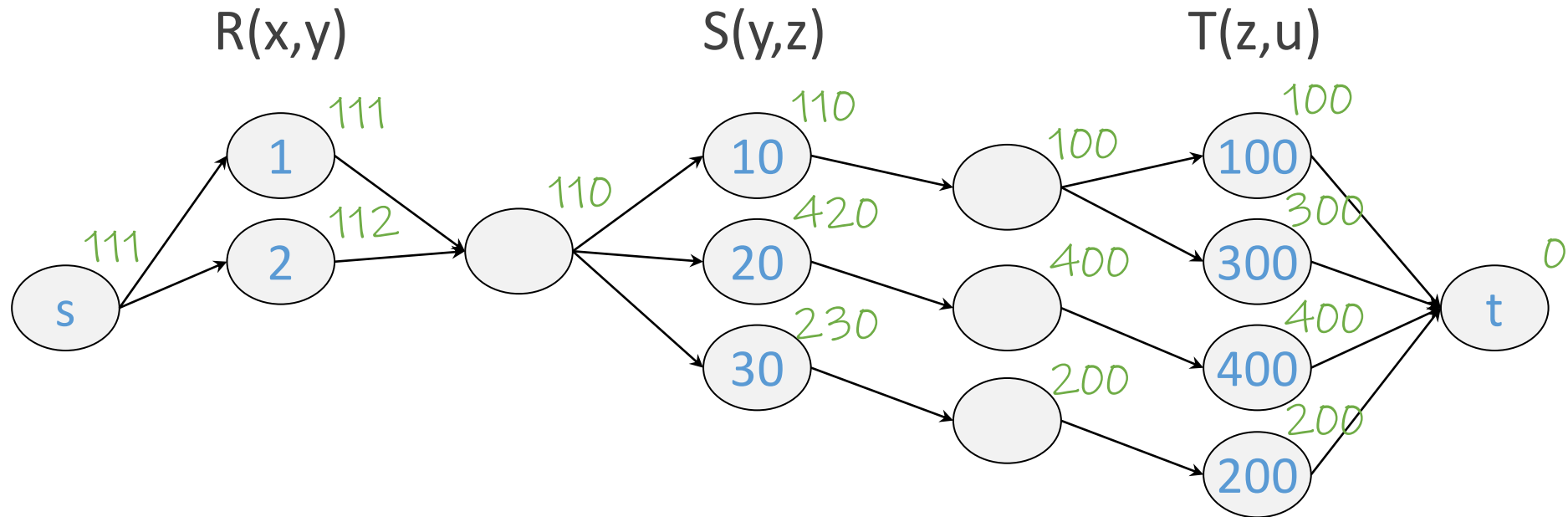
Factorized Representation



- We start from the tuple-level factorization
- It has already been semi-join reduced

(Here we assume the weights are on the nodes.
Equivalently, we could place the weights on the edges)

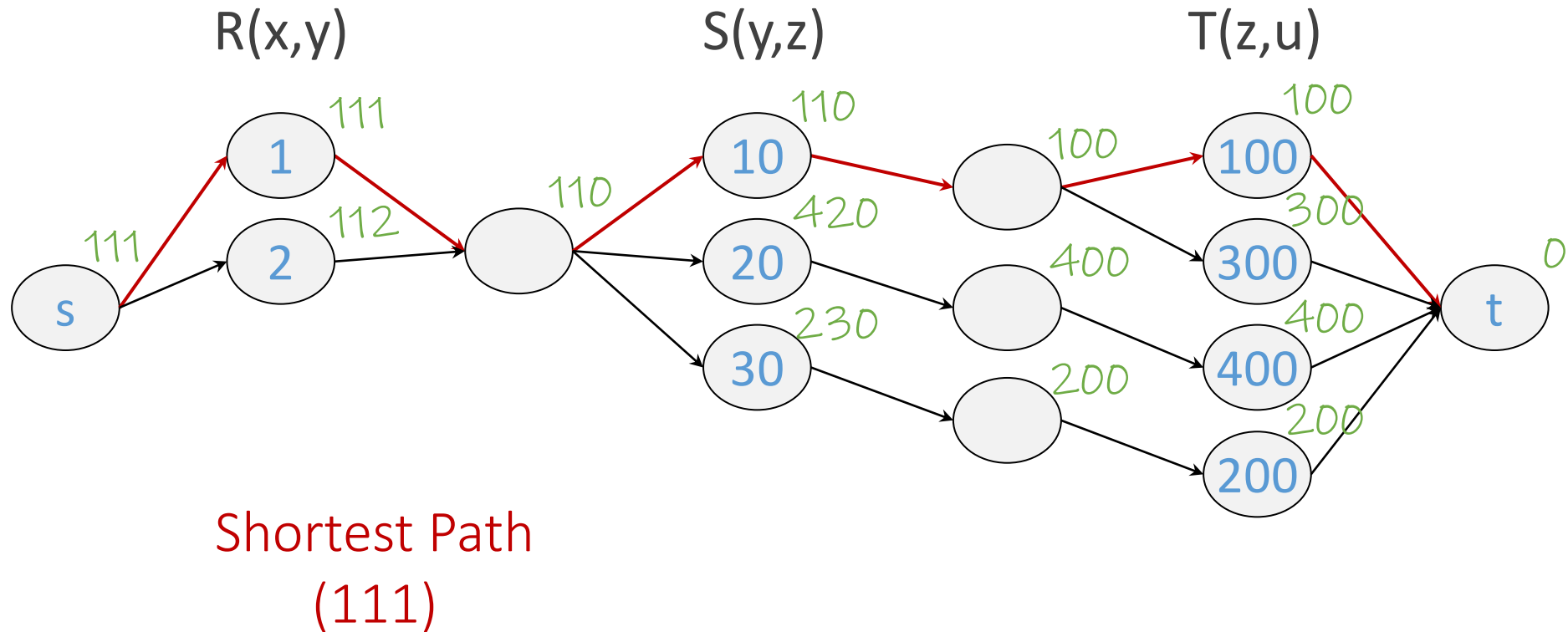
Top-1: Dynamic Programming



DP: Bottom-up (right-to-left)

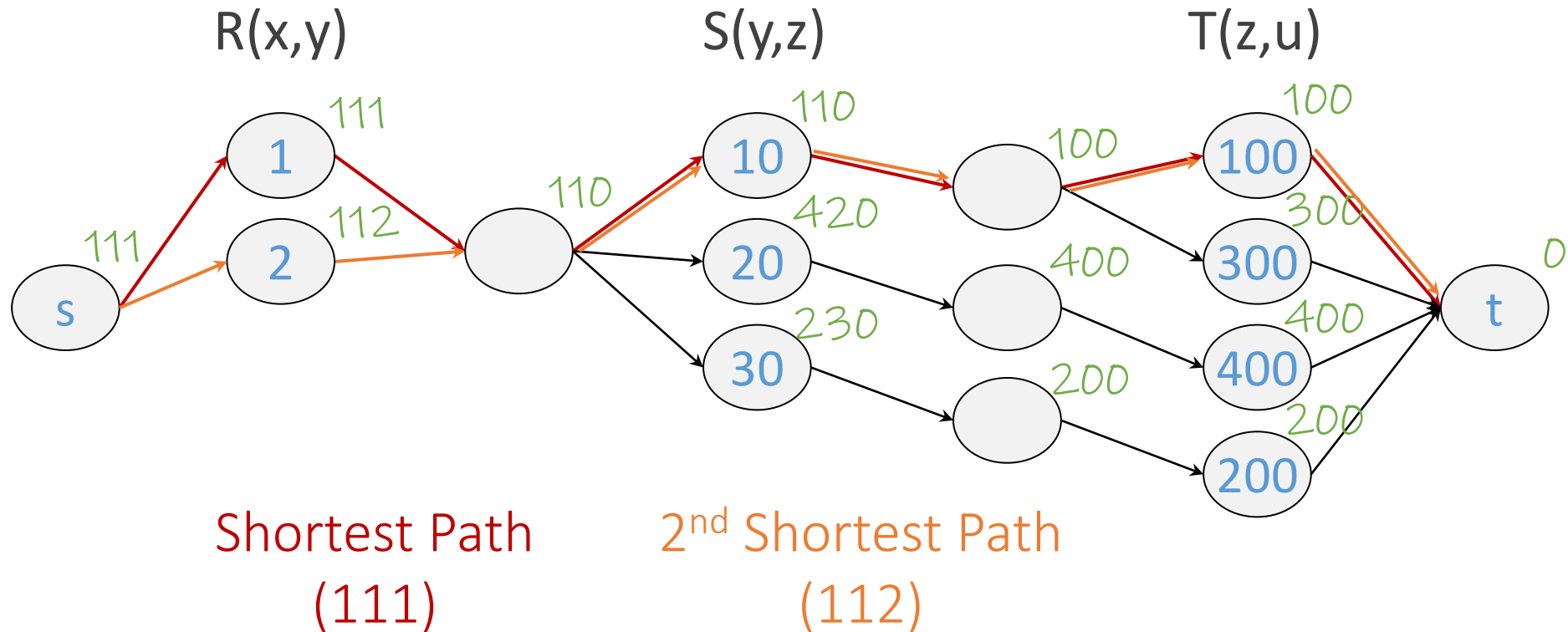
computes the optimal weight per node

DP as a Shortest Path Problem



DP computation equivalent to finding the shortest path in a graph

DP as a Shortest Path Problem



How do we find the k^{th} best solution to a DP problem?

- Rank-1 DP solution $\Rightarrow$ shortest path
- Rank- k DP solution $\Rightarrow$ k^{th} shortest path

Any-k DP Algorithms

Anyk-Part

Repeatedly **partitions** the solution space.
Relies on the Lawler-Murty procedure.

Wins when k is small.

$$TT(k) = O(n\ell + k(\log k + \ell))$$

Anyk-Rec

Recursively computes lower-rank paths
(suffixes) and reuses them.

Wins when k is large.

$$TT(k) = O(n\ell + k\ell \log n)$$

But can be even **faster than sorting!**

TTL for Cartesian Product: $O(n^\ell(\log n + \ell))$

• for sorting

Anyk-Part+

Combines the best of both worlds.

Wins over Anyk-Rec for small k.

Wins over Anyk-Part for large k.

$$TT(k) = O(n\ell + k(\log \min\{n, k\} + \ell))$$

(for free-connex CQ,
sum-of-weights ranking)

n : data size
 ℓ : query size

Tziavelis, Ajwani, Gatterbauer, Riedewald, Yang. Optimal Algorithms for Ranked Enumeration of Answers to Full Conjunctive Queries. PVLDB'20

<https://doi.org/10.14778/3397230.3397250> Extended report: <https://arxiv.org/abs/1911.05582>

Tziavelis, Gatterbauer, Riedewald. Any-k Algorithms for Enumerating Ranked Answers to Conjunctive Queries. arXiv'22 <https://arxiv.org/abs/2205.05649>

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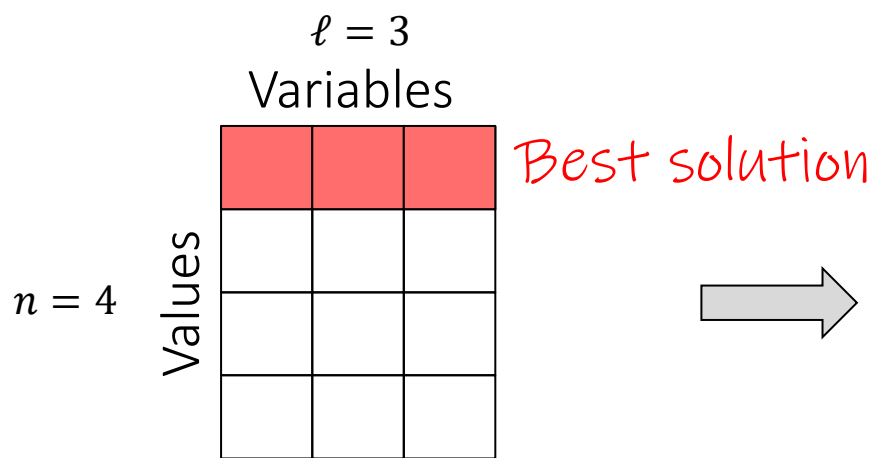
Ingredient 1: Lawler-Murty Procedure

[Lawler 72]: generic procedure for ranked enumeration

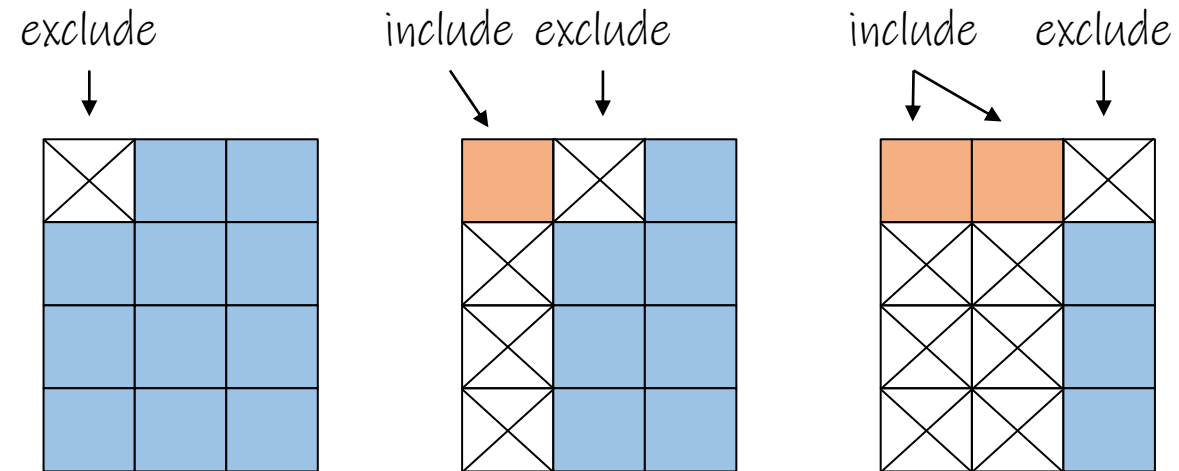
- Repeatedly partitions the solution space
- Applicable to a wide range of problems
- Generalization of an earlier algorithm of [Murty 68]

Solution space =
Subset of Cartesian Product

Original Solution Space



Disjoint Subspaces for next best solution



[Lawler 72] Lawler. A procedure for computing the k best solutions to discrete optimization problems and its application to the shortest path problem.

