Updated 3/1/2024

Topic 2: Complexity of Query Evaluation
Unit 1: Conjunctive Queries
Lecture 14

Wolfgang Gatterbauer

CS7240 Principles of scalable data management (sp24)

https://northeastern-datalab.github.io/cs7240/sp24/

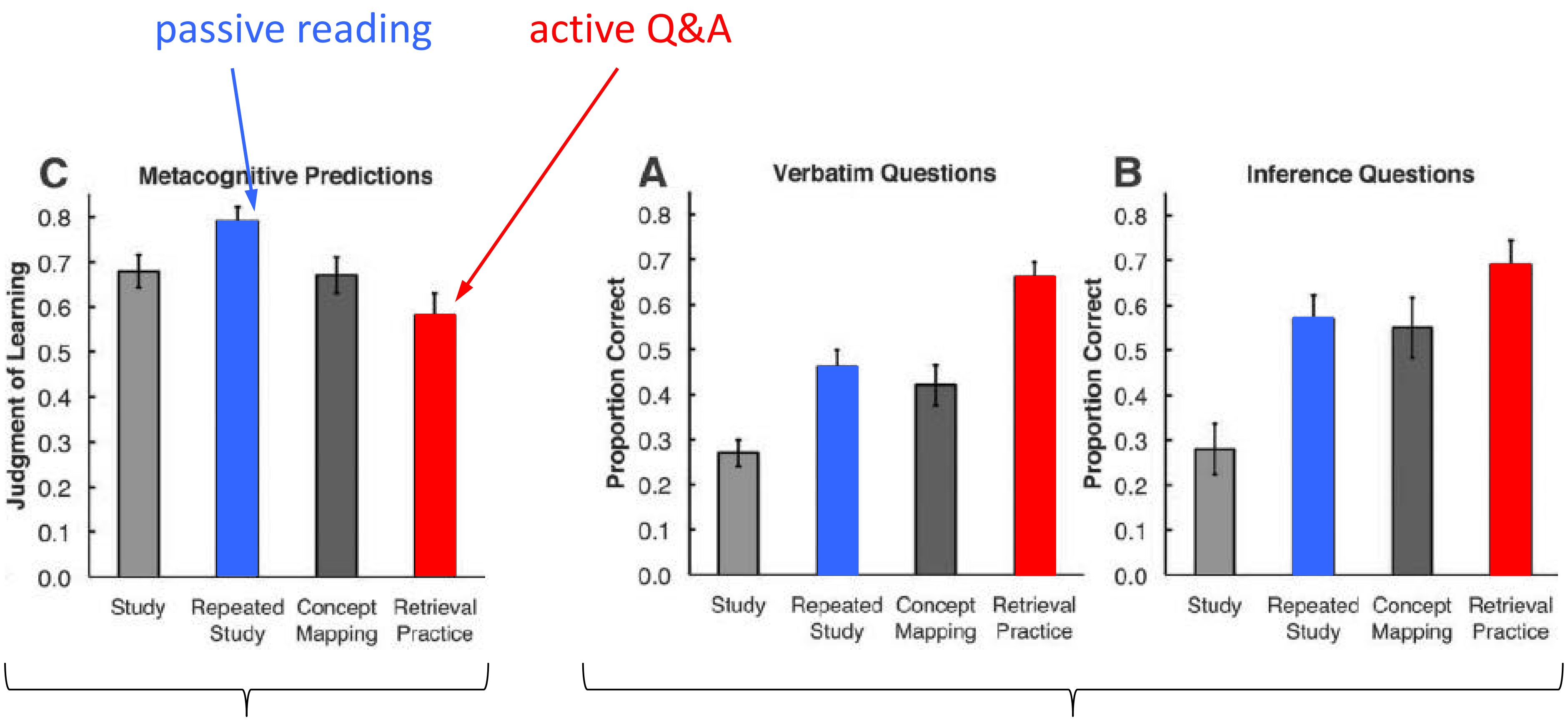
3/1/2024

Pre-class conversations

- Last class summary
- Clingo: can't export to CSV (in contrast to souffle)
- Faculty candidates (THU Feb 29, WED March 20)

- Today:
  - Complexity of query evaluation
  - Homomorphisms

Studying new material: "Under which study condition do you think you learn better?"

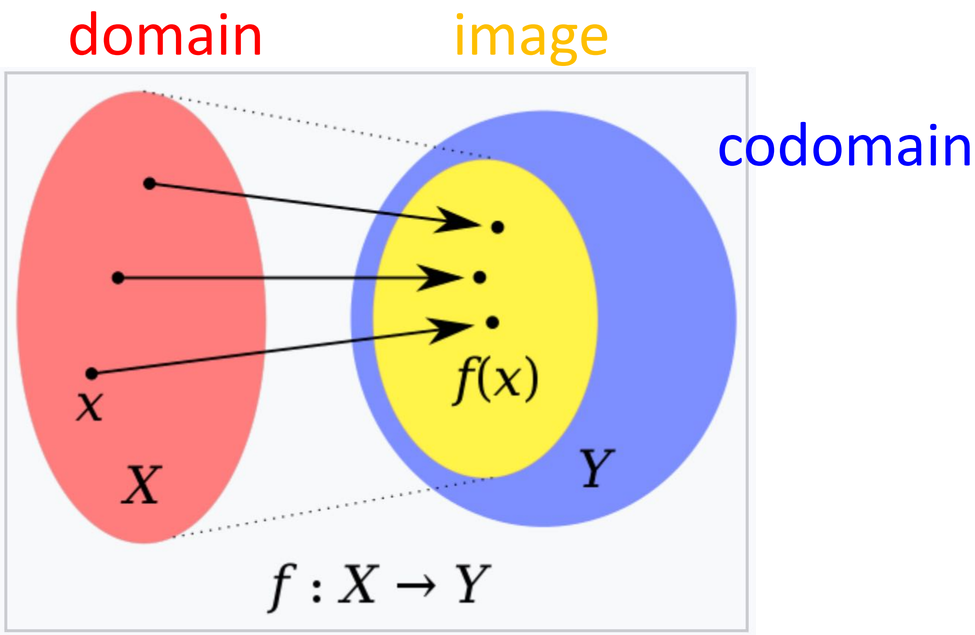


Judged performance
(=what people think)

Actual performance
(=what is actually working)

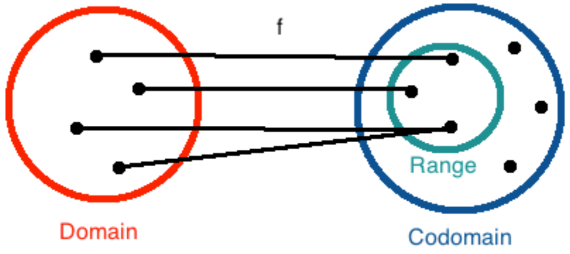
Data from: Karpicke & Blunt, "Retrieval Practice Produces More Learning than Elaborative Studying with Concept Mapping," Science, 2011. https://doi.org/10.1126/science.1199327

Domain, codomain, range = image



A function f from X to Y. The blue oval Y is the codomain of f. The yellow oval inside Y is the image of f, and the red oval X is the domain of f.

- A codomain of a function is a set into which all of the output of the function is constrained to fall. It is the set Y in the notation f: X → Y.
- The set of all elements of the form f(x), where x ranges over the elements of the domain X, is called the image (sometimes called range) of f. The image of a function is a subset of its codomain so it might not coincide with it.
  - A function that is not surjective has elements y in its codomain for which the equation f(x) = y does not have a solution.



Sources: https://en.wikipedia.org/wiki/Codomain, http://mathonline.wikidot.com/different-types-of-functions
Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

Topic 2: Complexity of Query Evaluation & Reverse Data Management

- **Lecture 14 (Fri 3/1):** T2-U1 Conjunctive Queries
- Spring break (Tue 3/5, Fri 3/8)
- **Lecture 15 (Tue 3/12):** T2-U1/2 Conjunctive & Beyond Conjunctive Queries
- **Lecture 16 (Fri 3/15):** T2-U1/2 Conjunctive & Beyond Conjunctive Queries
- **Lecture 17 (Tue 3/19):** T2-U3 Provenance
- **Lecture 18 (Fri 3/22):** T2-U3 Provenance
- **Lecture 20 (Tue 3/26):** T2-U4 Reverse Data Management

Pointers to relevant concepts & supplementary material:

- **Unit 1. Conjunctive Queries**: Query evaluation of conjunctive queries (CQs), data vs. query complexity, homomorphisms, constraint satisfaction, query containment, query minimization, absorption: [Kolaitis, Vardi'00], [Vardi'00], [Kolaitis'16], [Koutris'19] L1 & L2
- **Unit 2. Beyond Conjunctive Queries**: unions of conjunctive queries, bag semantics, nested queries, tree pattern queries: [Kolaitis'16], [Tan+'14], [Gatterbauer'11], [Martens'17]
- **Unit 3. Provenance**: [Buneman+'02], [Green+'07], [Cheney+'09], [Green,Tannen'17], [Kepner+16], [Buneman, Tan'18], [Simons'23], [Dagstuhl'24]
- **Unit 4. Reverse Data Management**: update propagation, resilience: [Buneman+'02], [Kimelfeld+'12], [Freire+'15], [Makhija+'24]

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Three Fundamental Algorithmic Problems about Queries

Let L be a database query language.

- The Query Evaluation Problem:

  ?

- The Query Equivalence Problem:

  ?

- The Query Containment Problem:

  ?

Three Fundamental Algorithmic Problems about Queries

Let L be a database query language.

- The Query Evaluation Problem:
  - "Given a query q in L and a database instance D, evaluate $q(D)$"
  - That's the main problem in query processing.

- The Query Equivalence Problem:

?

- The Query Containment Problem:

?

Based on Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

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  - "Given two queries q_1 and q_2 in L, is it the case that $q_1 \equiv q_2$?"
    - i.e., is it the case that, for all (infinitely many) database instances D, we have that $q_1(D) = q_2(D)$?
  - This problem underlies query optimization: transform a given query to an equivalent more efficient one.

- The Query Containment Problem:

?

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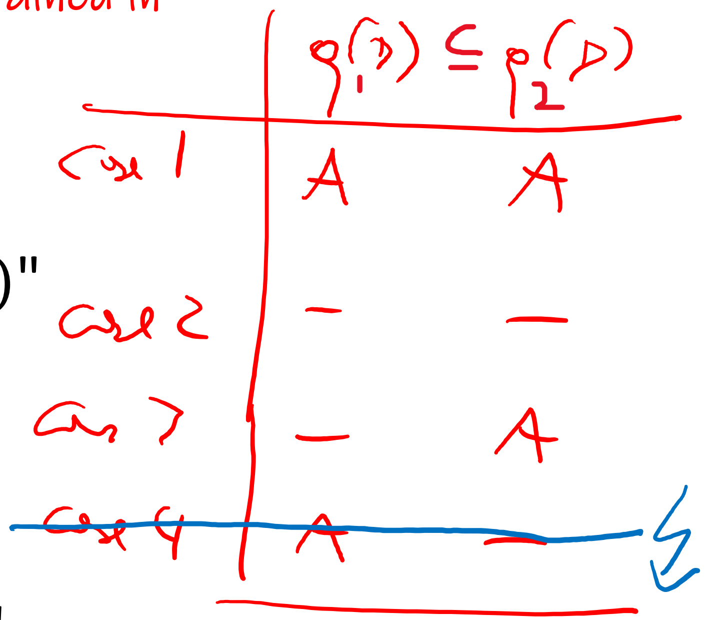
Three Fundamental Algorithmic Problems about Queries

Is every answer A contained in q_1 also contained in q_2?



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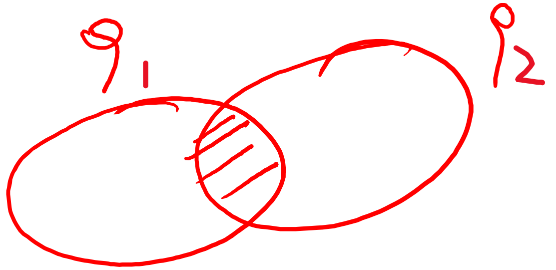
Boolean variant $q_1 \Rightarrow q_2$:
for all D: if $D \vDash q_1$, then $D \vDash q_2$

- The Query Containment Problem:
  - "Given two queries q_1 and q_2 in L, is it the case that $q_1(D) \subseteq q_2(D)$ for every D?"

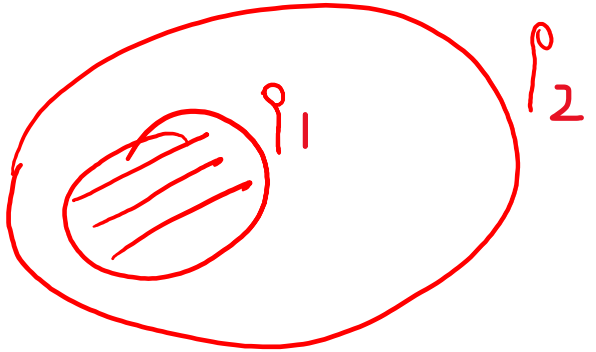
Why bother about Query Containment

- The Query Containment Problem and Query Equivalence Problem are closely related to each other:



  - $q_1 \equiv q_2$ if and only if

    ?



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- The Query Containment Problem and Query Equivalence Problem are closely related to each other:

  - $q_1 \equiv q_2$ if and only if
    - $q_1 \subseteq q_2$ and $q_1 \supseteq q_2$

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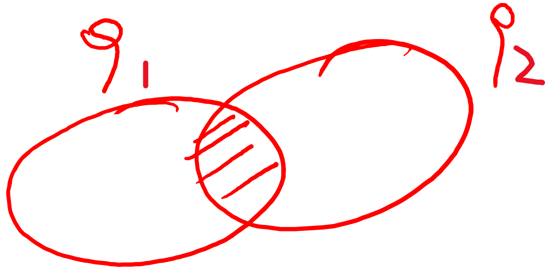

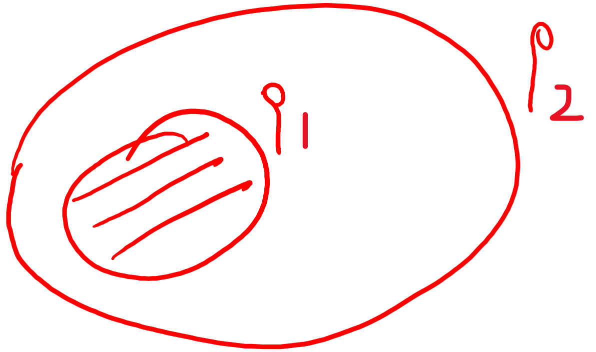

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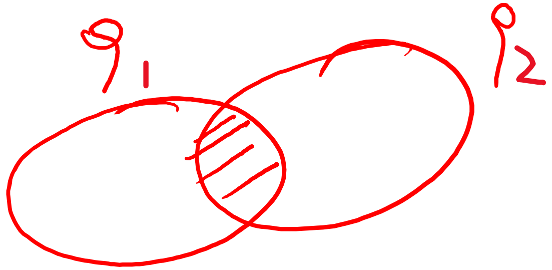

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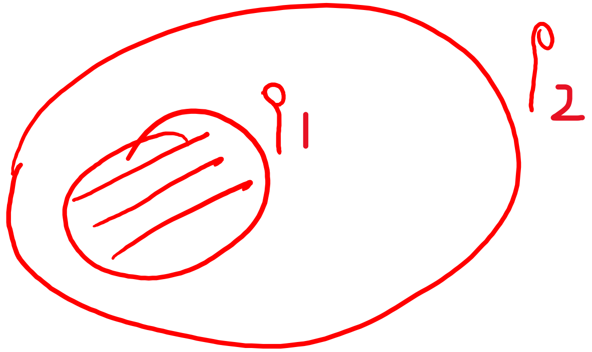

  - $q_1 \subseteq q_2$ if and only if
    - $q_1 \equiv (q_1 \cap q_2)$

Complexity of Equivalence and Containment

- Thm: The Query Equivalence Problem for relational calculus (RC) queries is...

?

Complexity of Equivalence and Containment

- Thm: The Query Equivalence Problem for relational calculus (RC) queries is...

  ... undecidable ☹

  A decision problem is undecidable if it is impossible to construct an algorithm that always leads to a correct yes-or-no answer.

- Proof: using Trakhtenbrot's Theorem (1949):
  - The Finite Validity Problem (problem of validity in FOL on the class of all finite models) is undecidable.

    a formula is valid if it comes out as true (or "satisfied") under all admissible assignments of meaning to that formula within the intended semantics for the logical language

what problem do we have to reduce to what other problem?

Tip: $A \preccurlyeq B$: reduction from A to B.
Means: B could be used to solve A. But A is hard ...