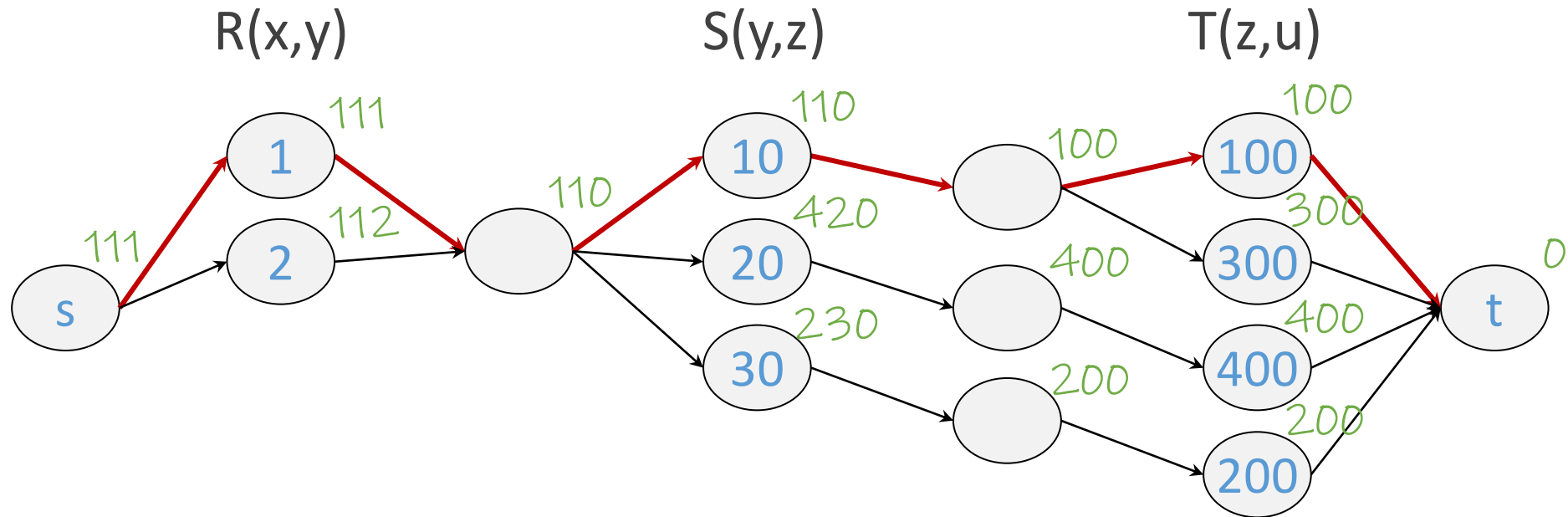
Management Science'72 <https://doi.org/10.1287/mnsc.18.7.401>

[Murty 68] Murty. An Algorithm for Ranking all the Assignments in Order of Increasing Cost. Operations Research'68 <https://doi.org/10.1287/opre.16.3.682>

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2nd Best Path

Top-1 Path



What can the 2nd best path be?

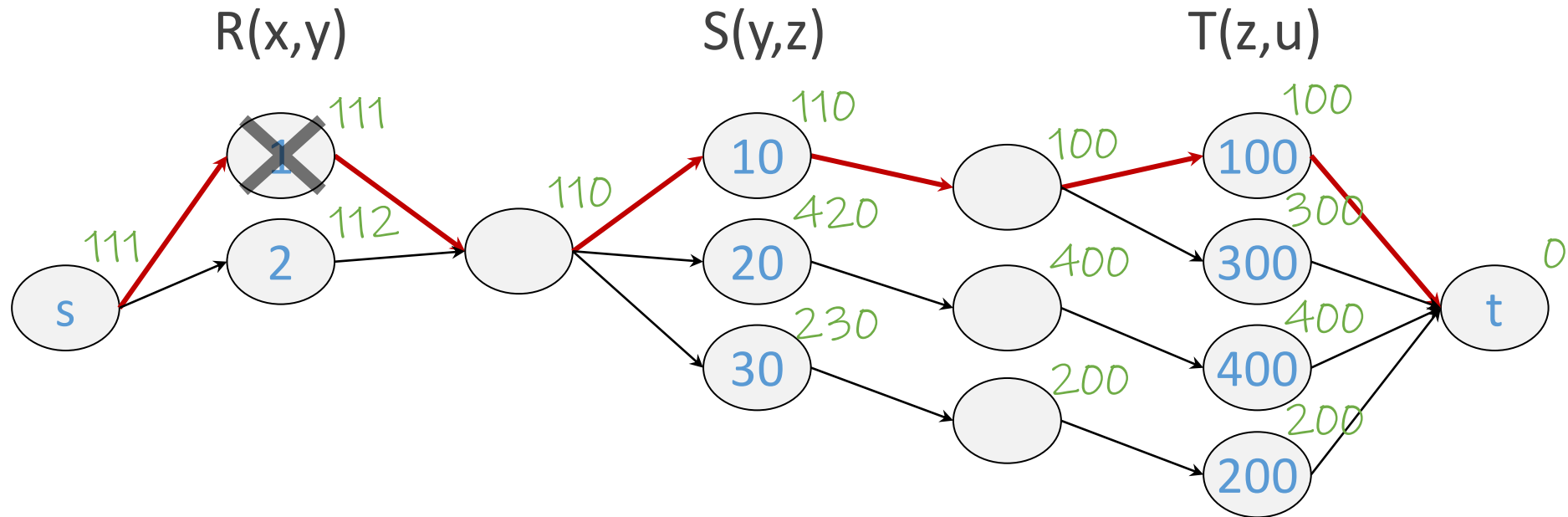
Option 1: Deviate in the first relation

Option 2: Keep the first decision. Deviate in the 2nd relation

Option 3: Keep the first and second decisions. Deviate in the 3rd relation

2nd Best Path

Top-1 Path



What can the 2nd best path be?

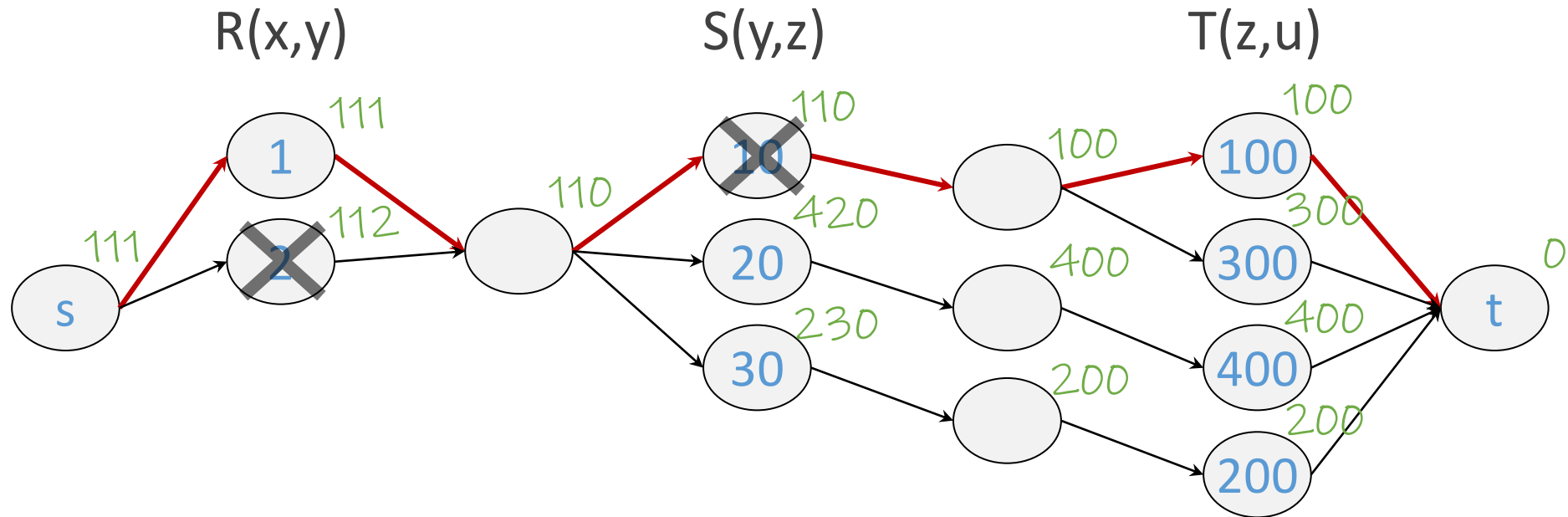
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2nd Best Path

Top-1 Path



What can the 2nd best path be?

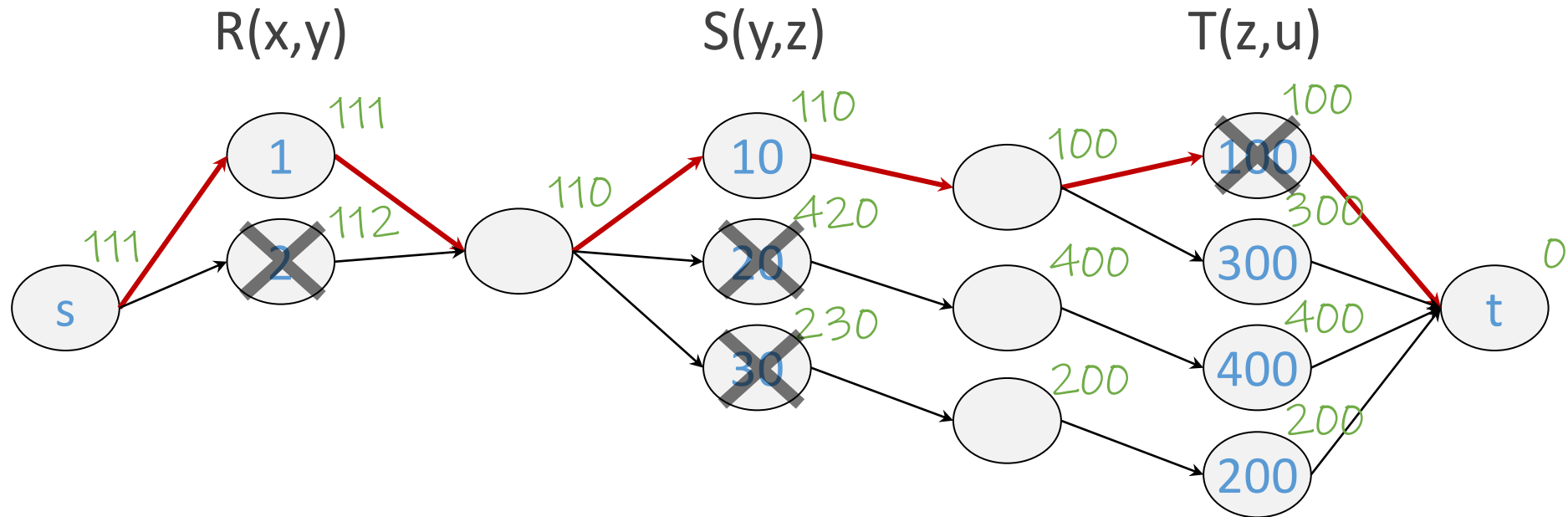
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2nd Best Path

Top-1 Path



What can the 2nd best path be?