Complexity of Equivalence and Containment

 

B

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  - Finite Validity Problem ≼ Query Equivalence Problem

how ?

Tip: A ≼ B: reduction from A to B.
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- Corollary: The Query Containment Problem for RC is undecidable.

how ?

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  - Finite Validity Problem $\preccurlyeq$ Query Equivalence Problem
    - Take a fixed finitely valid RC sentence ψ, and assume you can solve the query equivalence problem. Then for every RC sentence φ, we could solve validity: φ is finitely valid $\Leftrightarrow \varphi \equiv \psi$.

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- Corollary: The Query Containment Problem for RC is undecidable.
  - Proof: Query Equivalence $\preccurlyeq$ Query Containment, since

$$q_1 \equiv q_2 \Leftrightarrow (q_1 \subseteq q_2 \text{ and } q_2 \supseteq q_1)$$

Complexity of the Query Evaluation Problem

- The Query Evaluation Problem for Relational Calculus (RC):
  - Given a RC formula φ and a database instance D, find $\varphi^{adom}(D)$.

- Theorem: The Query Evaluation Problem for Relational Calculus is ...

  ... PSPACE-complete.
  - PSPACE: decision problems, can be solved using an amount of memory that is polynomial in the input length (~ in polynomial amount of space).
  - PSPACE-complete: PSPACE + every other PSPACE problem can be transformed to it in polynomial time (PSPACE-hard)

- Proof: We need to show both
  - This problem is in PSPACE.
  - This problem is PSPACE-hard. (We only focus on this task for Boolean RC queries)

Complexity of the Query Evaluation Problem

- Theorem: The Query Evaluation Problem for Boolean RC is PSPACE-hard.
- Reduction uses QBF (Quantified Boolean Formulas):
  - Given QBF $\forall x_1 \exists x_2 \; .... \forall x_k \psi$, is it true or false
  - (notice every variable is quantified = bound at beginning of sentence; no free variables)
- Proof shows that QBF ≼ Query Evaluation for Relational Calculus
  - Given QBF $\forall x_1 \exists x_2 \; .... \forall x_k \psi$,
  - Let V and P be two unary relations and D be the database instance with V(0), V(1), P(1)
  - Obtain ψ^* from ψ by replacing every occurrence of x_i by $P(x_i)$, and $\neg x_i$ by $\neg P(x_i)$
  - Then the following statements are equivalent:
    - $\forall x_1 \exists x_2 \; .... \forall x_k \psi$ is true
    - $\forall x_1 [V(x_1) \rightarrow \exists x_2 [V(x_2) \wedge \ldots \forall x_k [V(x_k) \rightarrow \psi^*]]\ldots]$ is true on D

Vardi's Taxonomy of the Query Evaluation Problem

Definition: Let **L** be a database query language.

- The combined complexity of **L** is the decision problem $\mathbf{P}_{\varphi,\mathbf{D}}$:
  - given an **L**-sentence φ and a database instance **D**, is φ true on **D**?
  - In symbols, does $\mathbf{D} \vDash \varphi$ (does **D** satisfy φ)?

- The data complexity of **L** is the family of the following decision problems $\mathbf{P}_{\varphi}$, where φ is a fixed L-sentence:
  - given a database instance **D**, does $\mathbf{D} \vDash \varphi$?

- The query complexity of **L** is the family of the following decision problems $\mathbf{P}_{\mathbf{D}}$, where **D** is a fixed database instance:
  - given an **L**-sentence φ, does $\mathbf{D} \vDash \varphi$?

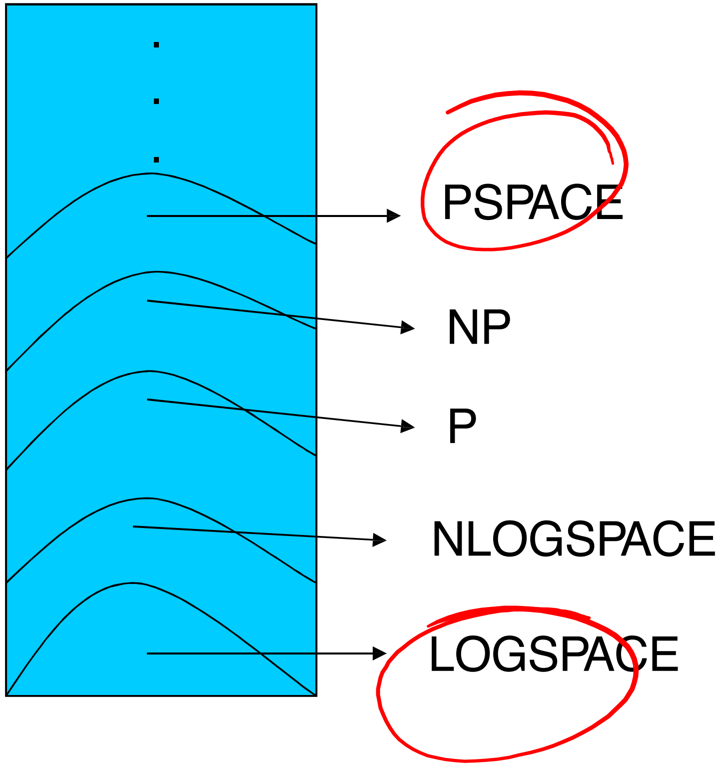
Vardi's Taxonomy of the Query Evaluation Problem

Vardi's "empirical" discovery:

- For most query languages L:
  - The data complexity of L is of lower complexity than both the combined complexity of L and the query complexity of L.
  - The query complexity of L can be as hard as the combined complexity of L.

Taxonomy of the Query Evaluation Problem for Relational Calculus

Complexity Classes



The Query Evaluation Problem
for Relational Calculus

Problem	**Complexity**
Combined Complexity	PSPACE-complete
Query Complexity	• in PSPACE • can be PSPACE-complete
Data Complexity	In LOGSPACE

Source: Phokion Kolaitis

Sublanguages of Relational Calculus

- Question: Are there interesting sublanguages of relational calculus for which the Query Containment Problem and the Query Evaluation Problem are “easier” than the full relational calculus?

- Answer:
  - Yes, the language of Conjunctive Queries (CQs) is such a sublanguage.
    - Think about a single "SELECT FROM WHERE" query block in SQL.
    - Usually only equijoins (but no comparison predicates like "R.A < S.B")are allowed
  - Moreover, conjunctive queries are the most frequently asked queries against relational databases.

Conjunctive Queries (CQs)

- DEFINITION: A CQ is a query expressible by a DRC formula in prenex normal form built from atomic formulas $R(y_1,\ldots,y_n)$, and $\wedge$ and $\exists$ only.
  - $\{ (x_1,\ldots,x_k) \mid \exists z_1 \ldots \exists z_m \; \phi(x_1, \ldots,x_k, z_1,\ldots,z_k) \}$,
  - where $\phi(x_1, \ldots,x_k, z_1,\ldots,z_k)$ is a conjunction of atomic formulas of the form $R(y_1,\ldots,y_m)$.
  - Prenex formula: prefix (quantifiers & bound variables), then quantifier-free part
- Equivalently, a CQ is a query expressible by a RA expression of the form
  - $\pi_X(\sigma_\Theta(R_1 \times \ldots \times R_n))$, where
  - Θ is a conjunction of equality atomic formulas (equijoin).
- Equivalently, a CQ is a query expressible by an SQL expression of the form
  - SELECT <list of attributes>
FROM <list of relation names>
WHERE <conjunction of equalities>

no inequalities (those can change complexities)
no selections (can be seen as preprocessing)

- Equivalently, a CQ can be written as a logic-programming (Datalog) rule:
  - $Q(x_1,\ldots,x_k)$:- $R_1(\mathbf{u}_1), \ldots, R_n(\mathbf{u}_n)$, where
  - Each $\mathbf{u}_i$ is a tuple of variables (not necessarily distinct). Each variable x_i occurs in the right-hand side of the rule. The variables occurring in the right-hand side (the body), but not in the left-hand side (the head) of the rule are existentially quantified (but the quantifiers are not displayed).

Conjunctive Queries (CQs)

- Every natural join is a conjunctive query with ...

  ... no existentially quantified variables
- Example: Given R(A,B,C), S(B,C,D)
  - R ⋈ S = {(x,y,z,w): R(x,y,z) ∧ S(y,z,w)}
  - q(x,y,z,w) :- R(x,y,z), S(y,z,w)

    (no variables are existentially quantified)
  - SELECT R.A, R.B, R.C, S.D
FROM R, S
WHERE R.B = S.B AND R.C = S.C
- Conjunctive queries are also known as SPJ-queries (SELECT-PROJECT-JOIN queries)

Examples of Conjunctive Queries

$E(S, T)$

START + END VERTEX

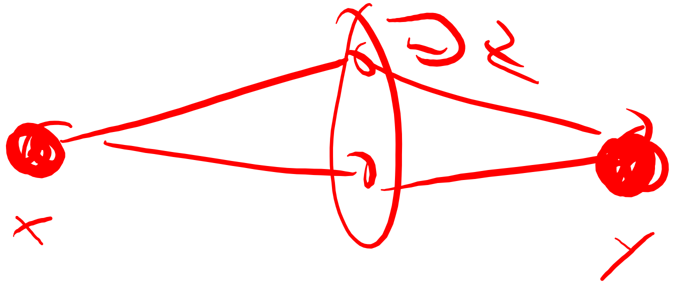
- Return paths of Length 2: (binary output)

  DRC: ?

  TRC: ?

  RA: ?

  Datalog: ?



Examples of Conjunctive Queries

$E(S,T)$

- Return paths of Length 2: (binary output)

DRC: $\{(x,y) \mid \exists z\,[E(x,z) \wedge E(z,y)]\}$

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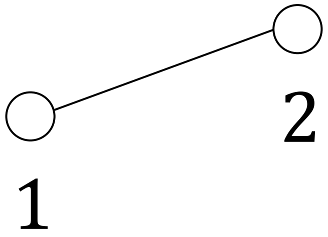
Datalog: ?



Is there a path of length 2 ?

Examples of Conjunctive Queries

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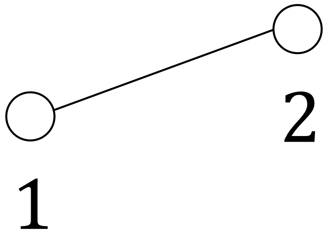
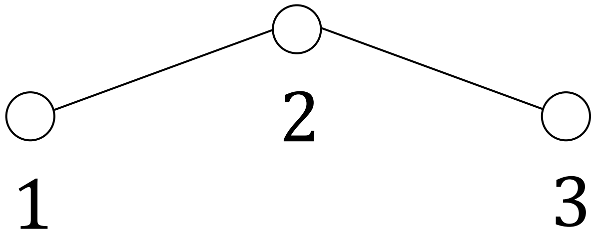


E

1	2
2	1

Is there a path of length 2 ?



$x \neq y$ not required!
Homomorphism vs.
Isomorphism (more on that later)

Examples of Conjunctive Queries

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Examples of Conjunctive Queries

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RA: $\pi_{\$1,\$4}(\sigma_{\$2=\$3}(E \times E))$ unnamed perspective

Datalog: ?

Examples of Conjunctive Queries

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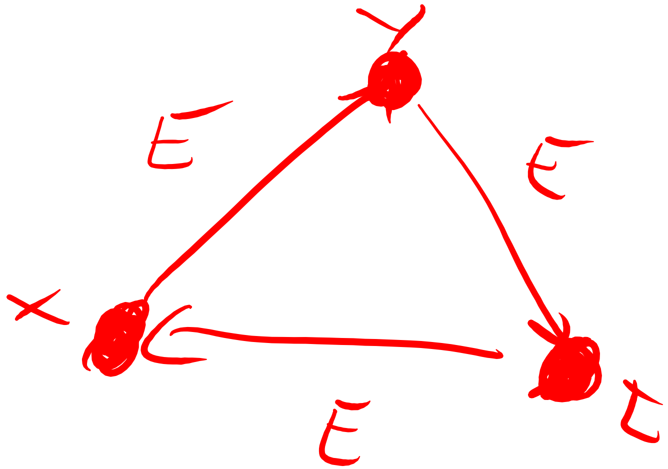
Datalog: $Q(\text{x,y}) :- E(x,z), E(z,y)$

- Is there a cycle of Length 3: (Boolean query)

DRC: ?

Datalog: ?



Examples of Conjunctive Queries

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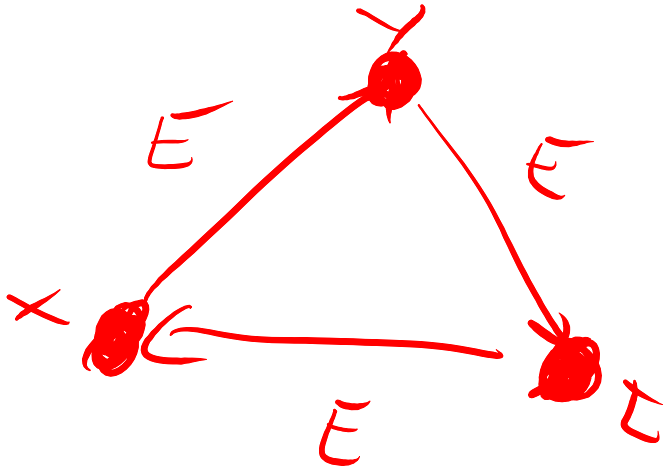
Datalog: ?



Examples of Conjunctive Queries

$E(S,T)$

- Return paths of Length 2: (binary output)

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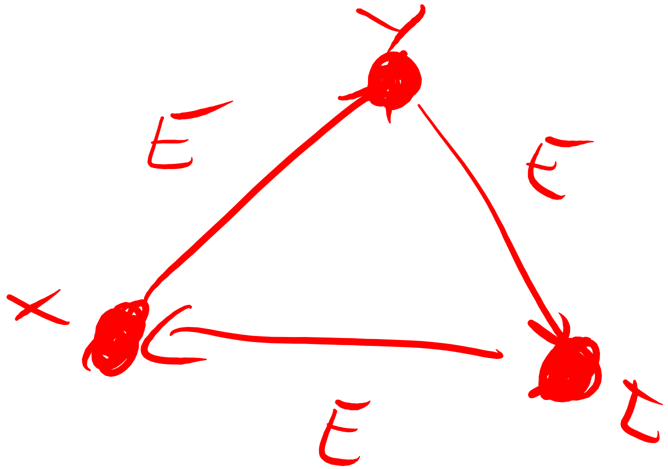
Datalog: $Q(\text{x,y}) :- E(x,z), E(z,y)$

- Is there a cycle of Length 3: (Boolean query)

DRC: $\exists x\, \exists y\, \exists z\, [E(x,y) \land E(y,z) \land E(z,x)]\}$

Datalog: $Q\text{()} :- E(x,y), E(y,z), E(z,x)$

alternative variant with "directed" cycle and arcs



Summary

- Relational Algebra (RA) and Relational Calculus (RC) have "essentially" the same expressive power (recall Codd's theorem from T1-U3)

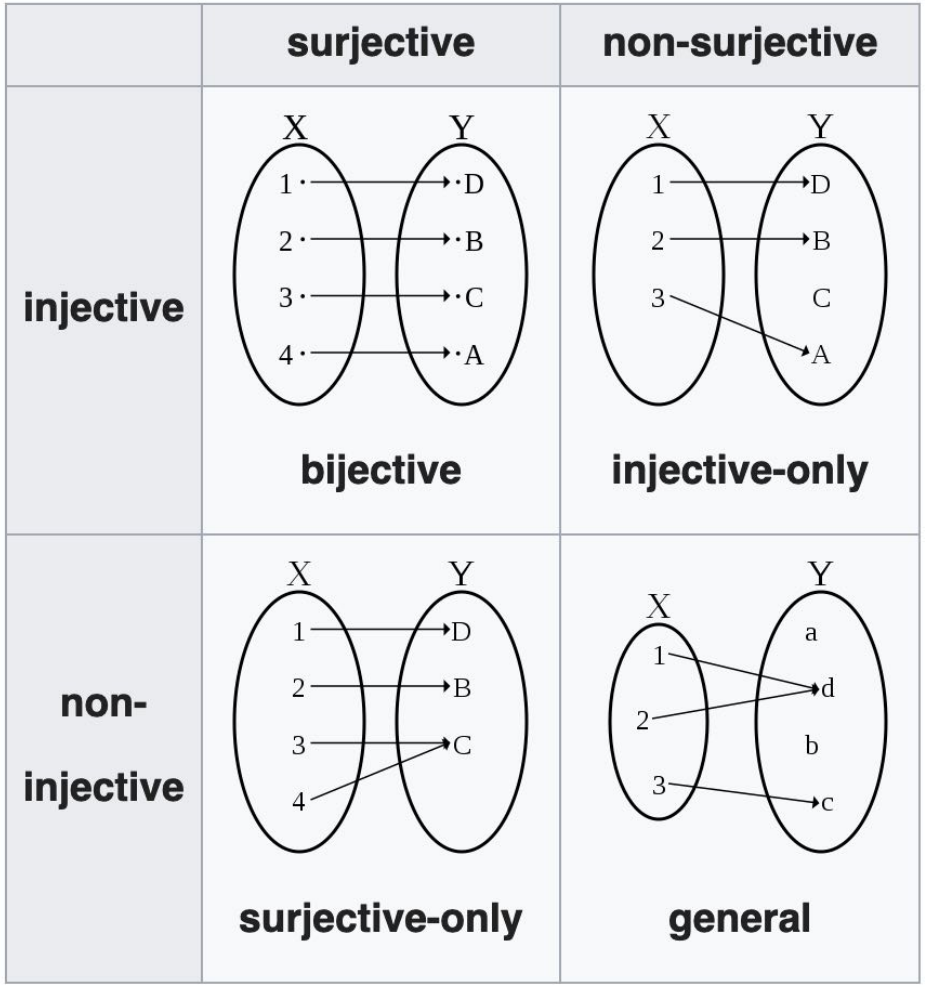
- The Query Equivalence Problem for Relational Calculus is undecidable.
  - Therefore also the Query Containment Problem

- The Query Evaluation Problem for Relational Calculus:
  - Data Complexity is in LOGSPACE (and thus very efficient)
  - Query Complexity and Combined Complexity are PSPACE-complete

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Injective, Surjective, and Bijective functions $f: X \to Y$



Function ?