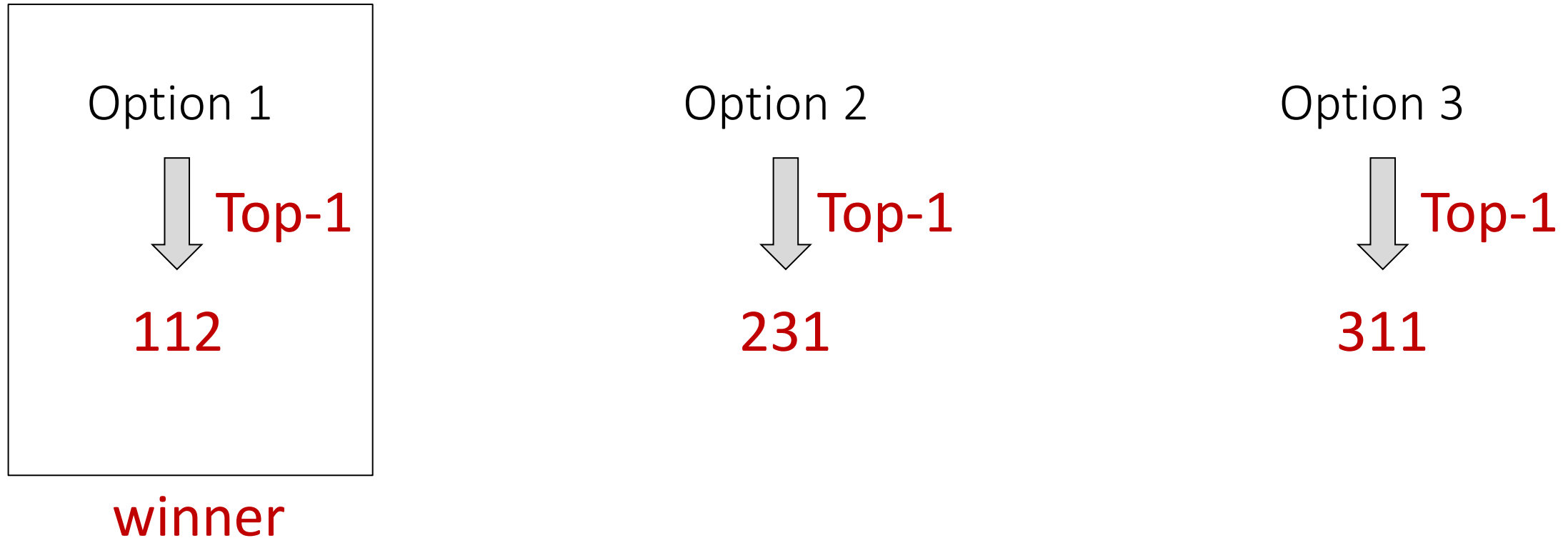
Option 1: Deviate in the first relation

Option 2: Keep the first decision. Deviate in the 2nd relation

Option 3: Keep the first and second decisions. Deviate in the 3rd relation

2nd Best Path

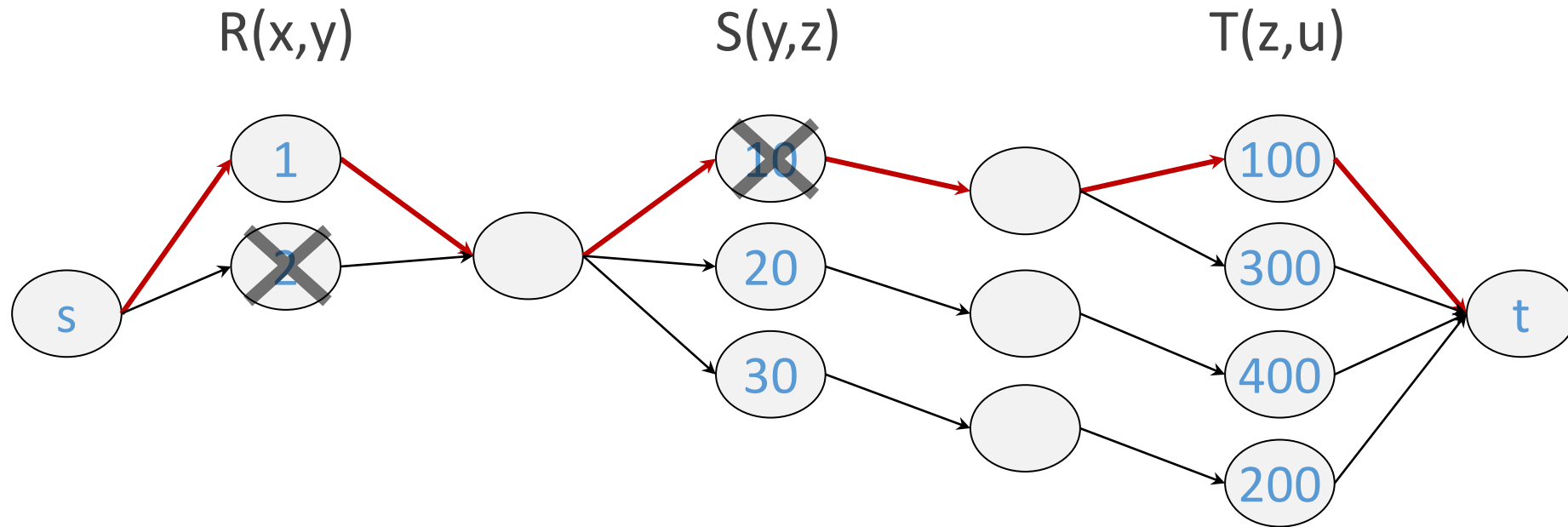
- Each option gives a different subspace (subgraph here)
- For each one, solve the top-1 problem and compare them



The winner is then partitioned further for the 3rd best path

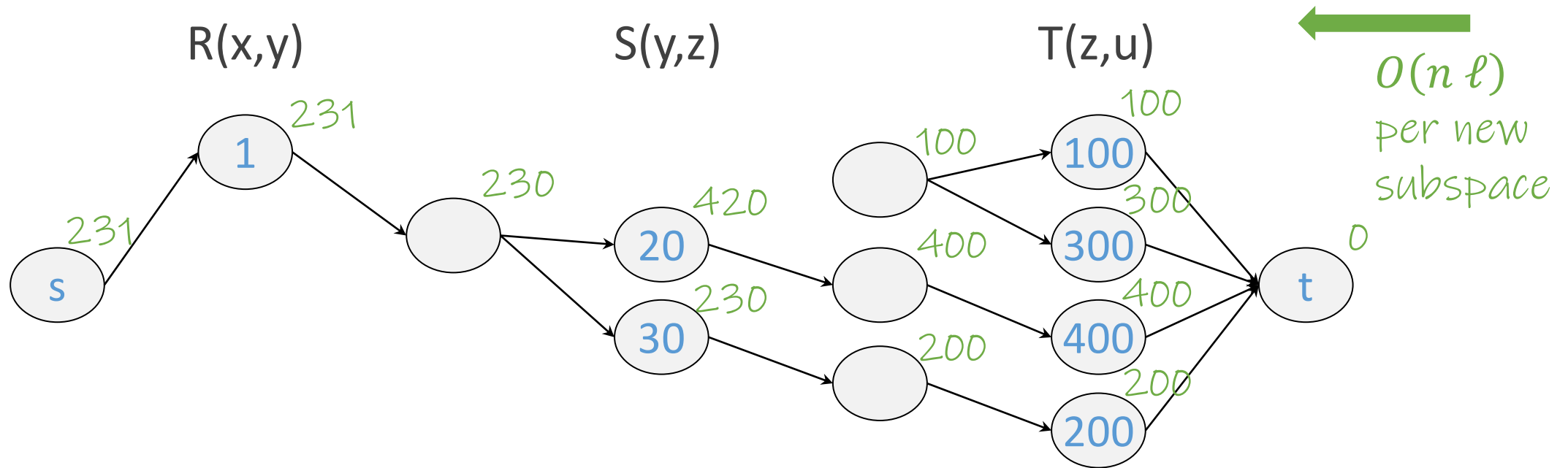
Anyk-Part: Default

- How do we find the best solution in each subspace?
- Default approach: Run shortest path algorithm from scratch



Anyk-Part: Default

- How do we find the best solution in each subspace?
- Default approach: Run shortest path algorithm from scratch



[Kimelfeld+ 06]: Ranked enumeration with linear delay in every iteration

- Gives the **best-known guarantee—linear preprocessing and delay—for CQs with projections that are not free-connex!**

[Kimelfeld+ 06] Kimelfeld, Sagiv. Incrementally Computing Ordered Answers of Acyclic Conjunctive Queries. NGITS'06 https://doi.org/https://doi.org/10.1007/11780991_13

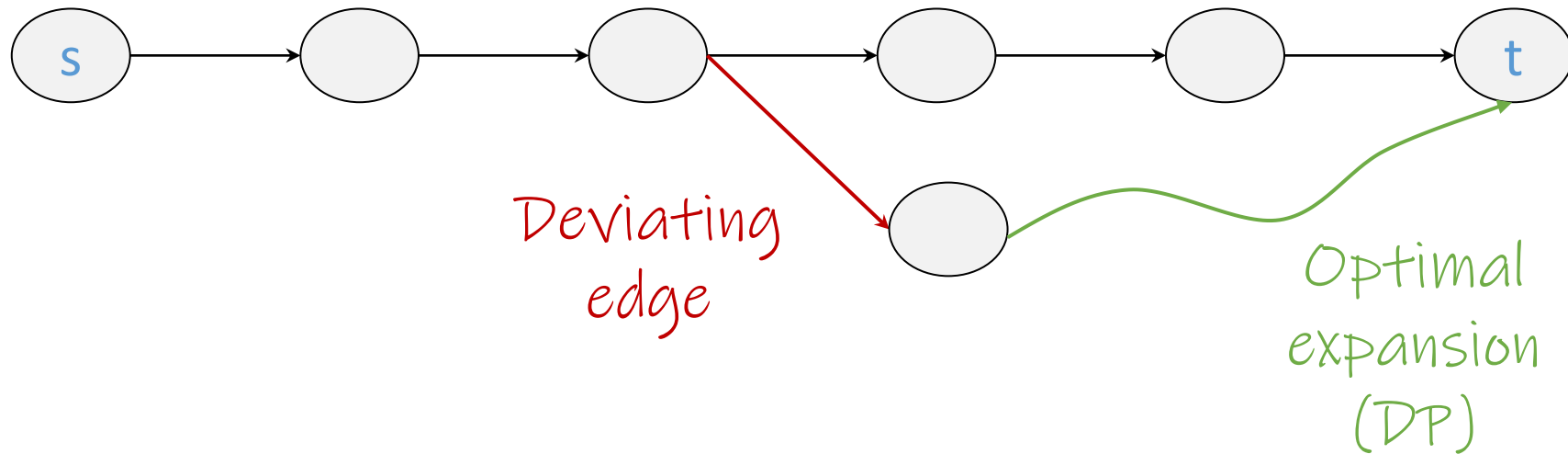
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Ingredient 2: Deviations

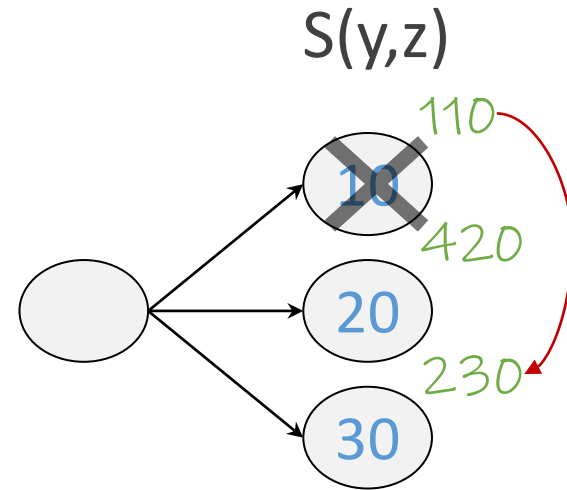
How can we compute the winner of each subspace faster?

=> By exploiting the concept of **deviations** [HP 59]

The k^{th} -shortest path is a deviation of one of the top-($k-1$) shortest paths



Comparing deviations



Find the successor:
the next-best deviating edge
according to the DP weights

Then expand optimally

Incremental sorting as subroutine:

- Heapsort
- IQS
- ...