Injective function ?

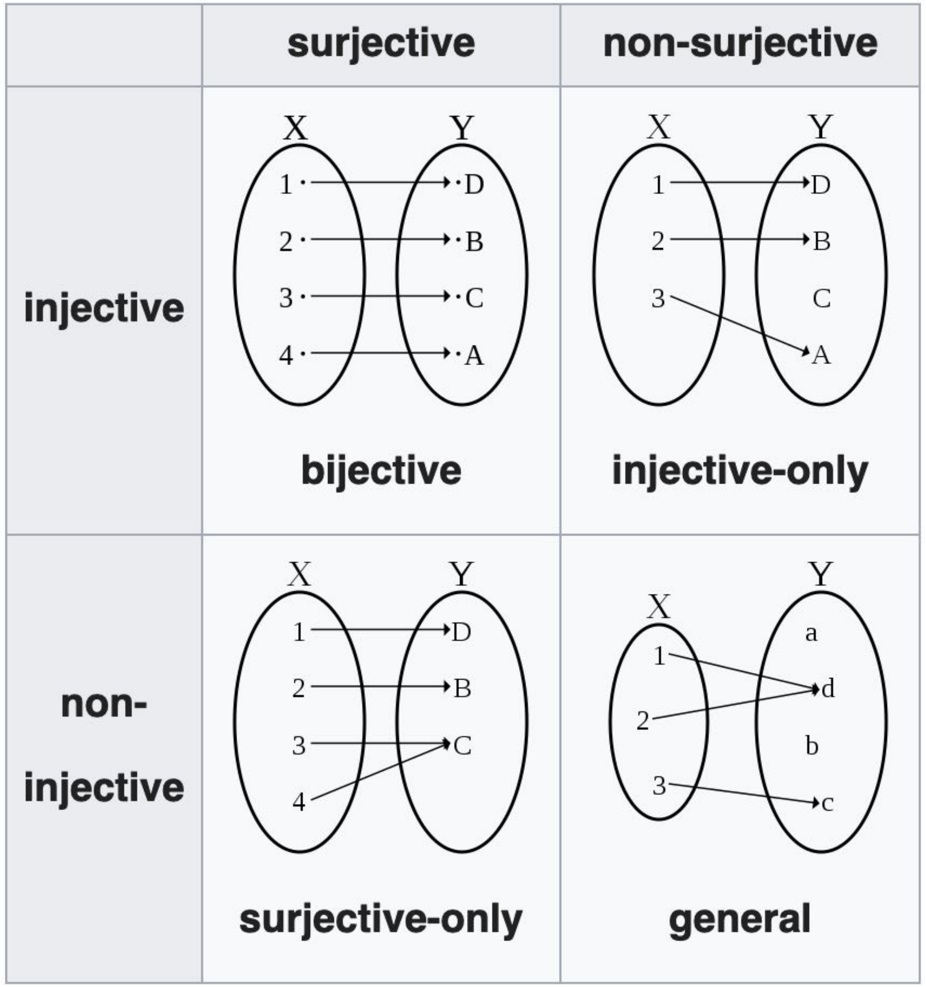
Surjective function ?

Bijective function ?

Source: https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection
Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

Injective, Surjective, and Bijective functions

$f: X \to Y$



Function maps each argument (element from its domain) to exactly one image (element in its codomain)

$\forall x \in X, \exists! y \in Y[y = f(x)]\}$

Injective function

?

$\exists! y \in Y[P(y)]$

$\exists y \in Y[P(y) \wedge \forall y' \in Y[P(y') \Rightarrow y = y']]$

$\exists y \in Y[P(y) \wedge \neg\exists y' \in Y[P(y') \wedge y \neq y']]$

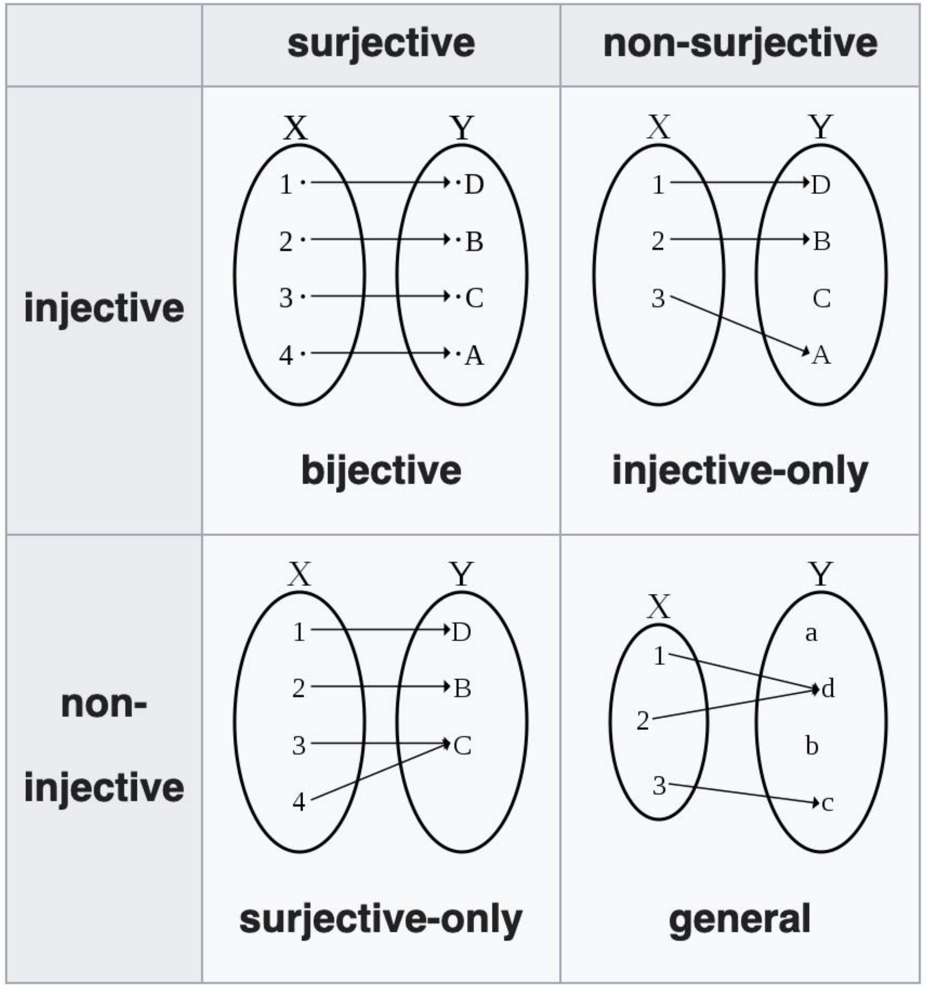
Surjective function

?

Bijective function

?

Injective, Surjective, and Bijective functions $f: X \rightarrow Y$



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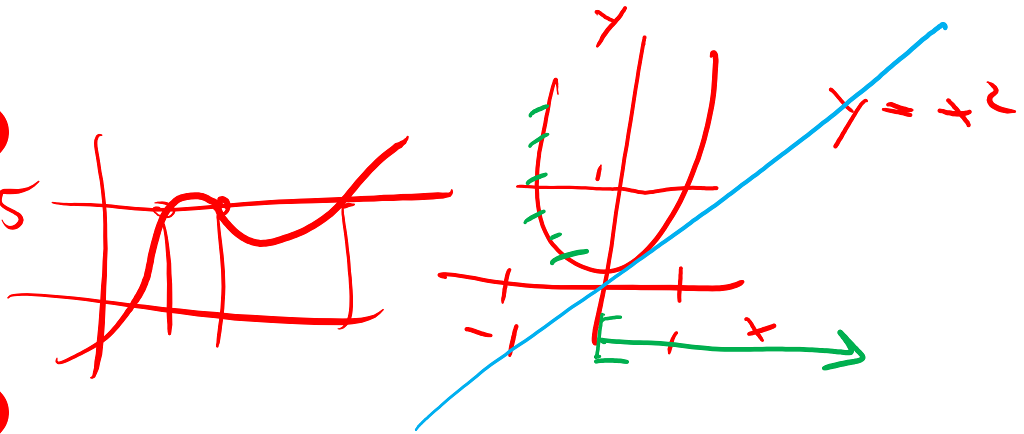
$\exists y \in Y[P(y) \wedge \neg \exists y' \in Y[P(y') \wedge y \neq y']]$

Function maps each <u>argument</u> (element from its <u>domain</u>) to exactly one <u>image</u> (element in its <u>codomain</u>)
$\forall x \in X, \exists! \, y \in Y[y = f(x)]\}$

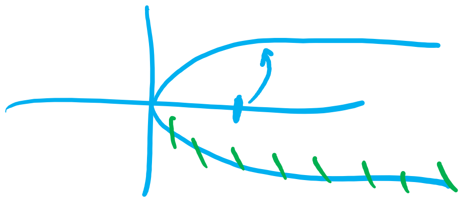
Injective function ("one-to-one"): each element of the codomain is mapped to by <u>at most one</u> element of the domain (i.e. distinct elements of the domain map to distinct elements in the codomain)
$\ldots \wedge \forall x, x' \in X. [x \neq x' \Rightarrow f(x) \neq f(x')]$

logical transpose without inequality: $\ldots \wedge \forall x, x' \in X. [f(x) = f(x') \Rightarrow x = x']$

Surjective function ?

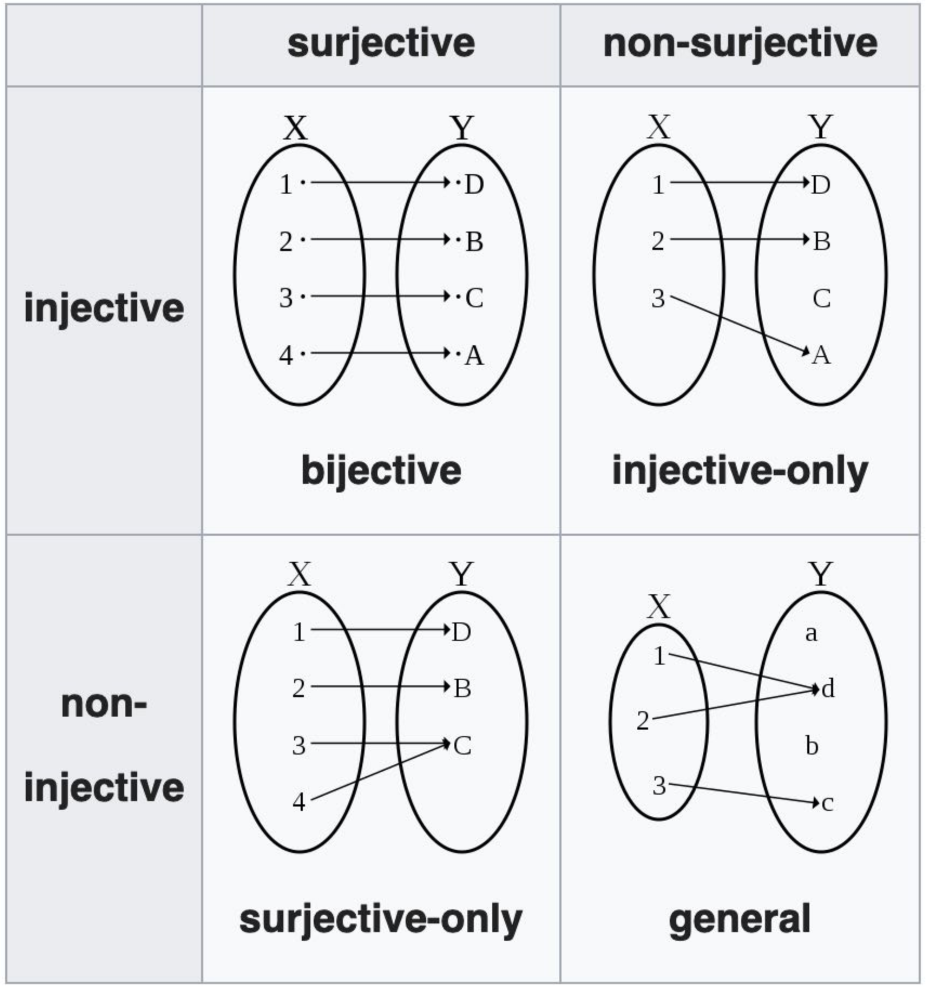


Bijective function ?



Source: https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection

Injective, Surjective, and Bijective functions $f: X \to Y$



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logical transpose without inequality: $\ldots \land \forall x, x' \in X. [f(x) = f(x') \Rightarrow x = x']$

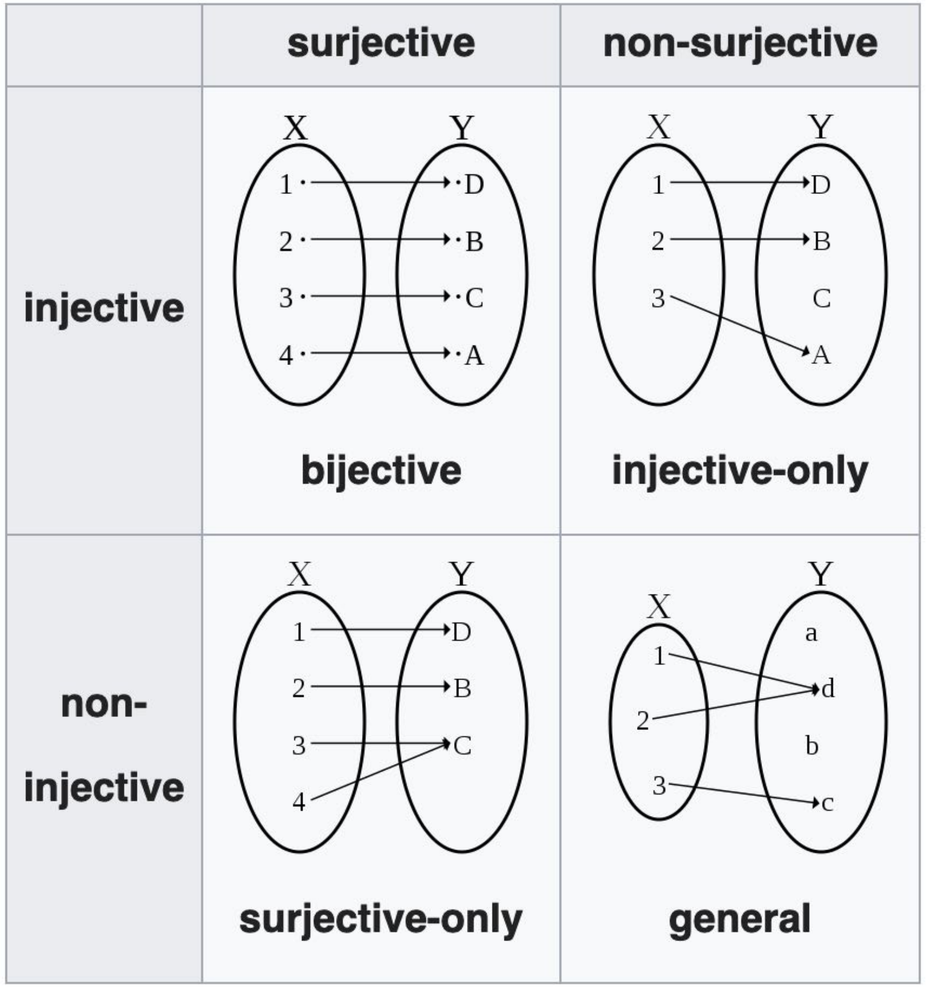
Surjective function ("onto"): each element of the codomain is mapped to by at least one element of the domain (i.e. the image and the codomain of the function are equal)

$\ldots \land \forall y \in Y, \exists x \in X[y = f(x)]$

Bijective function ?