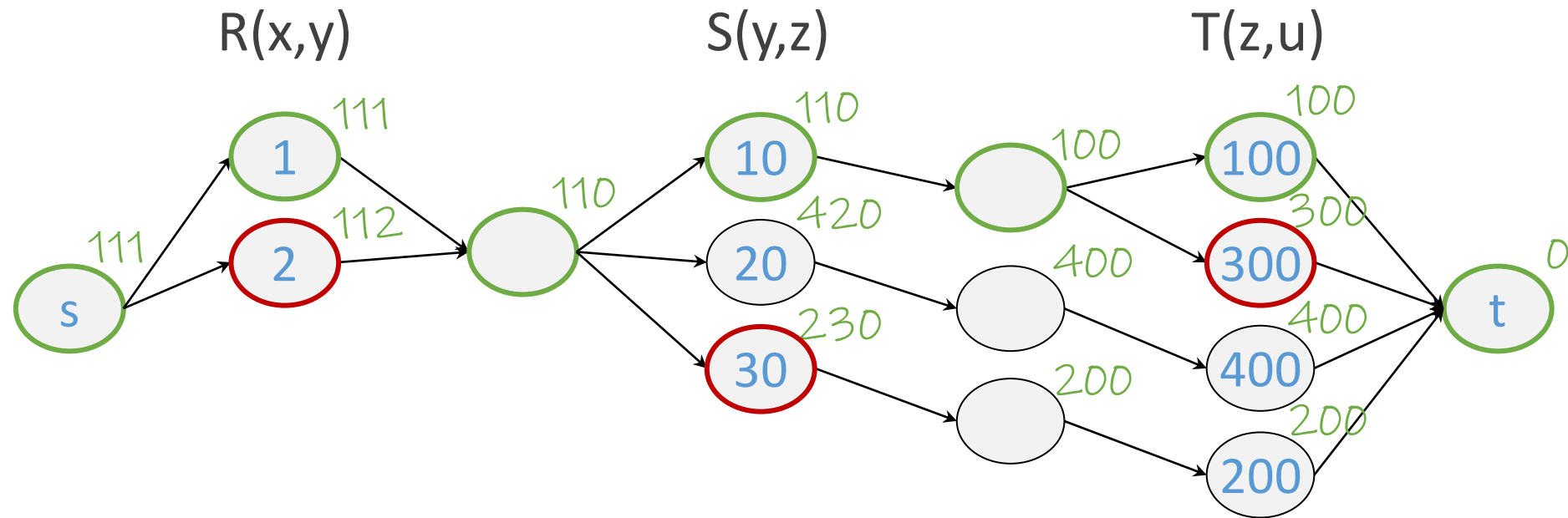


Variants

- Eager
- All
- Lazy
- Take2
- Quick

Anyk-Part: End-to-end Example

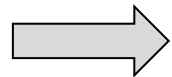
Priority Queue



Initialize with
"empty" path
(= Top-1)

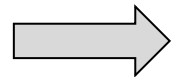
$\langle \rangle$

Expand
optimally



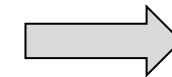
$\langle 1 \rangle$

Expand
optimally



$\langle 1, 10 \rangle$

Expand
optimally

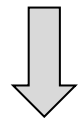


$\langle 1, 10, 100 \rangle$

(intermediate
blank nodes not
shown for
simplicity)

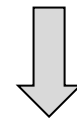
New candidates
not expanded yet,
push into PQ

Successor



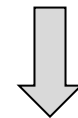
$\langle 2 \rangle$

Successor



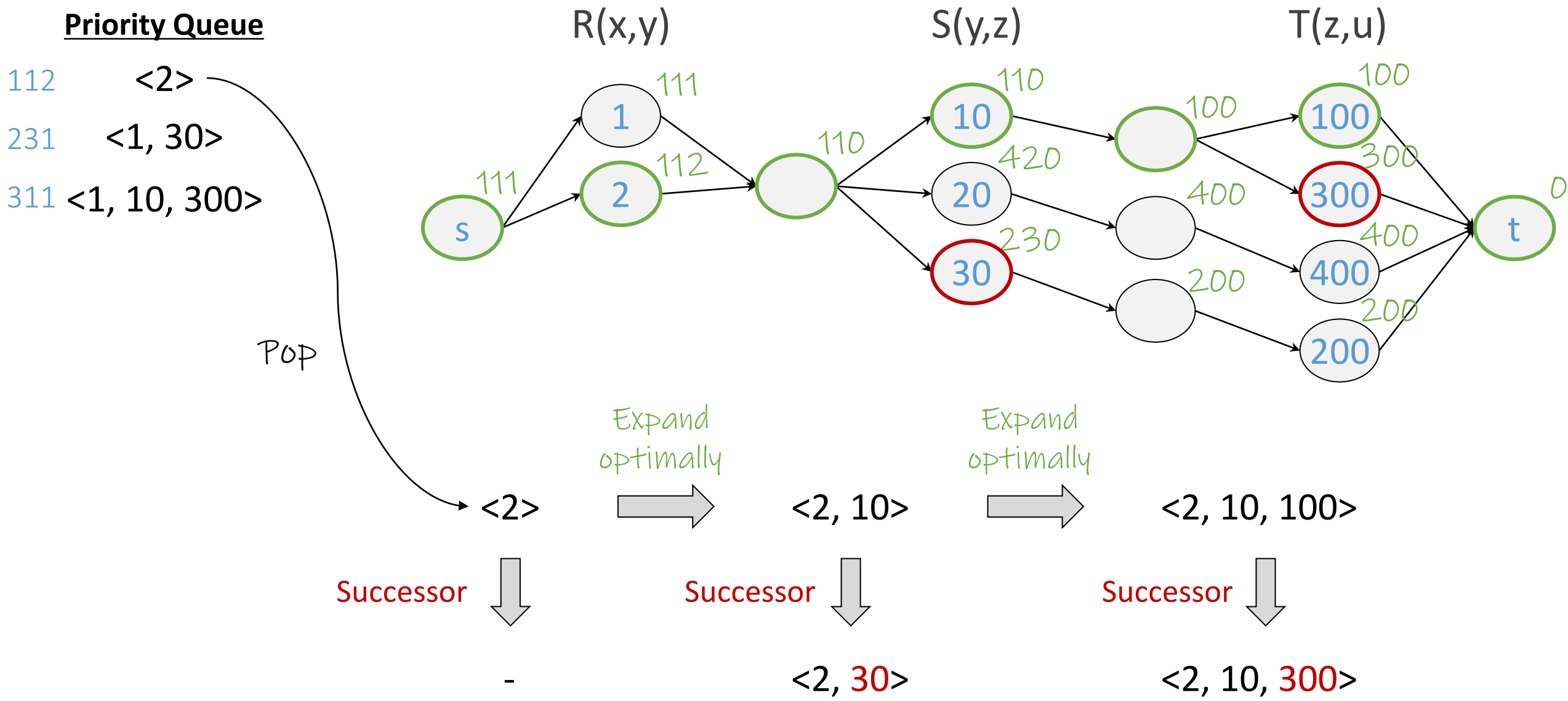
$\langle 1, 30 \rangle$

Successor



$\langle 1, 10, 300 \rangle$

Anyk-Part: End-to-end Example



More on the History of Anyk-Part

- [HP59]: Deviations
- [Lawler72, Murty68]: Lawler-Murty procedure
- [Kimelfeld06]: Ranked enumeration for CQs w/ linear delay
- [Chang+15]: Ranked enumeration of twig patterns
- [Yang+18]: Ranked enumeration of tree patterns
- [Tziavelis+20]: Ranked enumeration for full CQs (also projections in extended report)

[HP 59] Hoffman, Pavley. A method for the solution of the n th best path problem. JACM 1959. <https://doi.org/https://doi.org/10.1145/320998.321004>

[Murty 68] Murty. An Algorithm for Ranking all the Assignments in Order of Increasing Cost. Operations Research'68 <https://doi.org/10.1287/opre.16.3.682>

[Lawler 72] Lawler. A procedure for computing the k best solutions to discrete optimization problems and its application to the shortest path problem. Management Science'72 <https://doi.org/10.1287/mnsc.18.7.401>

[Kimelfeld+ 06] Kimelfeld, Sagiv. Incrementally Computing Ordered Answers of Acyclic Conjunctive Queries. NGITS'06 https://doi.org/https://doi.org/10.1007/11780991_13

[Chang+ 15] Chang, Lin, Zhang, Yu, Zhang, Qin. Optimal enumeration: Efficient top- k tree matching. PVLDB'15 <https://doi.org/10.14778/2735479.2735486>

[Yang+ 18] Yang, Ajwani, Gatterbauer, Nicholson, Riedewald, Sala. Any- k : Anytime Top- k Tree Pattern Retrieval in Labeled Graphs. WWW'18 <https://doi.org/https://doi.org/10.1145/3178876.3186115>

[T+ 20] Tziavelis, Ajwani, Gatterbauer, Riedewald, Yang. Optimal Algorithms for Ranked Enumeration of Answers to Full Conjunctive Queries. PVLDB'20 <https://doi.org/10.14778/3397230.3397250> Extended report: <https://arxiv.org/abs/1911.05582>

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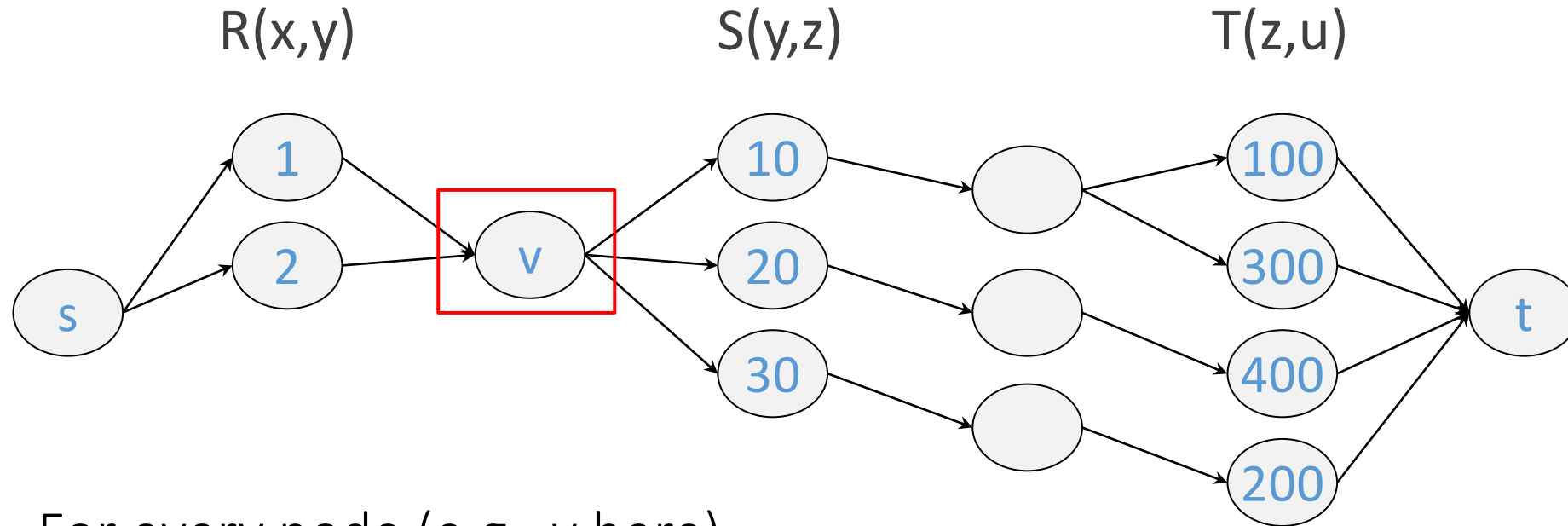
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Anyk-Rec: Example

Idea: Store ordering of lower-rank suffixes and reuse it as much as possible



For every node (e.g., v here)
we want to compute the
ranking of suffixes

$$\Pi_k(v) = k^{th} \text{ shortest path from node } v$$

$\Pi_1(v)$

$\Pi_2(v)$

$\Pi_3(v)$

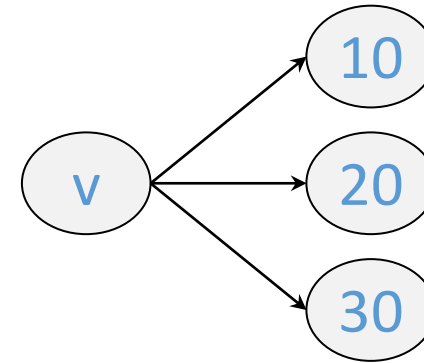
Anyk-Rec: Example

PQ

10	$v \circ \Pi_1(10)$ [110]
20	$v \circ \Pi_1(20)$ [420]
30	$v \circ \Pi_1(30)$ [230]