Source: https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection

Injective, Surjective, and Bijective functions $f: X \to Y$



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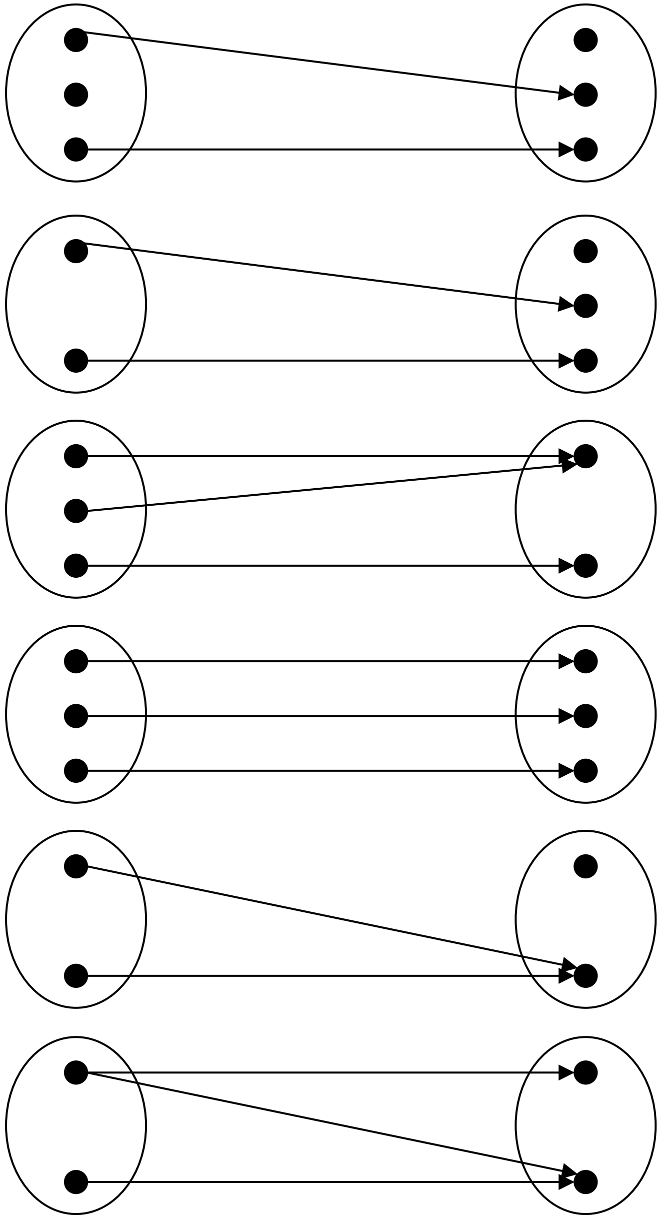
Surjective function ("onto"): each element of the codomain is mapped to by at least one element of the domain (i.e. the image and the codomain of the function are equal)
$\ldots \wedge \forall y \in Y, \exists x \in X[y = f(x)]$

Bijective function ("invertible"): each element of the codomain is mapped to by exactly one element of the domain (both injective and surjective)
$\ldots \wedge \forall y \in Y, \exists! x \in X[y = f(x)]\}$

Source: https://en.wikipedia.org/wiki/Bijection,_injection_and_surjection

Mappings: Injection, Surjection, and Bijection



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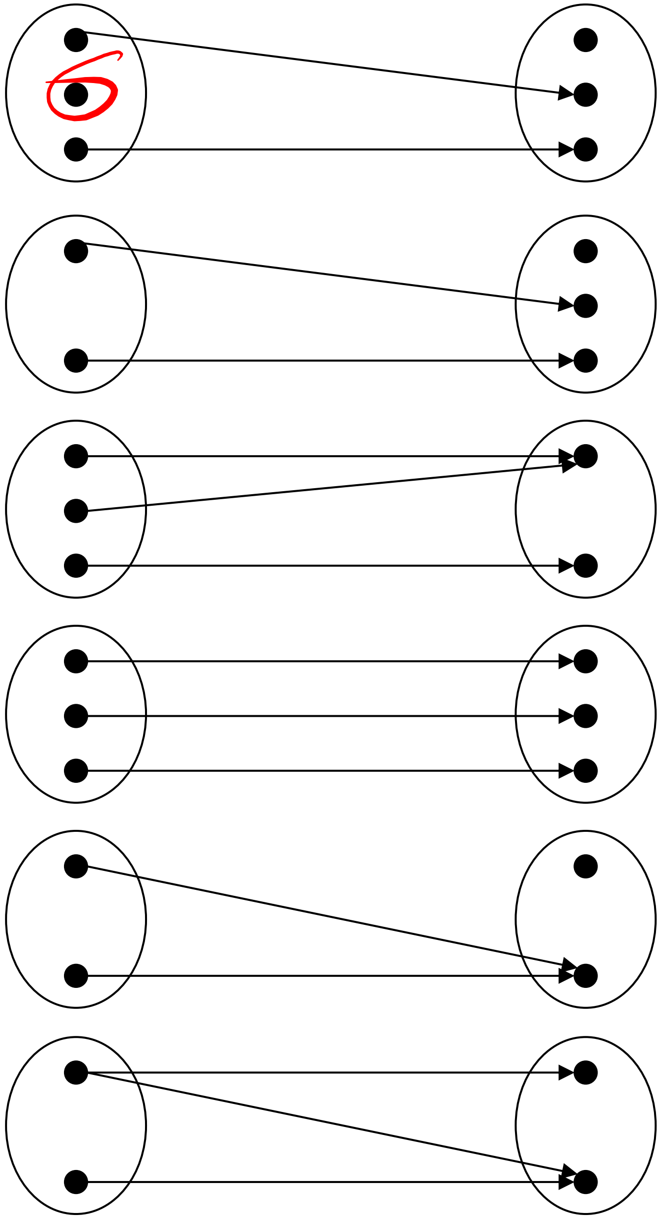
Mappings: Injection, Surjection, and Bijection



not a mapping (or function)!

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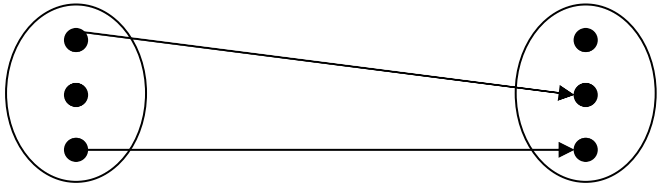
Mappings: Injection, Surjection, and Bijection



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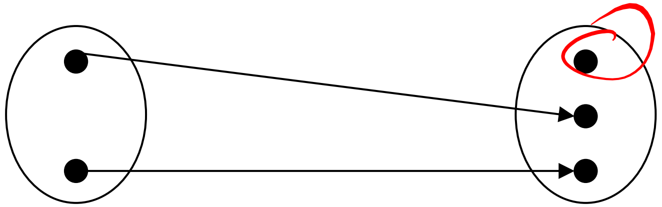
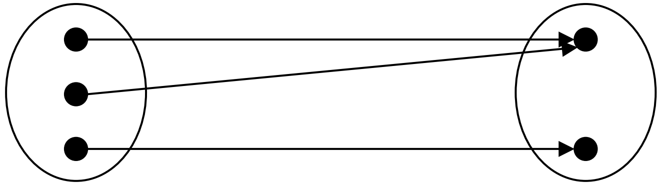


injective function (or one-to-one): maps distinct elements of its domain to <u>distinct elements of its codomain</u>



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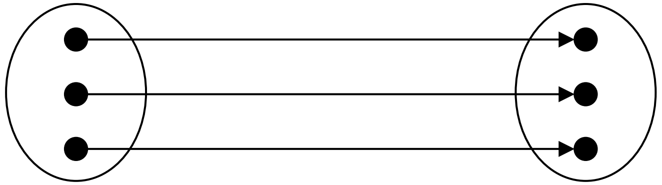

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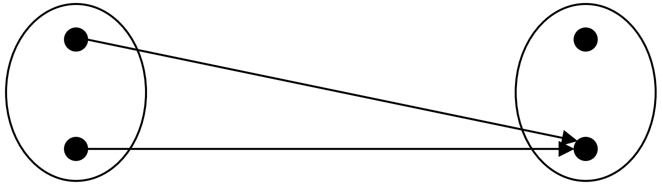

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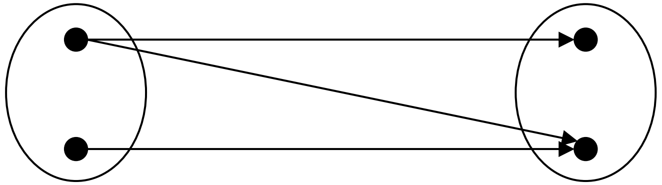

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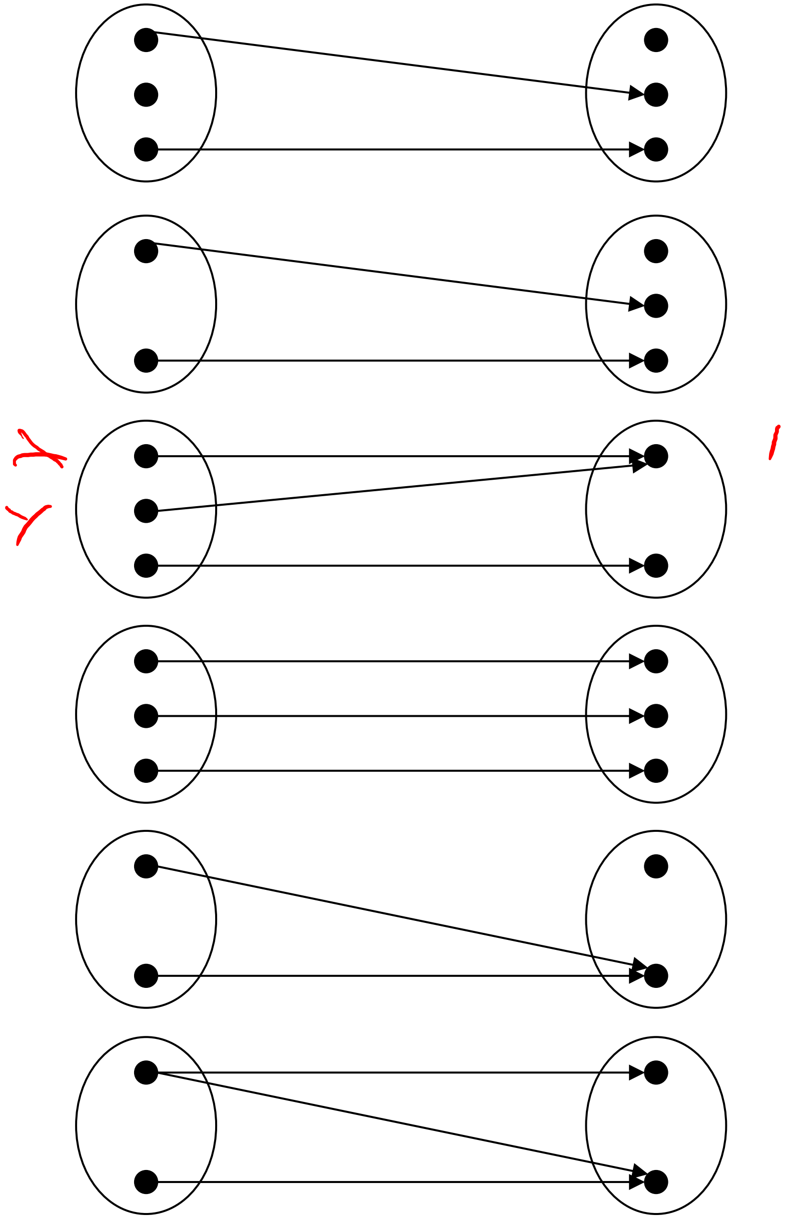
Mappings: Injection, Surjection, and Bijection



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surjective (or onto): every element y in the codomain Y of f has at least one element x in the domain that maps to it

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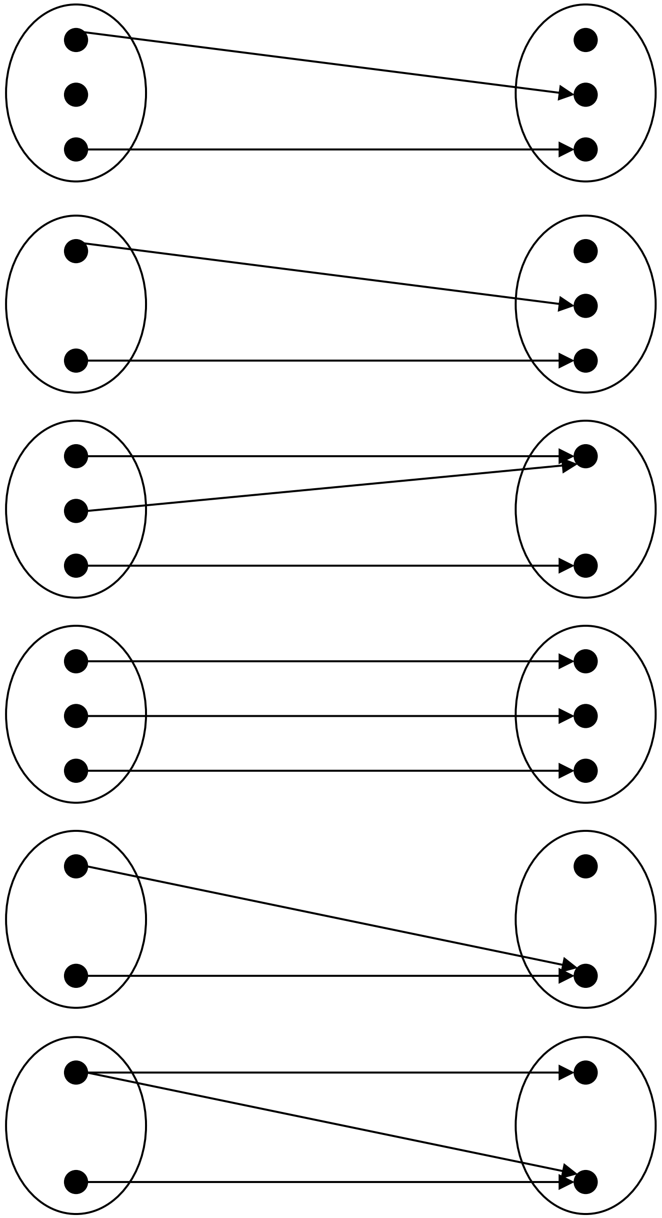
Mappings: Injection, Surjection, and Bijection



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injective & surjective = bijection

?