Size $O(n)$

One entry per outgoing edge



Size $O(n^\ell)$

Sorted List

Initially
Empty

Stores ordering of suffixes

Anyk-Rec: Example

PQ

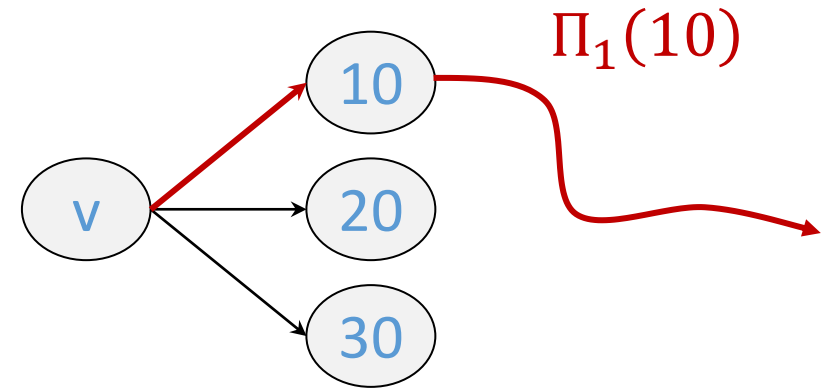
10	
20	$v \circ \Pi_1(20)$ [420]
30	$v \circ \Pi_1(30)$ [230]

POP

$v \circ \Pi_1(10)$ [110]

Sorted List

Initially
Empty



Anyk-Rec: Example

PQ

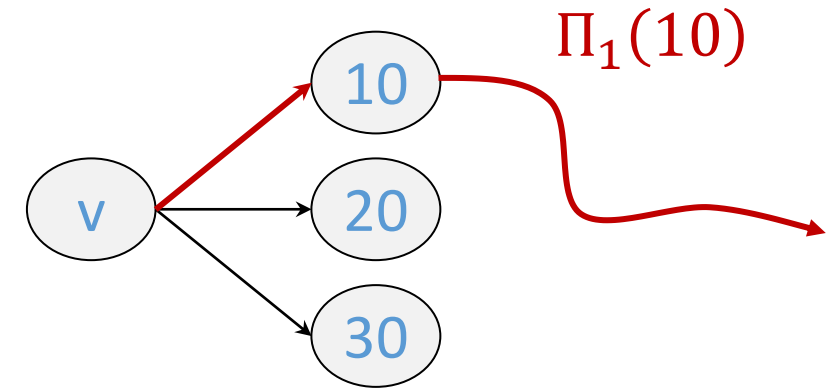
10	
20	$v \circ \Pi_1(20)$ [420]
30	$v \circ \Pi_1(30)$ [230]

$v \circ \Pi_1(10)$ [110]

Sorted List

$\Pi_1(v) = v \circ \Pi_1(10)$ [110]

Store



Anyk-Rec: Example

PQ

10	$v \circ \Pi_2(10)$ [310]
20	$v \circ \Pi_1(20)$ [420]
30	$v \circ \Pi_1(30)$ [230]

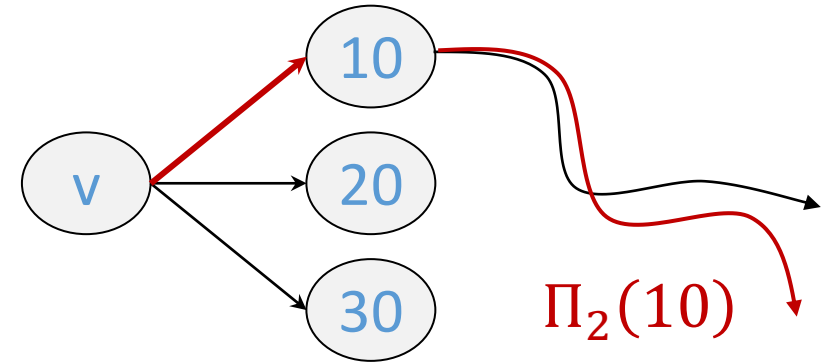
Replace

$v \circ \Pi_2(10)$ [310]

Computed **recursively**

Sorted List

$\Pi_1(v) = v \circ \Pi_1(10)$ [110]



Anyk-Rec: Example

PQ

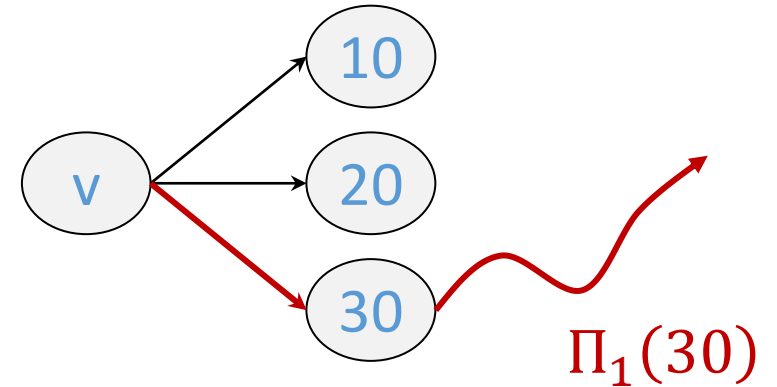
10	$v \circ \Pi_2(10)$ [310]
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30	

POP

$v \circ \Pi_1(30)$ [230]

Sorted List

$\Pi_1(v) = v \circ \Pi_1(10)$ [110]



Anyk-Rec: Example

PQ

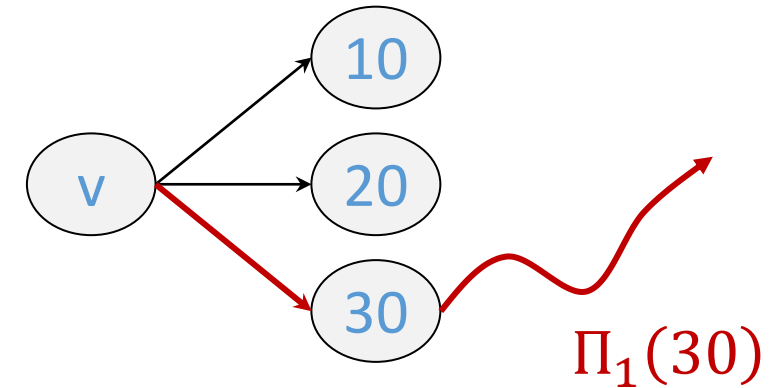
10	$v \circ \Pi_2(10)$ [310]
20	$v \circ \Pi_1(20)$ [420]
30	

$v \circ \Pi_1(30)$ [230]

Sorted List

$\Pi_1(v) = v \circ \Pi_1(10)$ [110]
↓
 $\Pi_2(v) = v \circ \Pi_1(30)$ [230]

Store



Anyk-Rec: Example

PQ

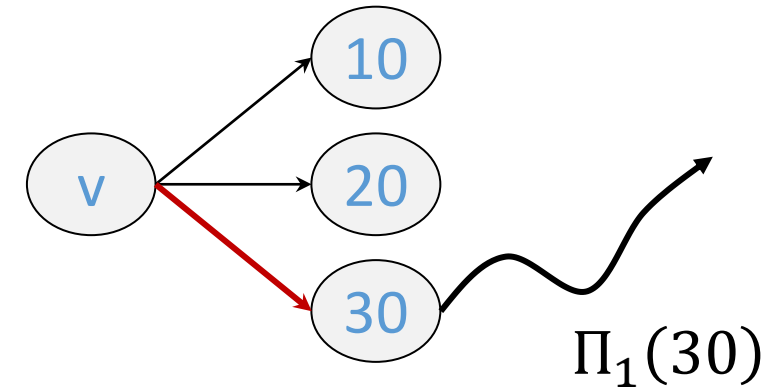
10	$v \circ \Pi_2(10)$ [310]
20	$v \circ \Pi_1(20)$ [420]
30	

$$v \circ \Pi_2(30) = \perp$$

Computed recursively

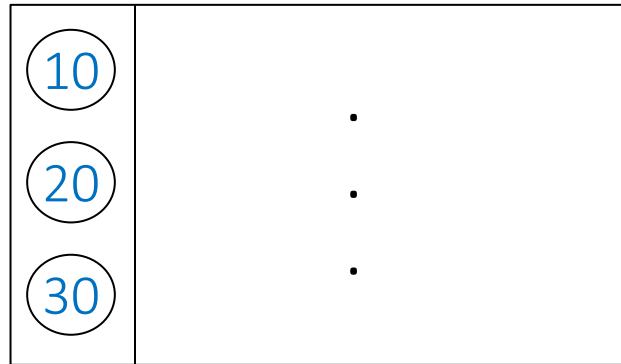
Sorted List

$$\begin{aligned} \Pi_1(v) &= v \circ \Pi_1(10) \text{ [110]} \\ &\quad \downarrow \\ \Pi_2(v) &= v \circ \Pi_1(30) \text{ [230]} \end{aligned}$$

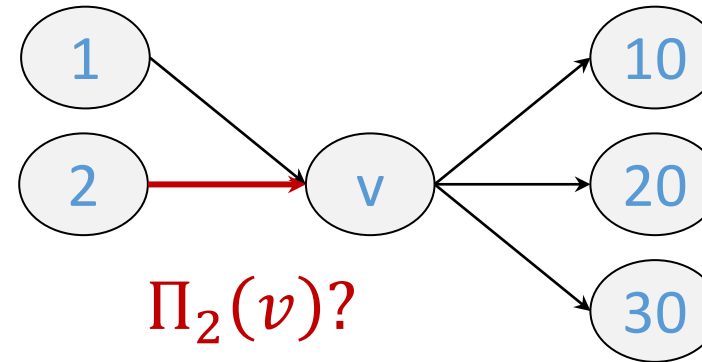


Anyk-Rec: Suffix Reusage

PQ



Later...



$\Pi_2(v)?$

Sorted List

$$\Pi_1(v) = v \circ \Pi_1(10) \text{ [110]}$$

↓

$$\Pi_2(v) = v \circ \Pi_1(30) \text{ [230]}$$

Reuse $\Pi_2(v)$ for different prefixes!

More on the History of Anyk-Rec

- [Bellman+ 60]: Keep the k best solutions per node
- [Dreyfus 69]: Recursive equations
- [Jiménez+ 99]: Top-down approach
- [Deep+ 19/21]: Application to conjunctive queries
- [Tziavelis+ 19/20]: Improved TTL guarantees

[Bellman+ 60] Bellman and Kalaba. “On k th best policies”. JSIAM’60 <https://doi.org/10.1137/0108044>

[Dreyfus 69] Dreyfus. An appraisal of some shortest-path algorithms. Operations research’69 <https://doi.org/10.1287/opre.17.3.395>

[Jiménez+ 99] Jiménez, Marzal. Computing the k shortest paths: A new algorithm and an experimental comparison. WAE’99 https://doi.org/10.1007/3-540-48318-7_4

[Deep+ 19/21] Deep and Koutris. Ranked enumeration of conjunctive query results. ArXiv’19/ICDT’21 <http://arxiv.org/abs/1902.02698>

[Tziavelis+ 19/20] Tziavelis, Ajwani, Gatterbauer, Riedewald, Yang. Optimal Algorithms for Ranked Enumeration of Answers to Full Conjunctive Queries. PVLDB’20 <https://doi.org/10.14778/3397230.3397250> Extended report: <https://arxiv.org/abs/1911.05582>

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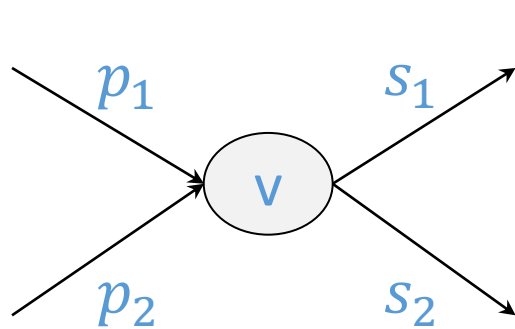
Outline Part 6

Part 6: Any- k

- Warm-up: Incremental QuickSort
- Any- k for joins
- AnyK-Part
- AnyK-Rec
- AnyK-Part+
- Experimental Results & Summary

Time-to-Last Faster than Sorting

- Anyk-Rec can be **faster than** (generic comparison-based) **sorting**!
- For Cartesian product with n^ℓ results:
 - Anyk-Rec TTL: $O(n^\ell (\log n + \ell))$
 - Sorting the join output: $O(n^\ell \log n \cdot \ell)$
- How is that possible?
 - By **reusing comparisons**!



$$\begin{array}{c} s_1 \leq s_2 \\ \downarrow \\ p_1 + s_1 \leq p_1 + s_2 \\ p_2 + s_1 \leq p_2 + s_2 \end{array}$$