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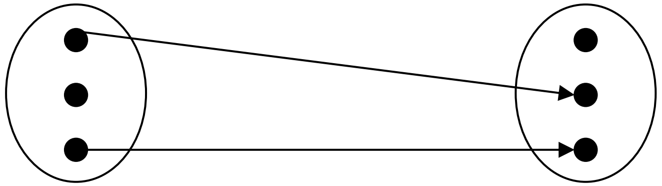
Mappings: Injection, Surjection, and Bijection



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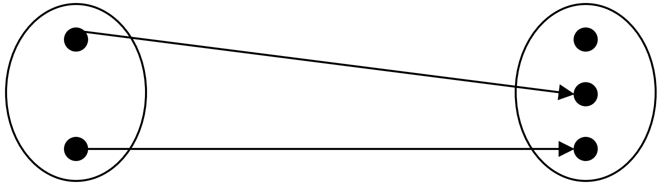
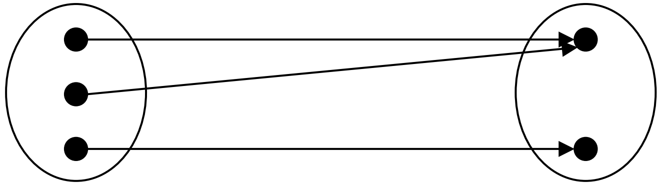
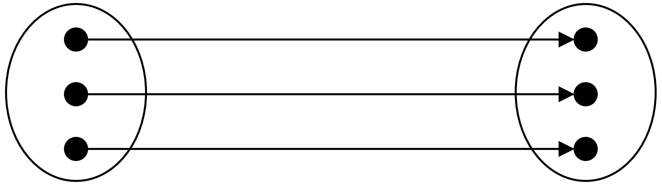


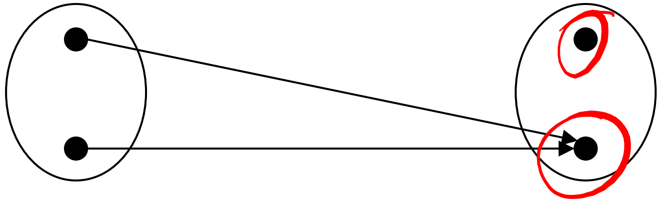
injective function (or one-to-one): maps distinct elements of its domain to <u>distinct elements of its codomain</u>



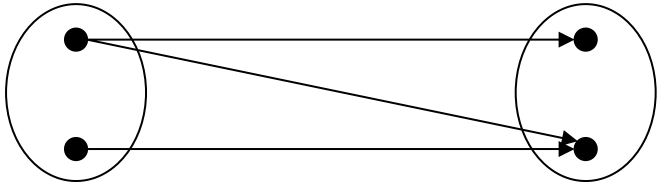
surjective (or onto): <u>every element y in the codomain Y</u> of f has at least one element x in the domain that maps to it



injective & surjective = bijection



neighter



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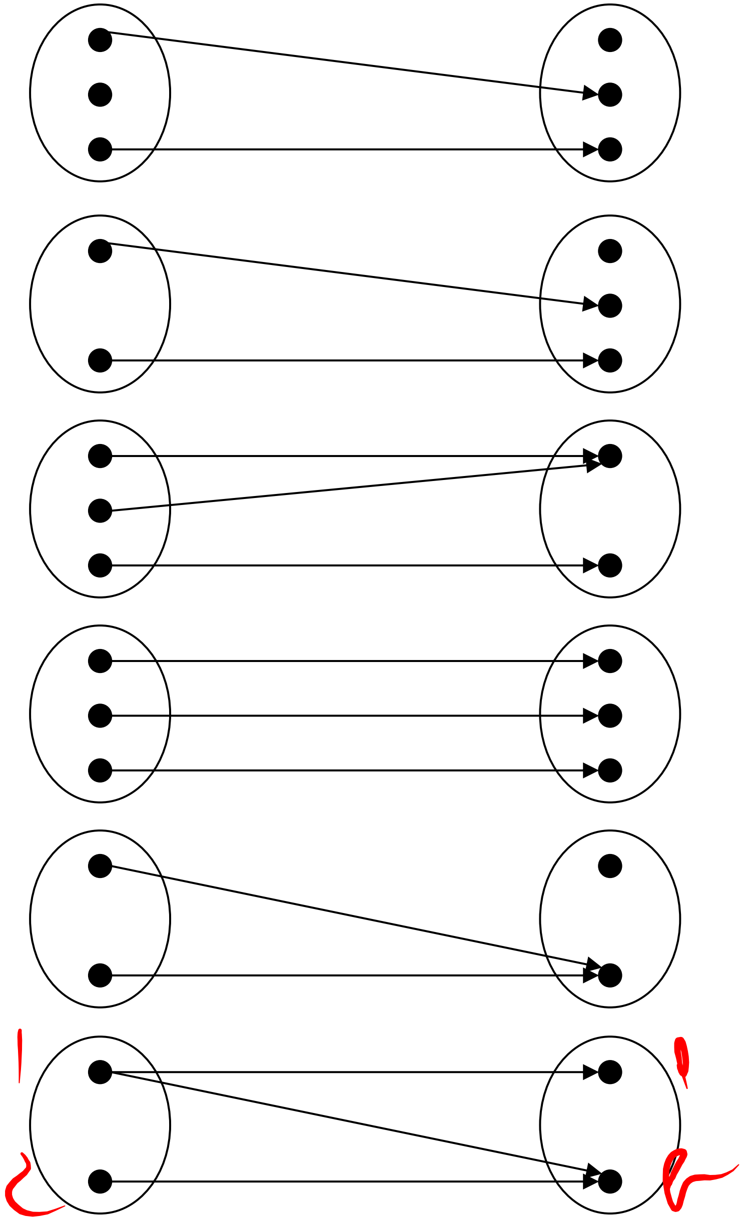
Mappings: Injection, Surjection, and Bijection



not a mapping (or function)!

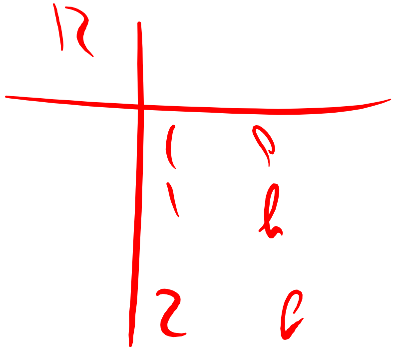
injective function (or one-to-one): maps distinct elements of its domain to distinct elements of its codomain

surjective (or onto): every element y in the codomain Y of f has at least one element x in the domain that maps to it

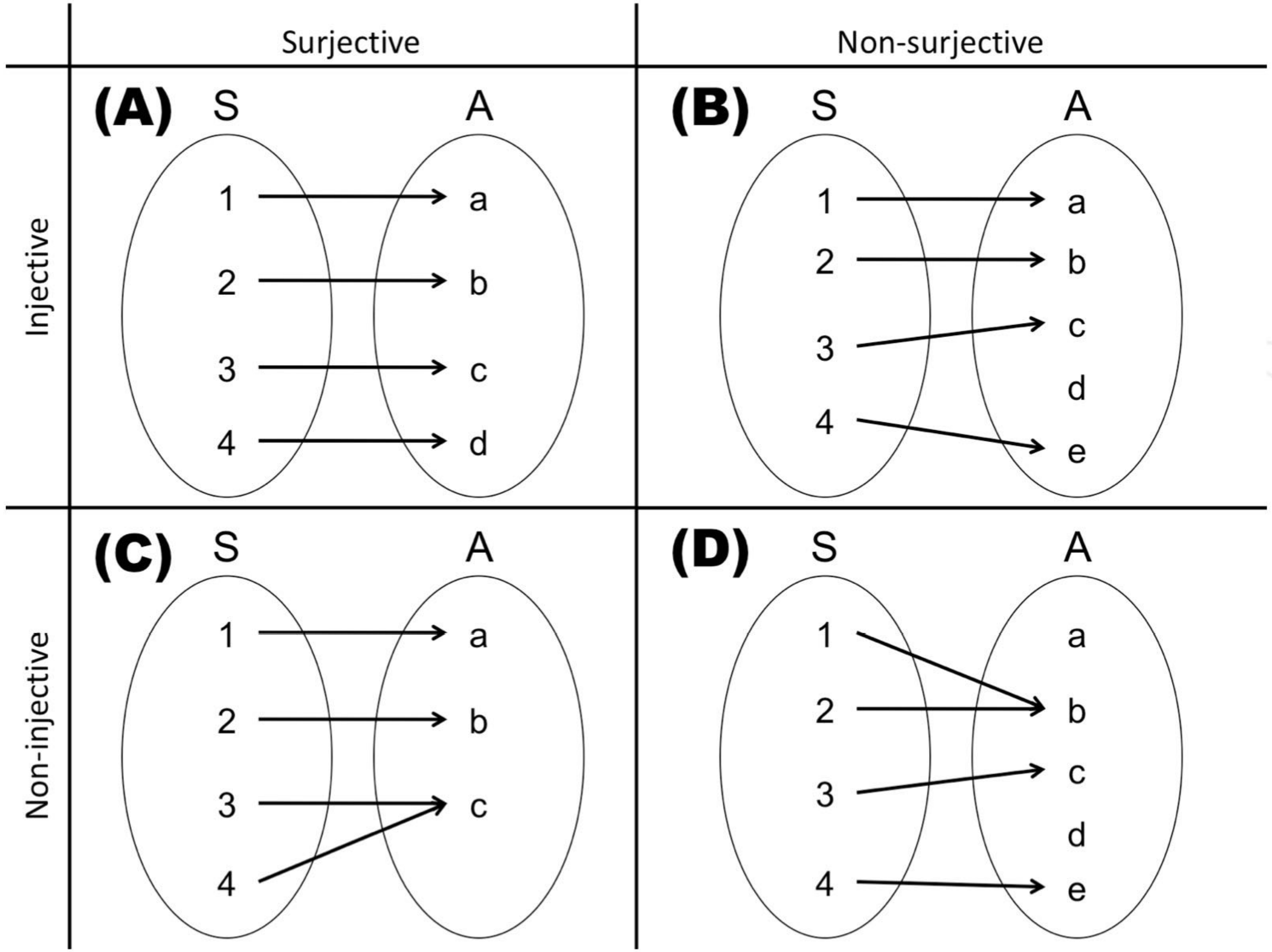
injective & surjective = bijection

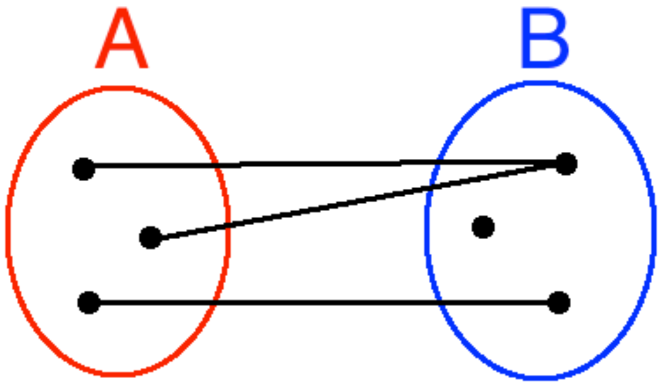
neighter

not even a mapping!

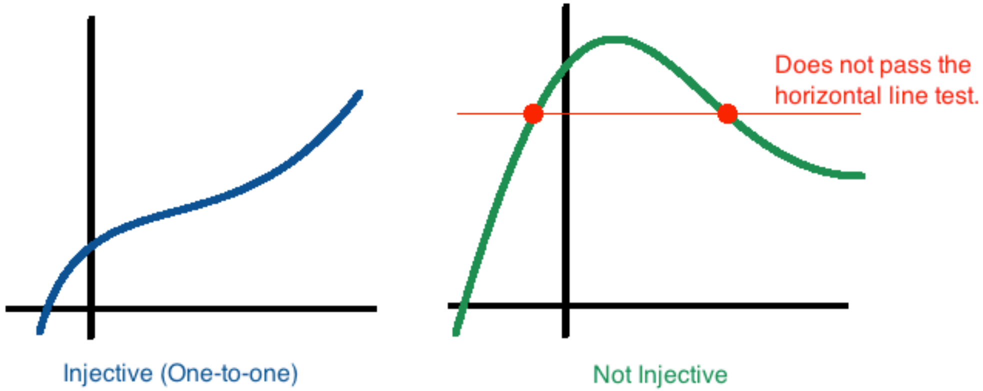


Bijection, Injection, and Surjection



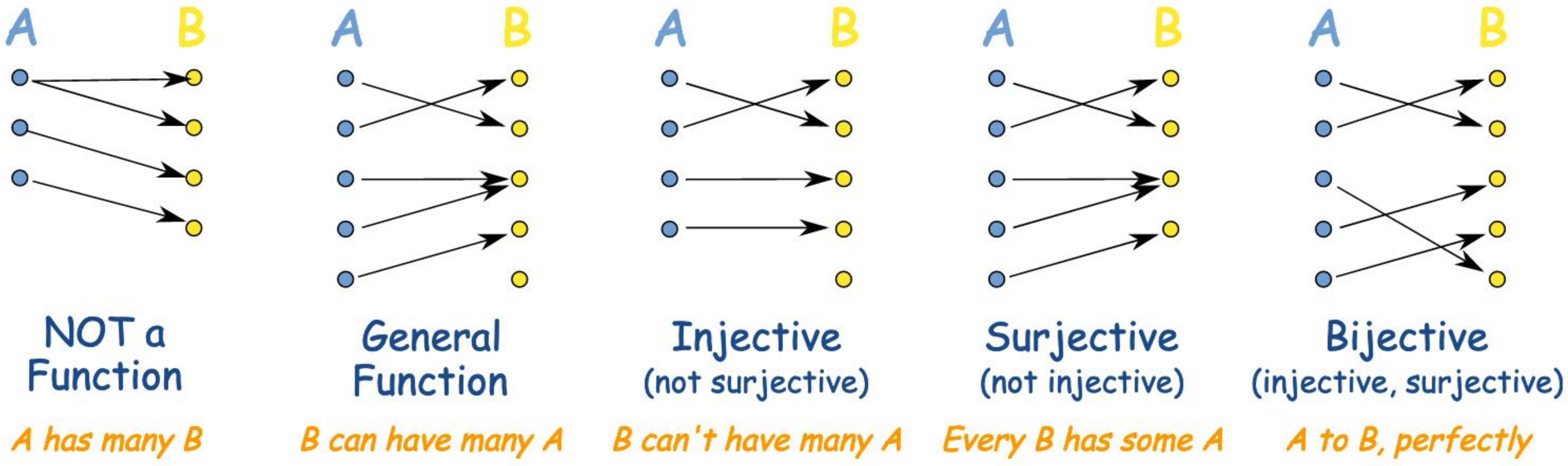


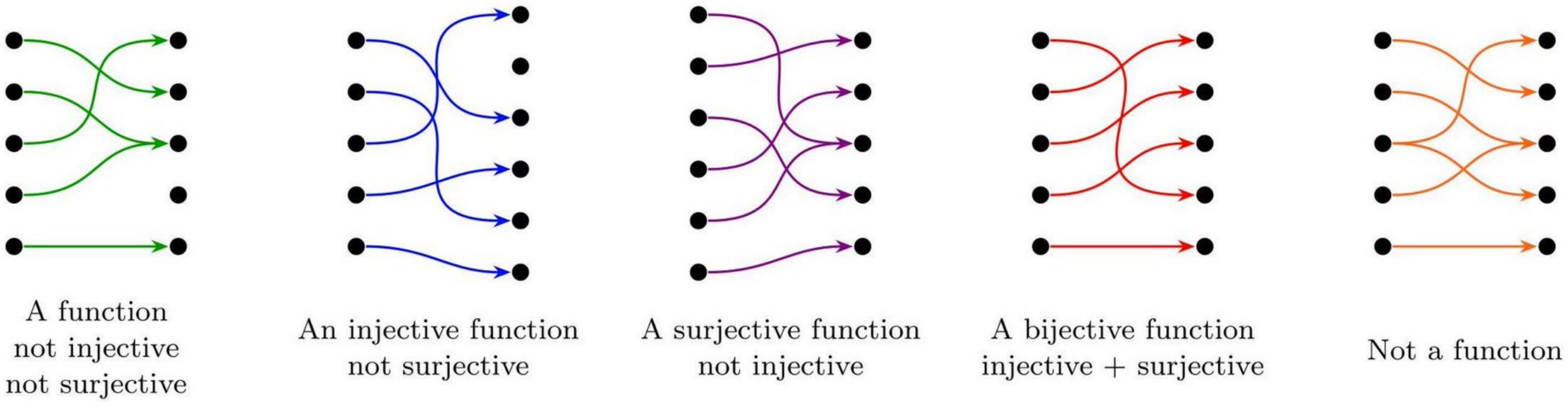
Neither Injective or Surjective
Two elements in set A maps to the same element in set B (not injective), and one element in set B is not in the image or range of the function that maps set A to B (not surjective).