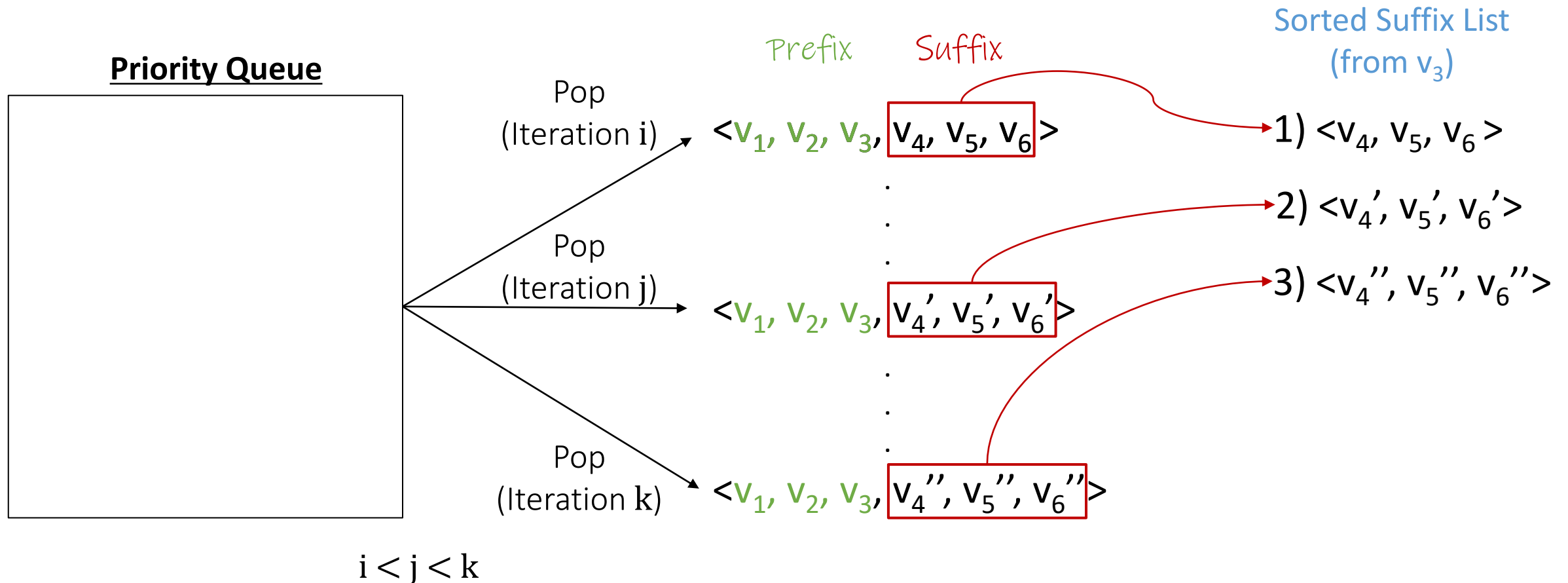
Suffix order $s_1 \rightarrow s_2$
reused for different
prefixes p_1, p_2

Anyk-Part+ : Extending Anyk-Part with Suffix-Order Reuse

- Still, Anyk-Rec has in general worse $TT(k)$ than Anyk-Part
 - The reason is that it manages many smaller PQs
 - In every iteration, it pops $O(\ell)$ times from PQs of size $O(n)$
- How can we do better?
- Extend Anyk-Part with the suffix-order reuse idea that Anyk-Rec exploits
- The result is Anyk-Part+, an algorithm that asymptotically beats both the other two

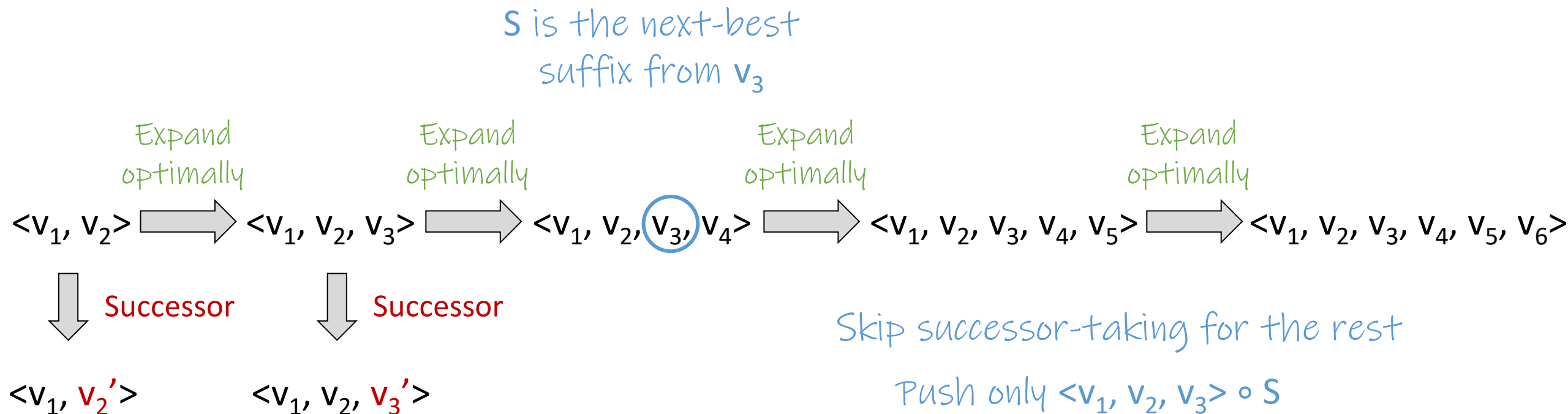
Anyk-Part+ : Finding the Suffix Order

- However, Anyk-Part doesn't directly have access to the suffix order
- We can discover it by monitoring the output of the (global) PQ



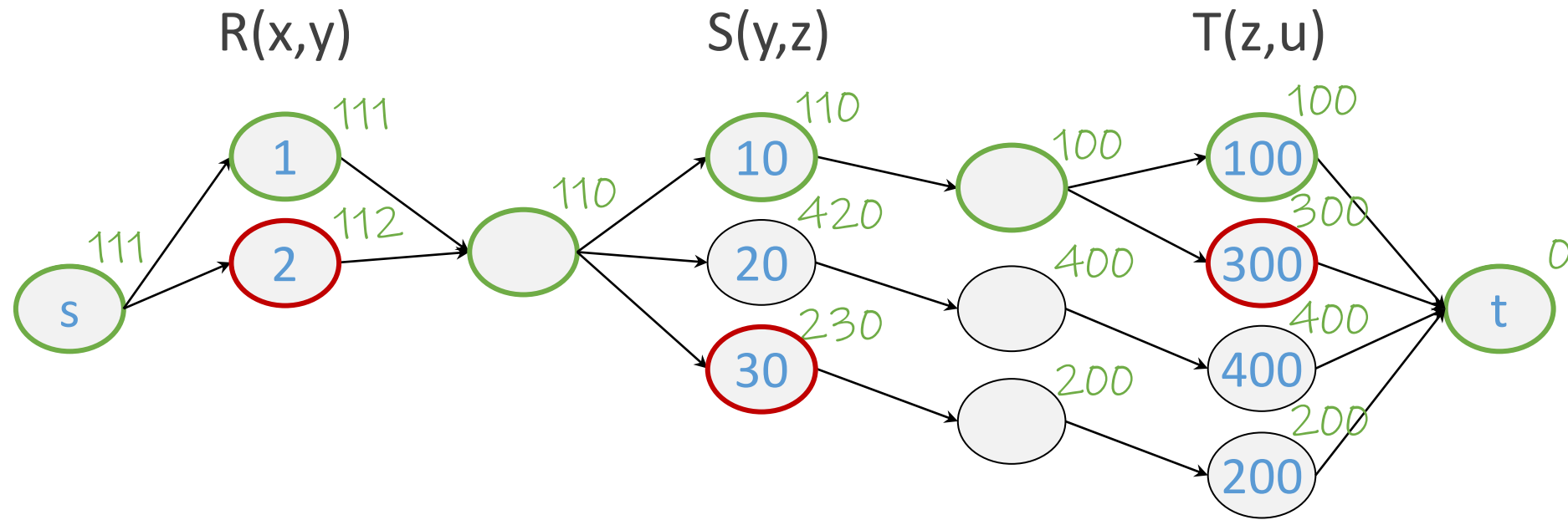
Anyk-Part+ : Reusing the Suffix Order

- The suffix order, when known, can be used to skip the successor-taking phase of Anyk-Part
- This reduces the number of candidates and drastically decreases the size of the PQ (now bounded by $n\ell$, which is linear)