Sources: http://mathonline.wikidot.com/injections-surjections-and-bijections, https://www.intechopen.com/books/protein-interactions/relating-protein-structure-and-function-through-a-bijection-and-its-implications-on-protein-structur,
Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

Bijection, Injection, and Surjection





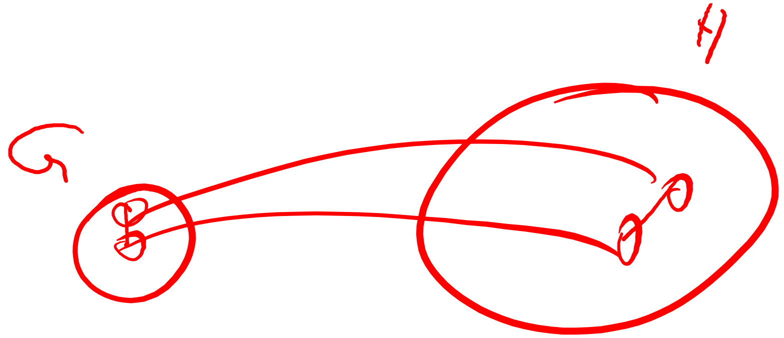
Sources: https://www.mathsisfun.com/sets/injective-surjective-bijective.html, https://twitter.com/jdhamkins/status/841318019397779456,
Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

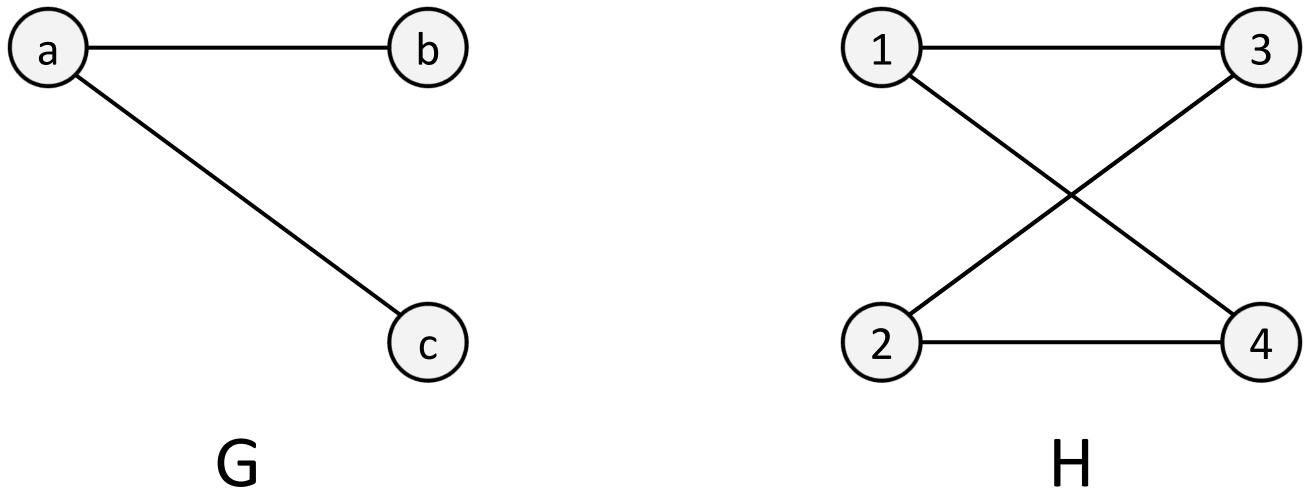
We make a detour to Graph matching

- Finding a correspondence between the nodes and the edges of two graphs that satisfies some (more or less stringent) constraints

Homomorphism

- A graph homomorphism h from graph $G(V_G,E_G)$ to $H(V_H,E_H)$, is a mapping from V_G to V_H such that $\{x,y\} \in E_G$ implies $\{h(x),h(y)\} \in E_H$
  - "edge-preserving": if two nodes in G are linked by an edge, then they are mapped to two nodes in H that are also linked





Is there a homomorphism from G to H ?

Homomorphism

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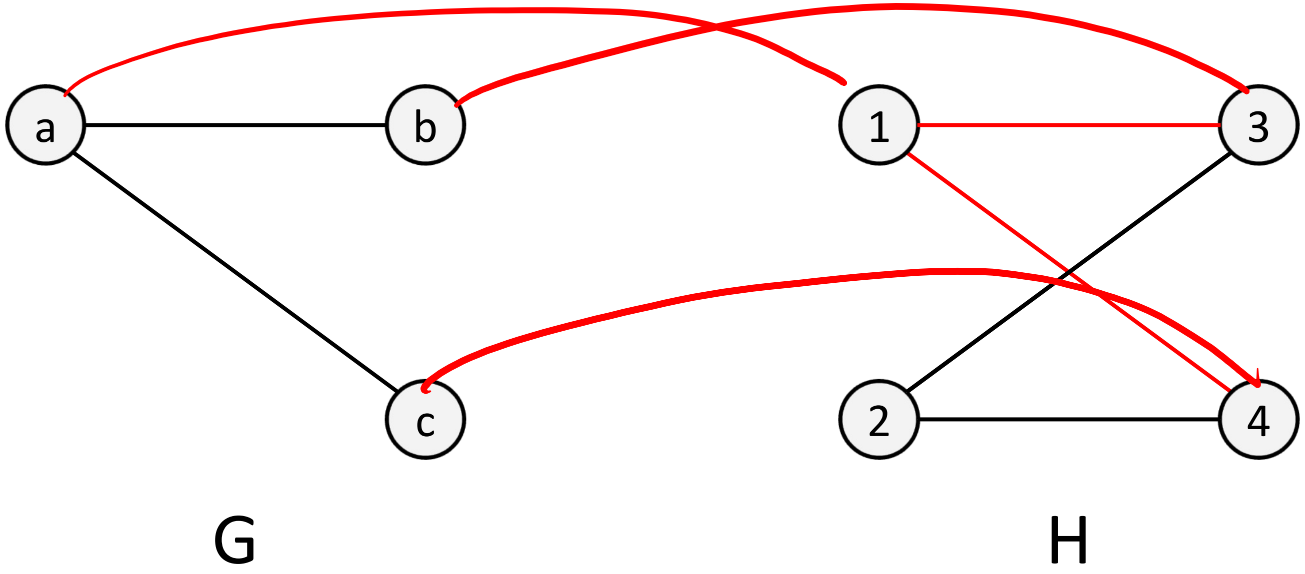

h: {(a,1), (b,3), (c,4)}

does not need to be surjective!

Homomorphism

- A graph homomorphism h from graph $G(V_G,E_G)$ to $H(V_H,E_H)$, is a mapping from V_G to V_H such that $\{x,y\} \in E_G$ implies $\{h(x),h(y)\} \in E_H$
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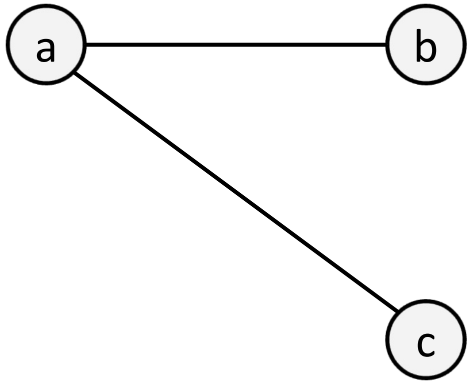
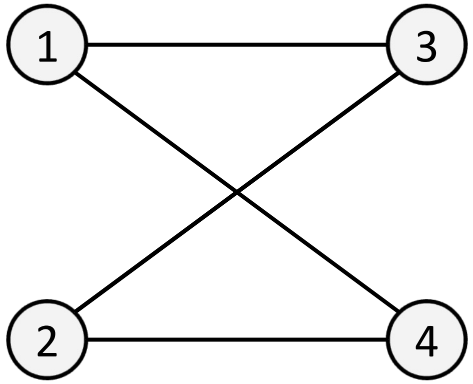
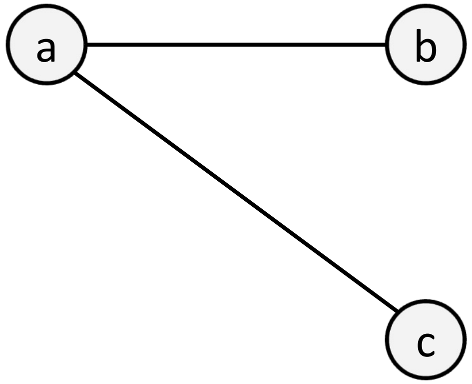


G



H



G

h: {(a,1), (b,3), (c,4)}

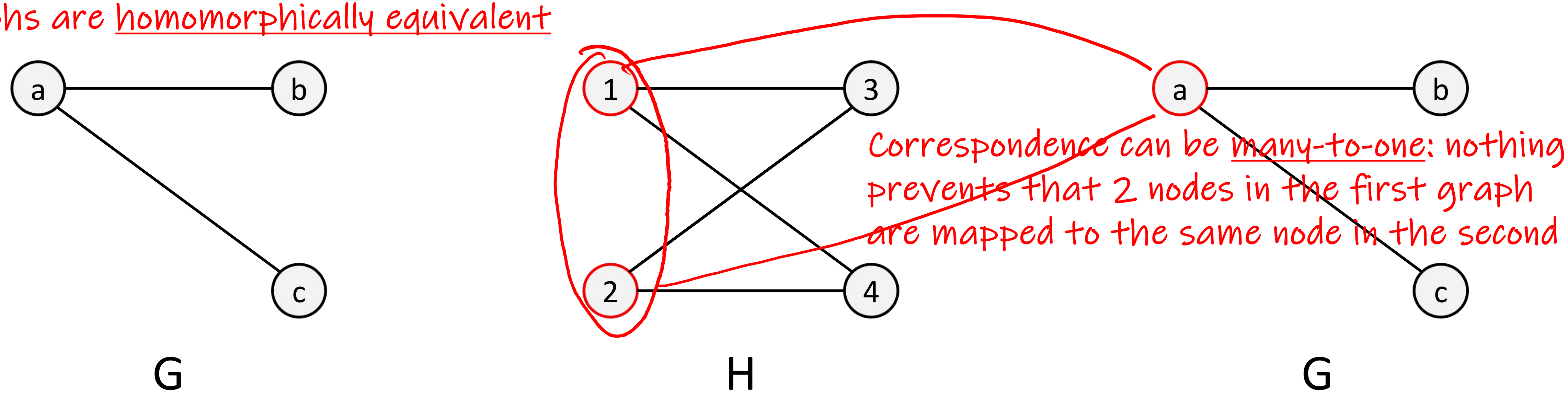
does not need to be surjective!

Is there a homomorphism from H to G ?

Homomorphism

- A graph homomorphism *h* from graph $G(V_G,E_G)$ to $H(V_H,E_H)$, is a mapping from V_G to V_H such that $\{x,y\} \in E_G$ implies $\{h(x),h(y)\} \in E_H$
  - "edge-preserving": if two nodes in *G* are linked by an edge, then they are mapped to two nodes in *H* that are also linked

Graphs are homomorphically equivalent

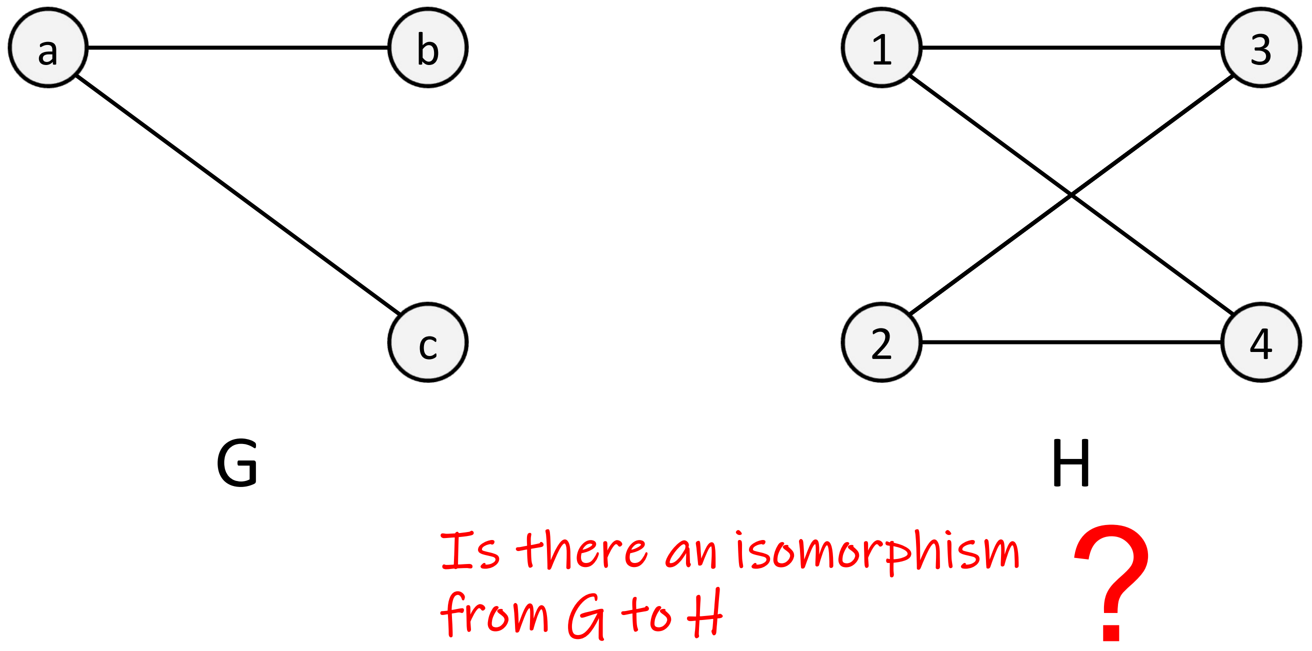


h: {(a,1), (b,3), (c,4)}
does not need to be surjective!

h: {(1,a), (2,a), (3,b), (4,c)}
does not need to be injective!

Graph Isomorphism

- Graphs $G(V_G,E_G)$ and $H(V_H,E_H)$ are isomorphic iff there is an invertible h from V_G to V_H s.t. $\{x,y\} \in E_G$ iff $\{h(u),h(v)\} \in E_H$
  - We need to find a one-to-one correspondence



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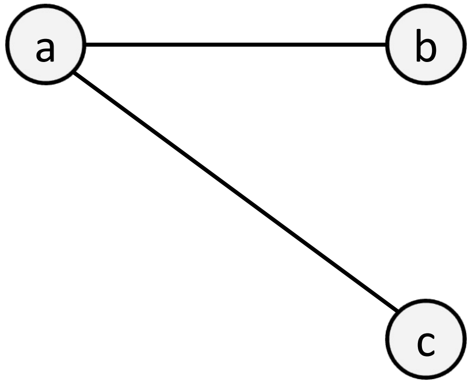
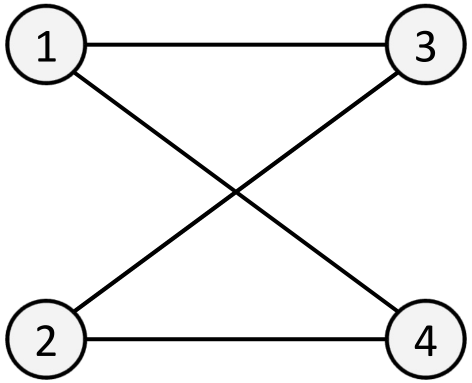


G



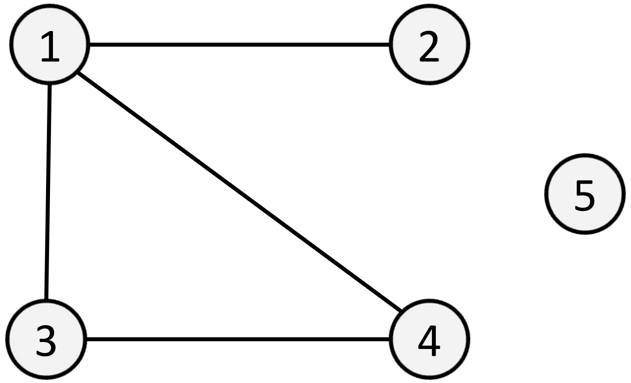
H

Is there an isomorphism from G to H? No!

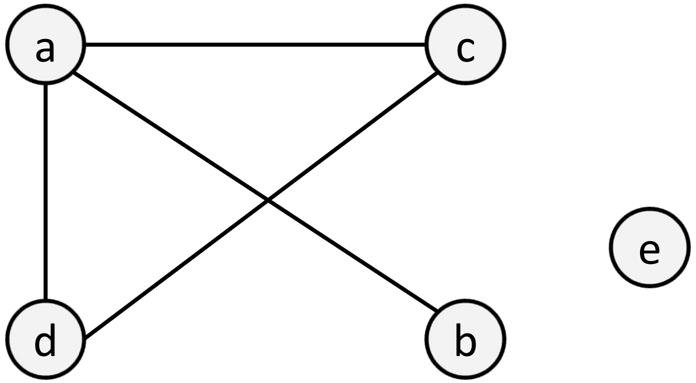
They are homomorphically equivalent, but not isomorphic!

Graph Isomorphism

- Graphs $G(V_G,E_G)$ and $H(V_H,E_H)$ are isomorphic iff there is an invertible h from V_G to V_H s.t. $\{x,y\} \in E_G$ iff $\{h(u),h(v)\} \in E_H$
  - We need to find a one-to-one correspondence



G



H

Is there an isomorphism from G to H?

?

Graph Isomorphism

- Graphs $G(V_G,E_G)$ and $H(V_H,E_H)$ are isomorphic iff there is an invertible h from V_G to V_H s.t. $\{x,y\} \in E_G$ iff $\{h(u),h(v)\} \in E_H$
  - We need to find a one-to-one correspondence

Is there an isomorphism from G to H?

Yes: h: {(1,a), (2,b), (3,d), (4,c), (5,e)}

bijection = surjective and injective mapping

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
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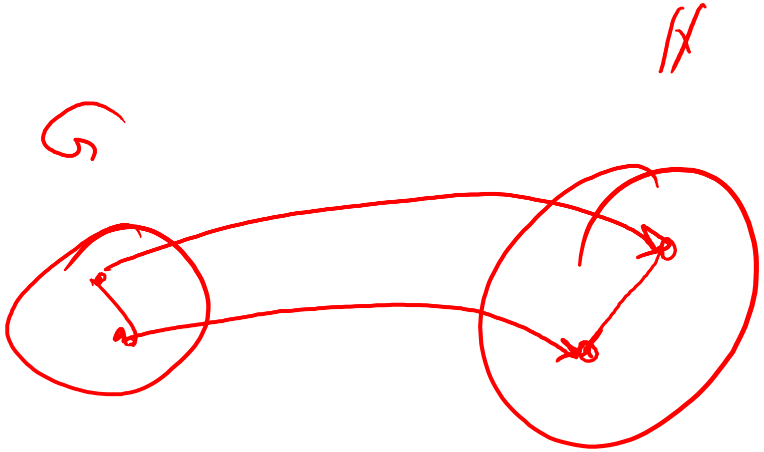
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Graph Homomorphism beyond graphs

Definition : *Let G and H be graphs. A homomorphism of G to H is a function f*: $V(G) \rightarrow V(H)$ such that

$$(x,y) \in E(G) \Rightarrow (f(x),f(y)) \in E(H).$$

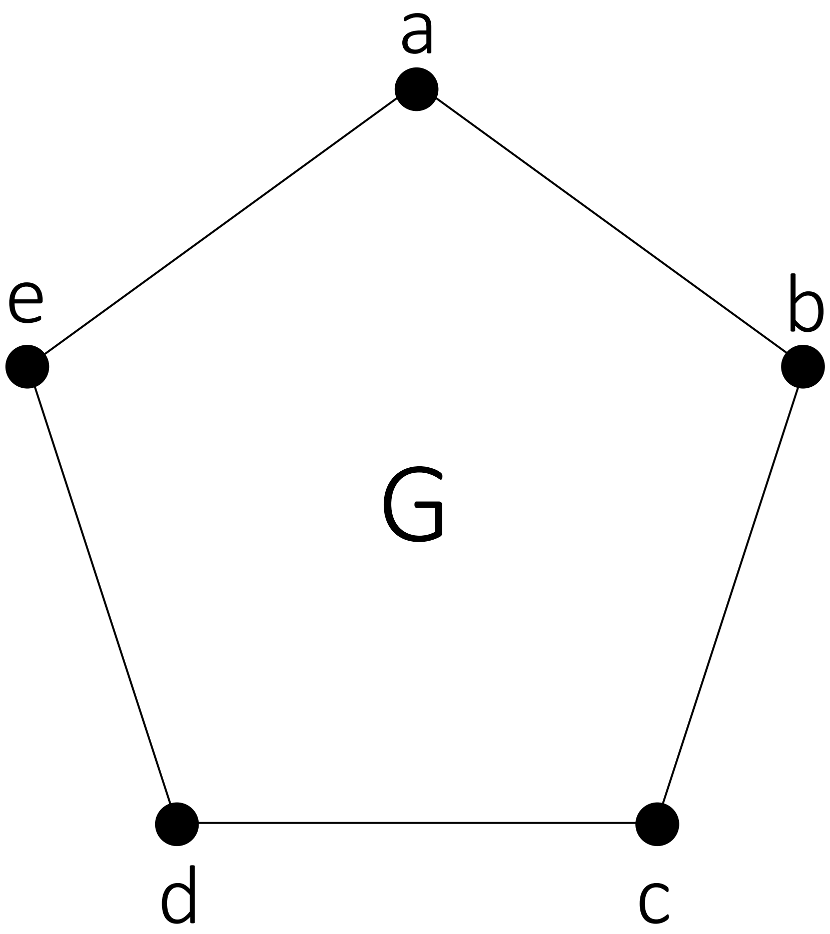
We sometimes write $G \rightarrow H$ ($G \nrightarrow H$) if there is a homomorphism (no homomorphism) of G to H



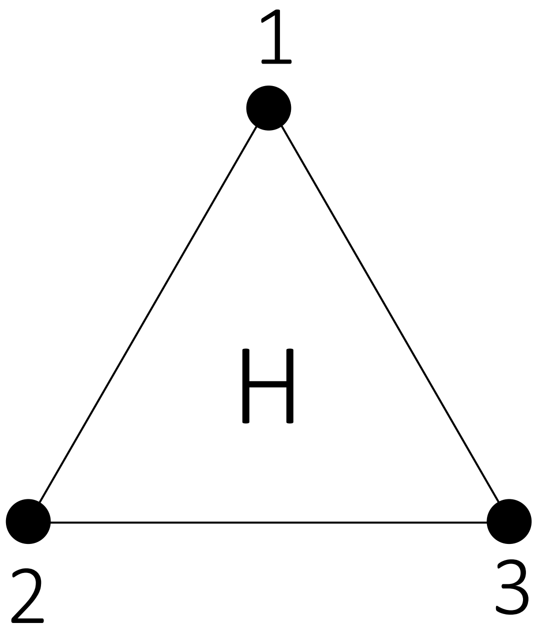
Definition of a homomorphism naturally extends to:

- digraphs (directed graphs)
- edge-colored graphs
- relational systems
- constraint satisfaction problems (CSPs)

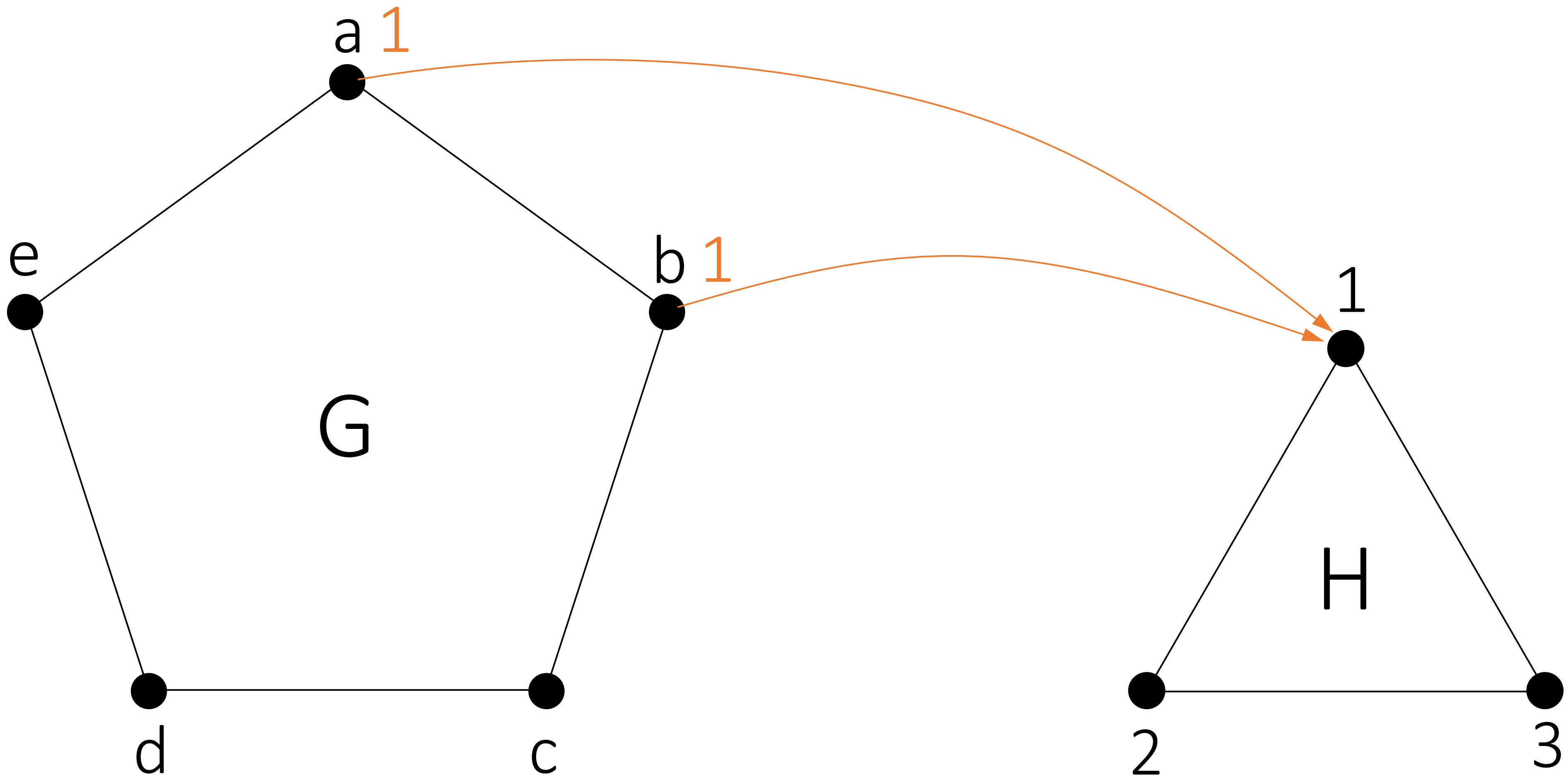
An example



3 "colors" of the vertices

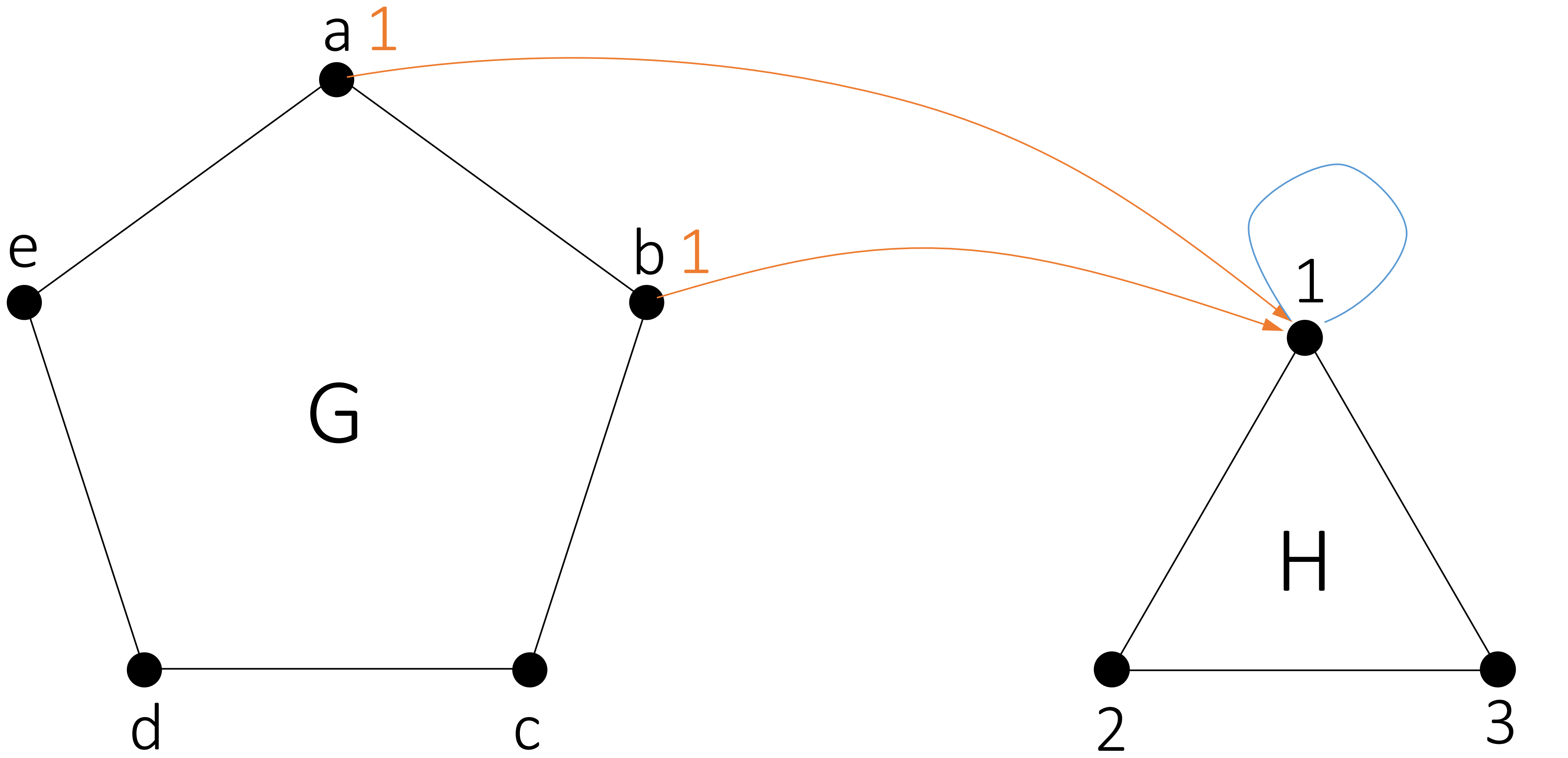


An example



Can this assignment be extended to a homomorphism?

An example

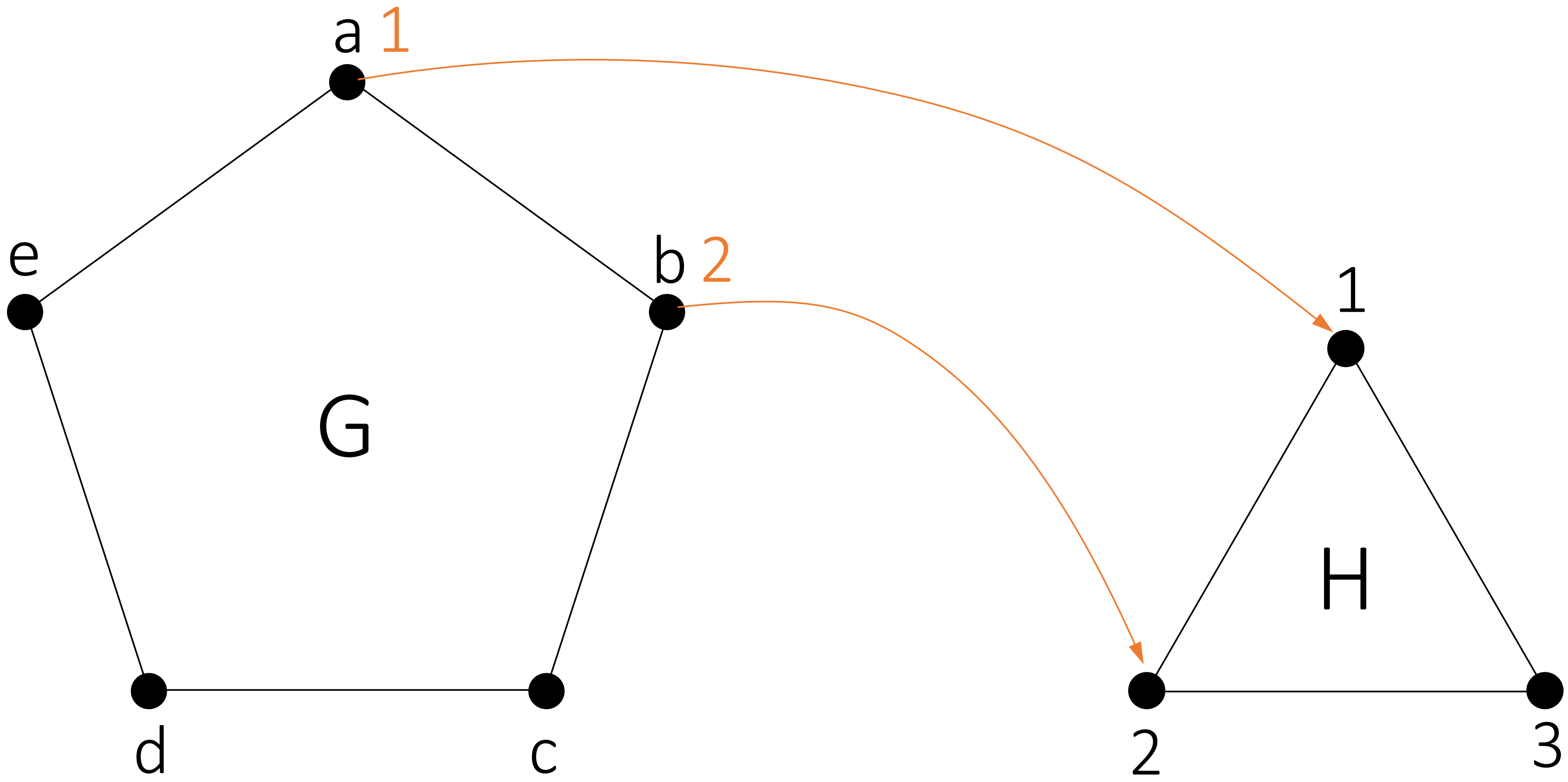


Can this assignment be extended to a homomorphism?