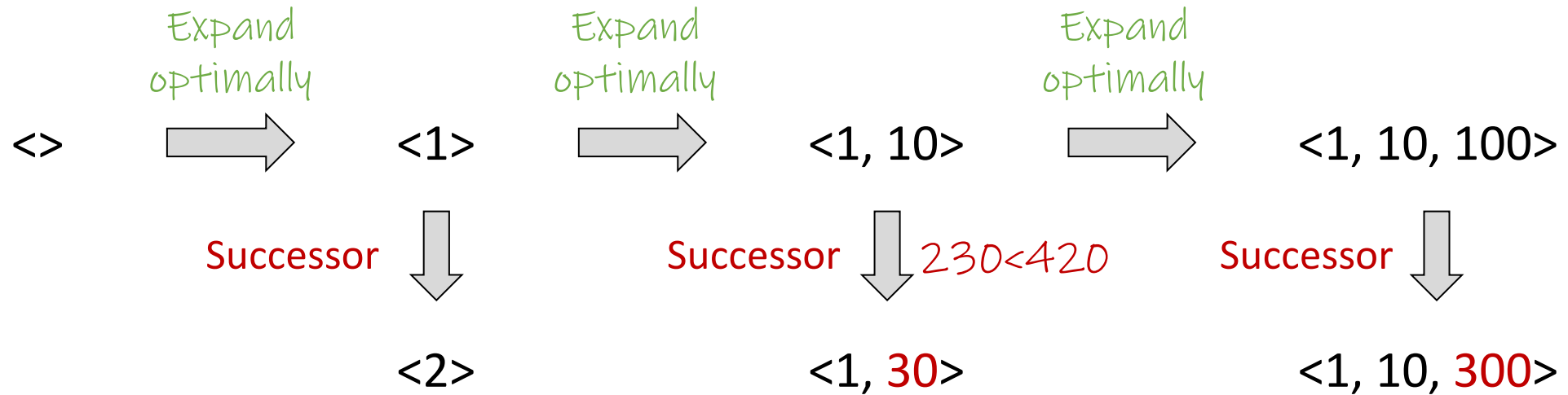


Anyk-Part+ Example

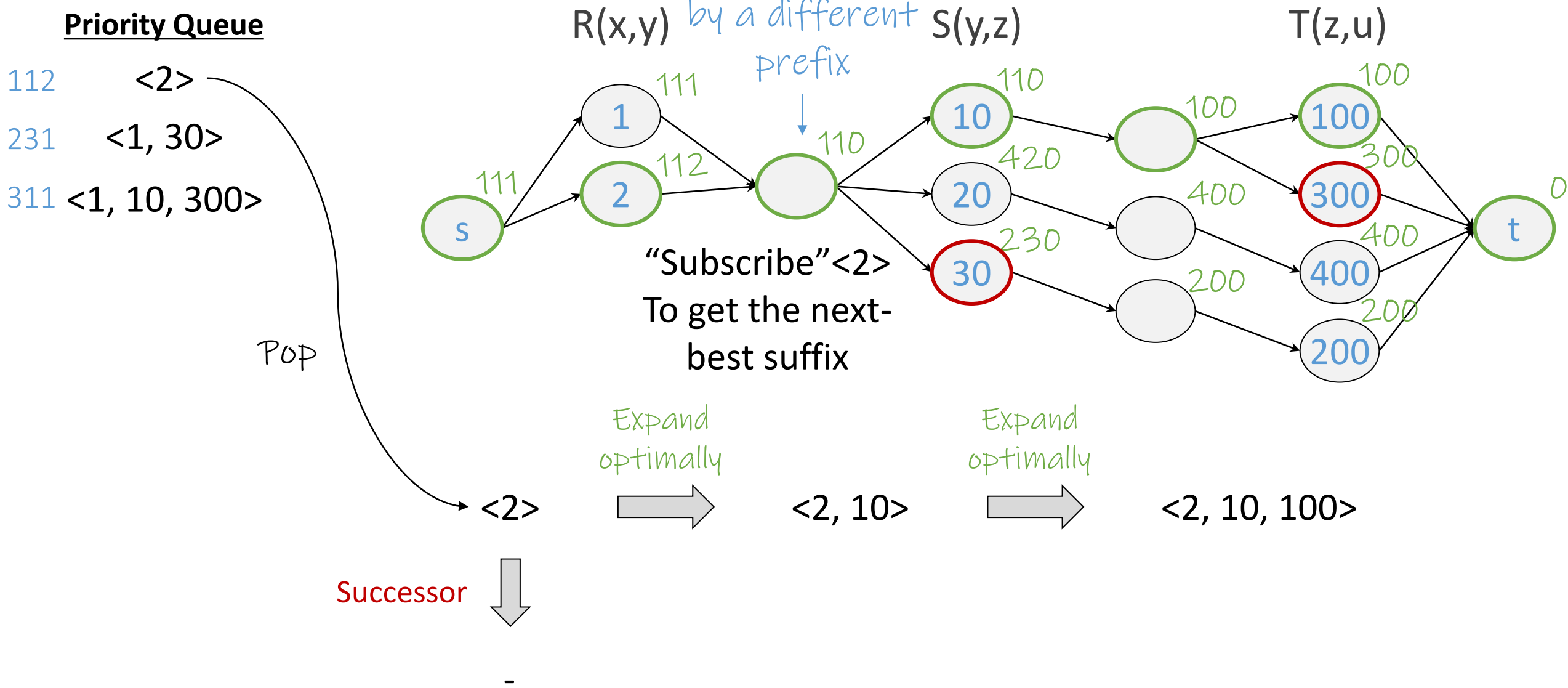
Priority Queue



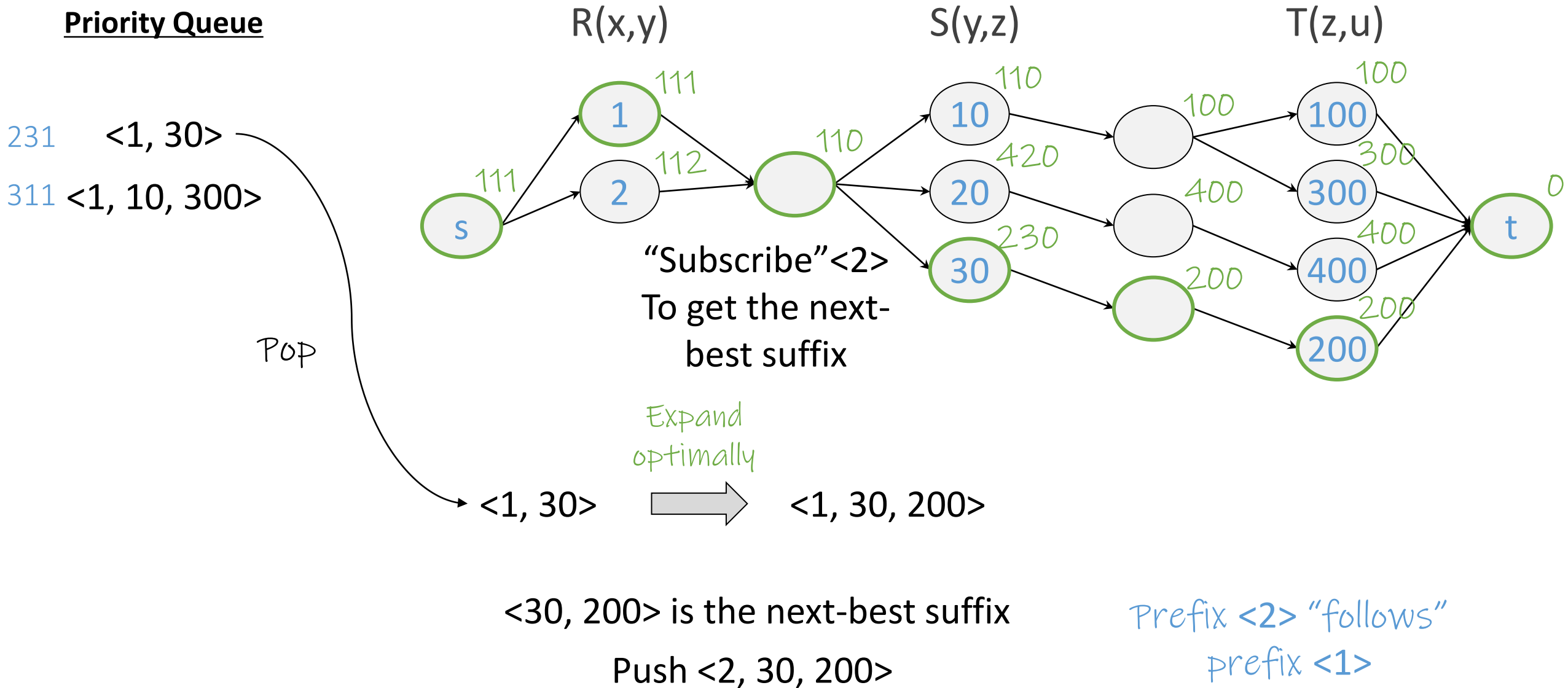
The first iteration is exactly the same as Anyk-Part



Anyk-Part+ Example



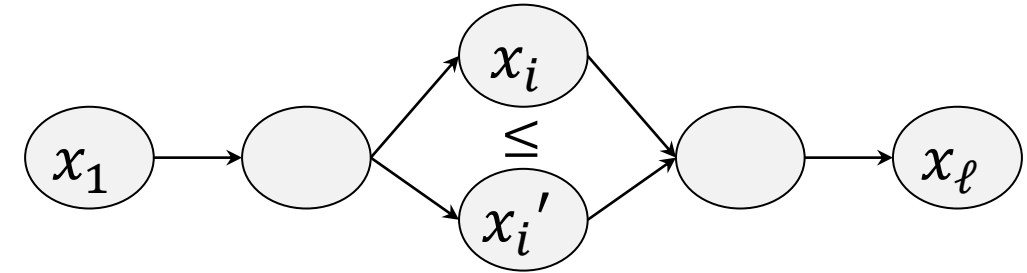
Anyk-Part+ Example



Monotonicity Properties

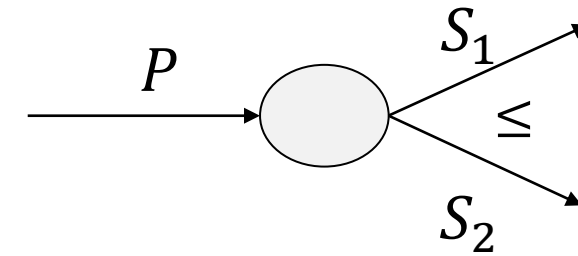
- Monotonicity [Fagin+ 03]

- $w(x_1, \dots, x_i, \dots, x_\ell) \leq w(x_1, \dots, x_i', \dots, x_\ell)$
if $w(x_i) \leq w(x_i')$



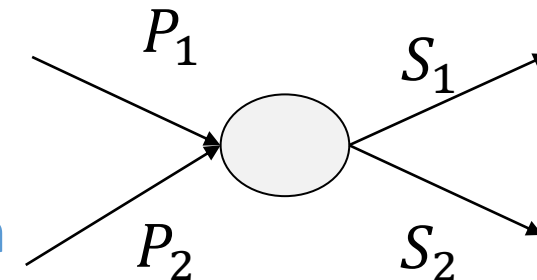
- Subset-Monotonicity [Kimelfeld+ 06]

- $w(P \uplus S_1) \leq w(P \uplus S_2)$
if $w(S_1) \leq w(S_2)$
- Allows us to compare partial solutions (DP)



- Strong subset-monotonicity [Tziavelis+ 22]

- $w(P_1 \uplus S_1) \leq w(P_1 \uplus S_2) \Rightarrow w(P_2 \uplus S_1) \leq w(P_2 \uplus S_2)$
if $w(P_1) \leq w(P_2)$
- Can reuse the ordering of partial solutions found from the ordering of complete solutions



Anyk-Part+

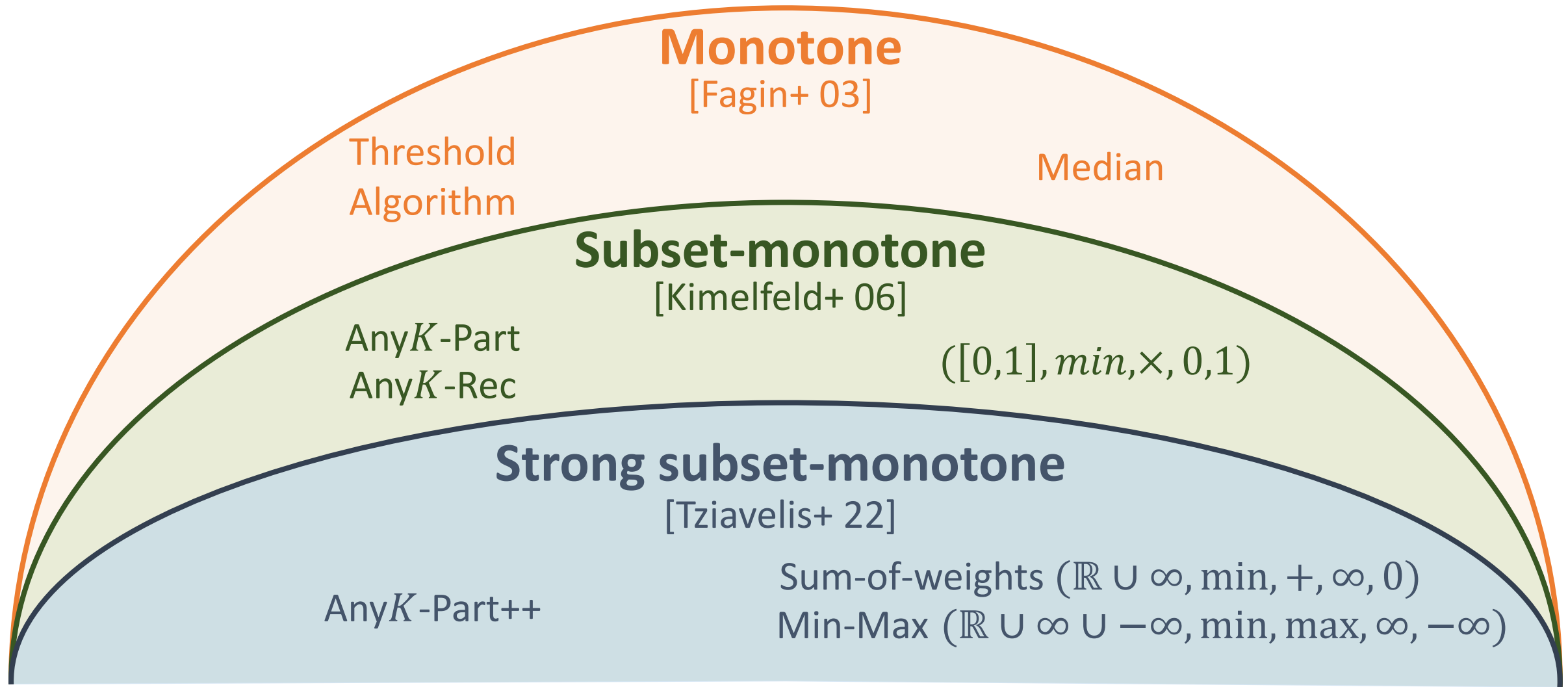
[Fagin+ 03] Fagin, Lotem, Naor. Optimal aggregation algorithms for middleware. JCSS 2003. <https://doi.org/10.1145/375551.375567>

[Kimelfeld+ 06] Kimelfeld, Sagiv. Incrementally Computing Ordered Answers of Acyclic Conjunctive Queries. NGITS'06 https://doi.org/10.1007/11780991_13

[Tziavelis+ 22] Tziavelis, Gatterbauer, Riedewald. Any-k Algorithms for Enumerating Ranked Answers to Conjunctive Queries. arXiv'22 <https://arxiv.org/abs/2205.05649>

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Monotonicity Properties Hierarchy



[Fagin+ 03] Fagin, Lotem, Naor. Optimal aggregation algorithms for middleware. JCSS 2003. <https://doi.org/10.1145/375551.375567>

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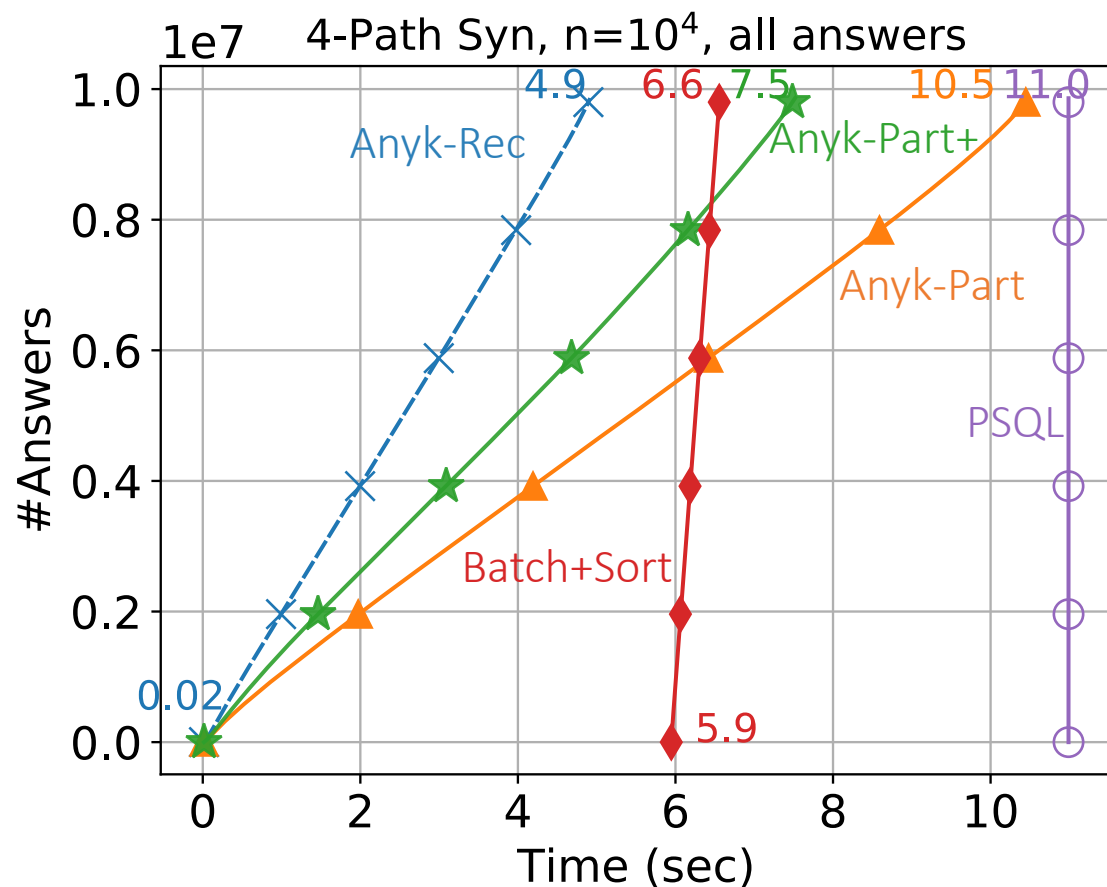
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Outline Part 6

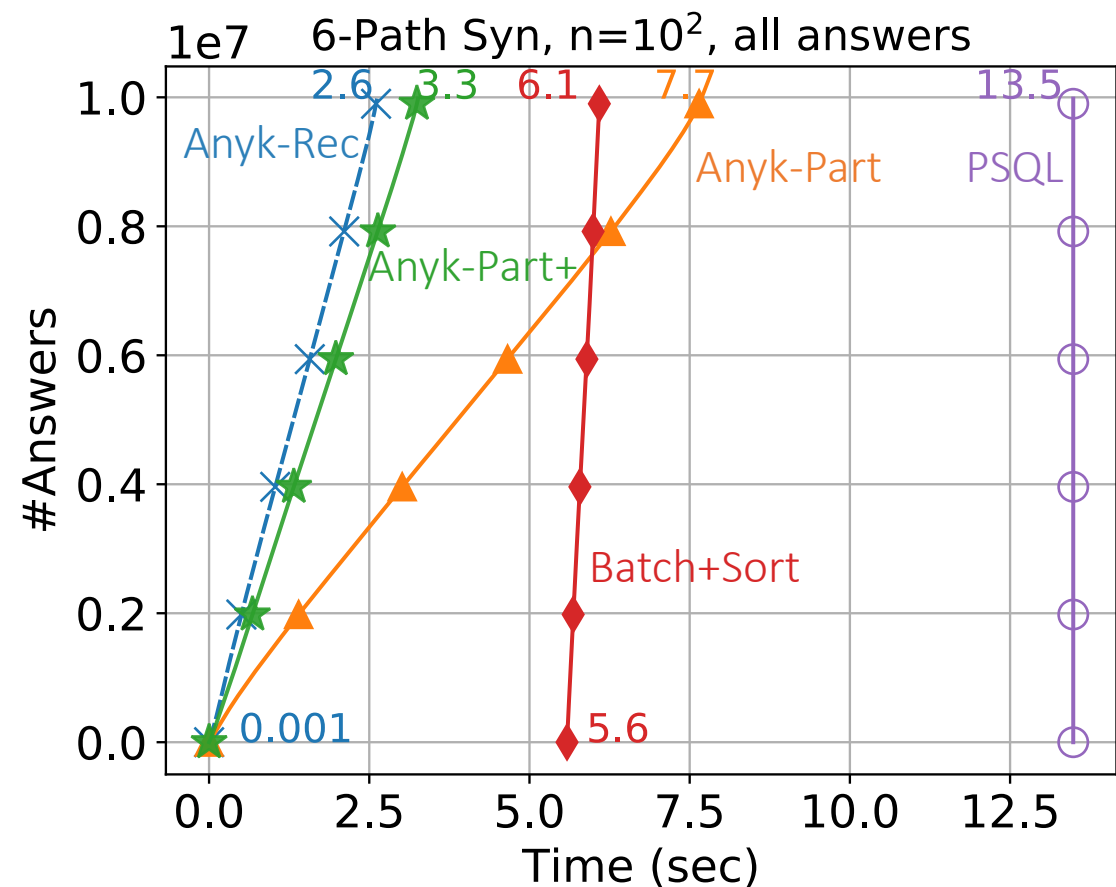
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Experimental Results

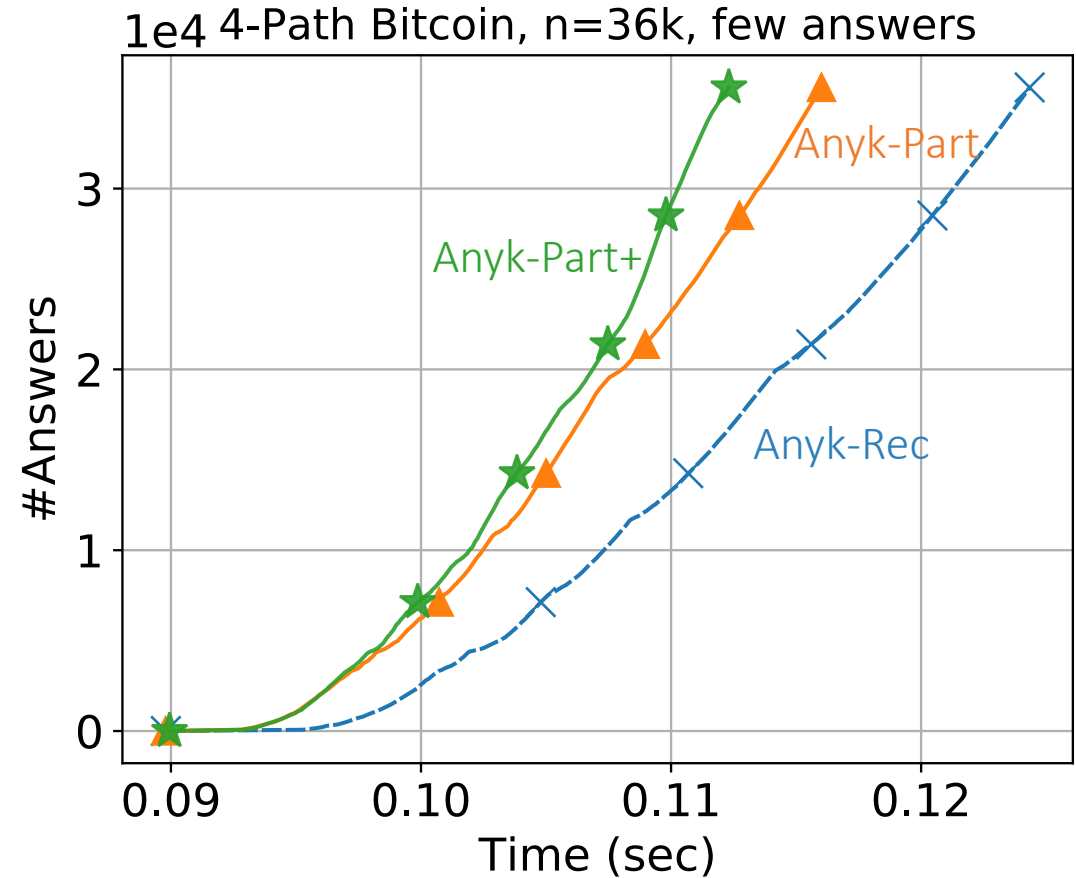
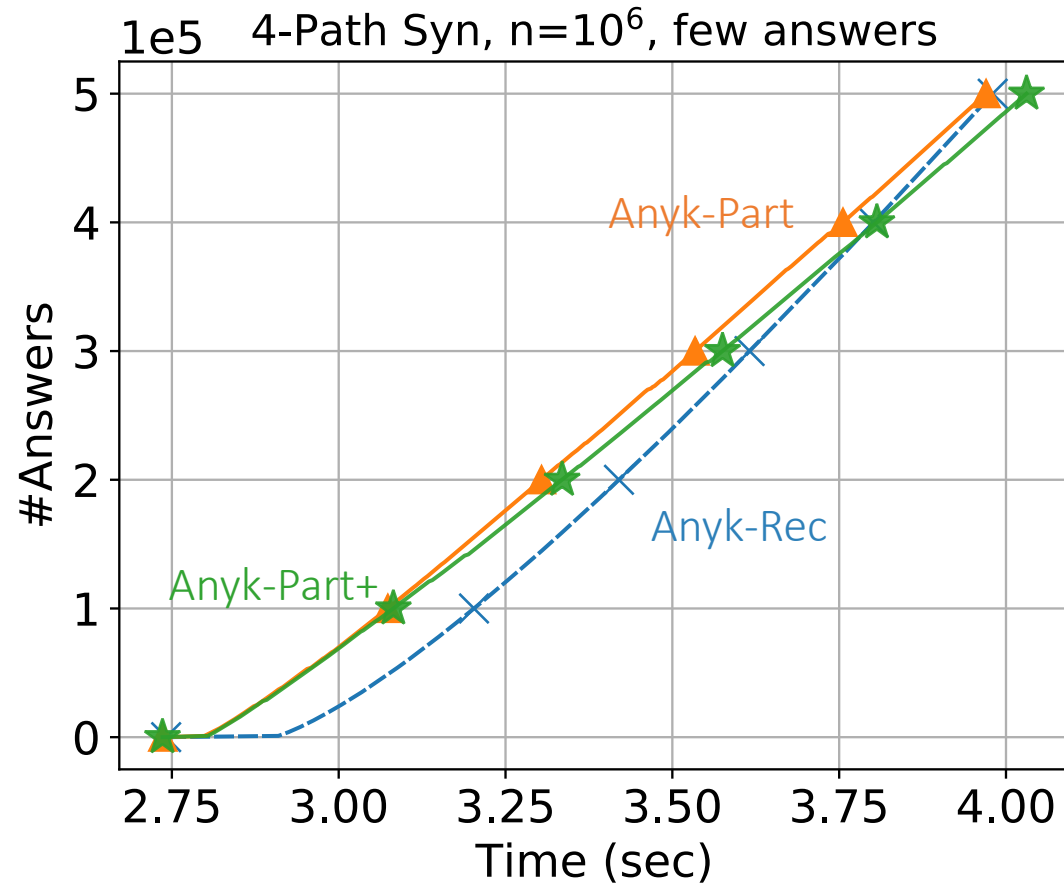


- Anyk starts much faster than Batch
- Anyk-Rec also finishes faster than Batch



- Anyk-Part+ reuses computation in a similar way to Anyk-Rec and is close for TTL

Experimental Results



- Anyk-Part is usually faster in the beginning

- Anyk-Part+ is close to Anyk-Part in the beginning and can be even faster

Summary

- What we saw: any- k algorithms for full CQs that have a path structure
 - $TT(k) = O(n + k \log k)$ in data complexity
 - Anyk-Part+ [Tziavelis+ 22] : The best-known algorithm when query complexity is also accounted for
 - We did not cover technical details for tree-structured CQs
- With projections:
 - $O(n + k \log k)$ guarantee extends to free-connex CQs [Tziavelis+ 19]
 - For arbitrary projections, the best-known guarantee is linear delay after linear preprocessing is possible [Kimelfeld+ 06]
- Any- k applies to any DP problem with the selective property (e.g., longest common subsequence)

[Kimelfeld+ 06] Kimelfeld, Sagiv. Incrementally Computing Ordered Answers of Acyclic Conjunctive Queries. NGITS'06 https://doi.org/10.1007/11780991_13

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