No, this assignment requires a loop on vertex 1 (in H)

Based upon an example from Rick Brewster's Graph homomorphism tutorial, 2006

An example



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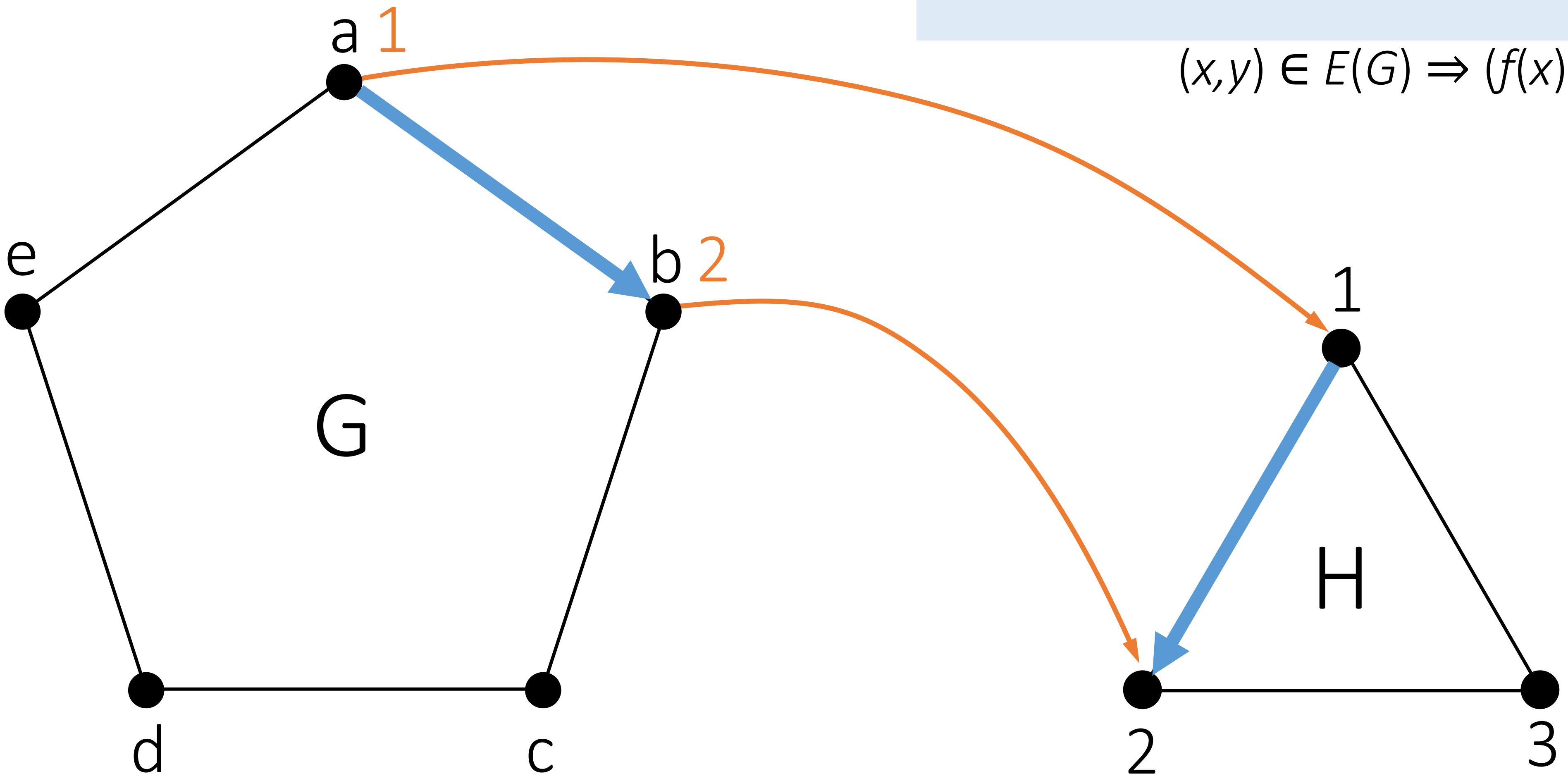

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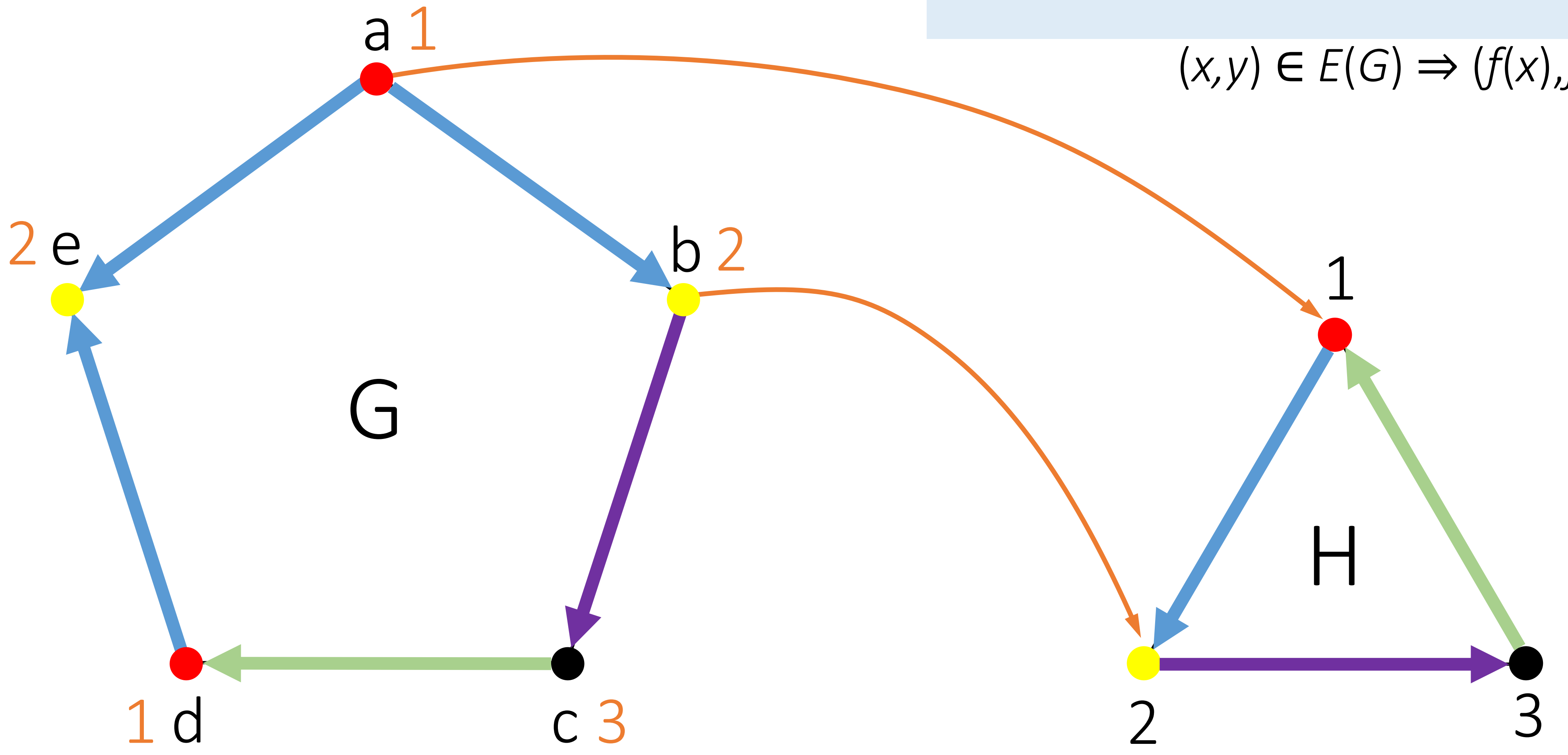
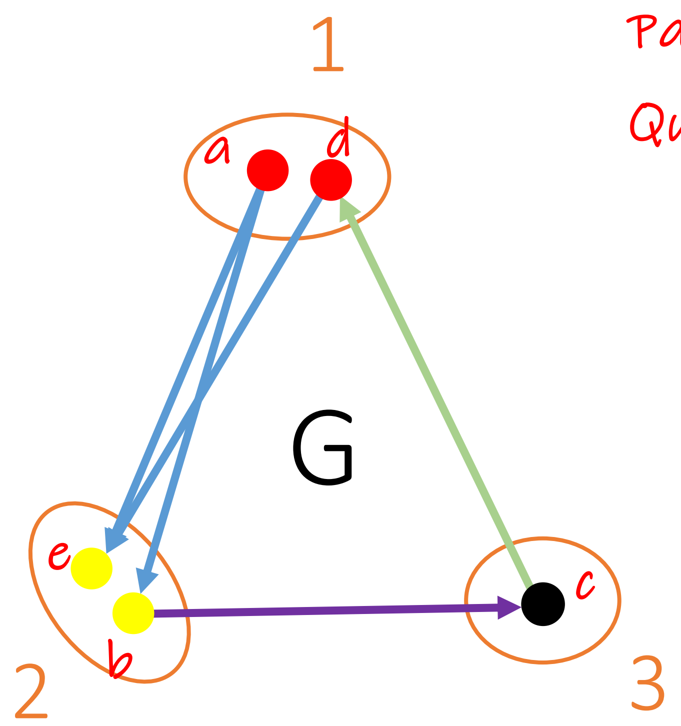


Based upon an example from Rick Brewster's Graph homomorphism tutorial, 2006

An example

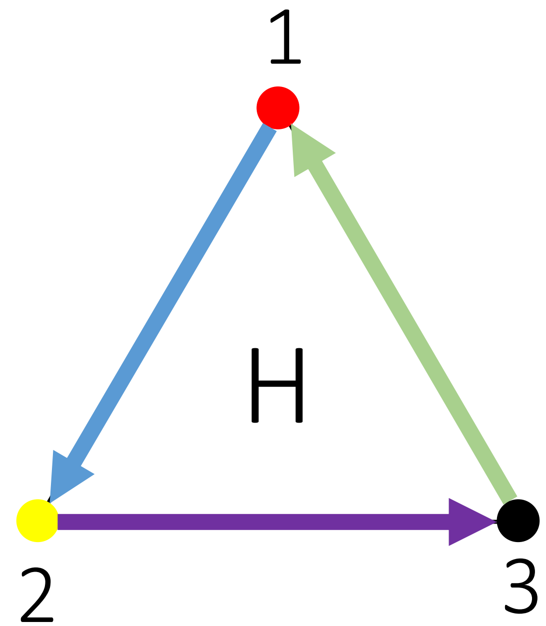
Basically a partitioning problem!

The quotient set of the partition (set of equivalence classes of the partition) is a subgraph of H.



Partition: {{a,d}, {b,e}, {c}}

Quotient set: {[a], [b], [c]}



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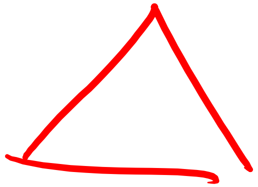
Some observations

When does G → K_3 hold? (K_3 = 3-clique = triangle)

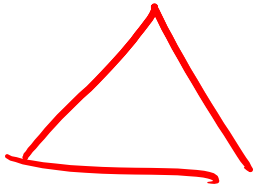


?

Some observations

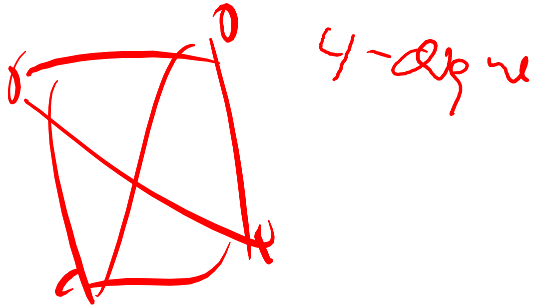
When does G → K_3 hold? (K_3 = 3-clique = triangle)

iff G is 3-colorable



When does G → K_d hold? (K_d = d-clique)

?



More on 3-coloring: https://en.wikipedia.org/wiki/Graph_coloring#Computational_complexity

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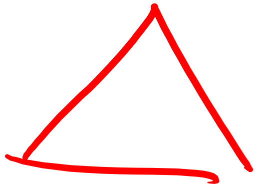

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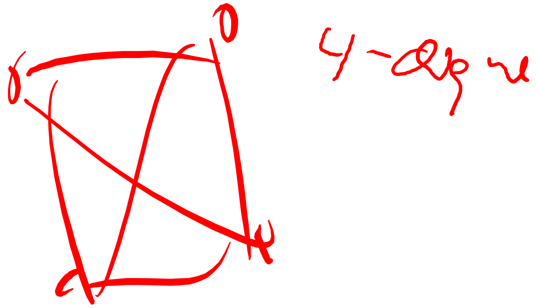

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Thus homomorphisms generalize colorings:

Notation: G → H is an H-coloring of G.

What is the complexity of testing for the existence of a homomorphism (in the size of G)?

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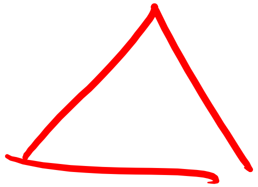

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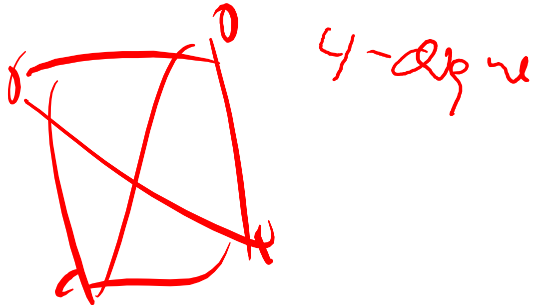

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NP-complete

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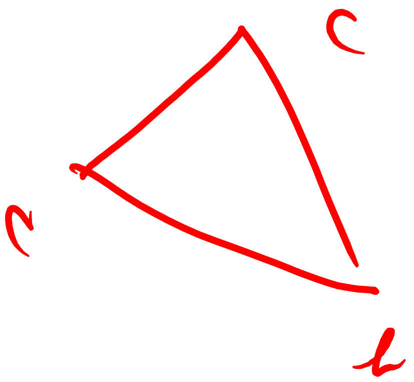
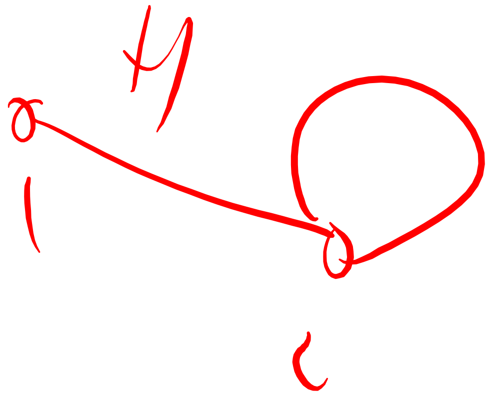
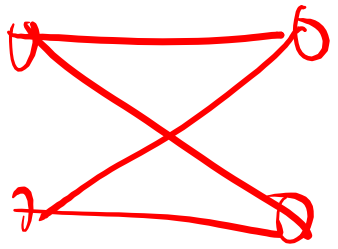
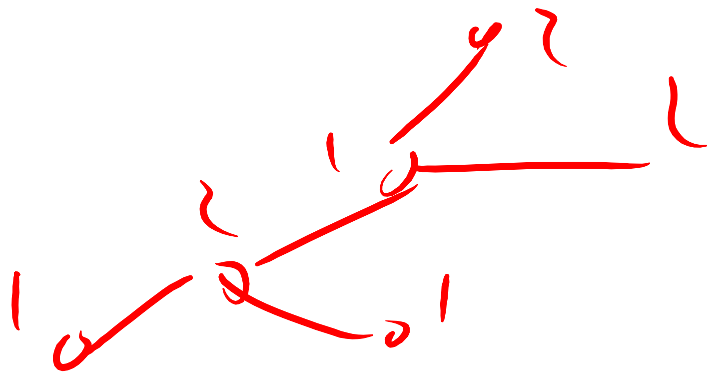
The complexity of H-coloring

H-coloring:
Let H be a fixed graph.
Instance: A graph G.
Question: Does G admit an H-coloring?



Theorem [Hell, Nesetril'90]:
If H is bipartite or contains a self-loop, then H-coloring is polynomial time solvable; otherwise, H is NP-complete.



[Hell, Nesetril'90]: Hell, Nešetřil. On the complexity of H-coloring. Journal of Combinatorial Theory, 1990. https://doi.org/10.1016/0095-8956(90)90132-J

Repeated variable names

In sentences with multiple quantifiers, distinct variables do not need to range over distinct objects! (cp. homomorphism vs. isomorphism)

$\exists x.\exists y.\, E(x,y) \quad \overset{\Rightarrow}{\Leftarrow} \quad \exists x.\, E(x,x)$

?

Which of formulas implies the other?

Repeated variable names

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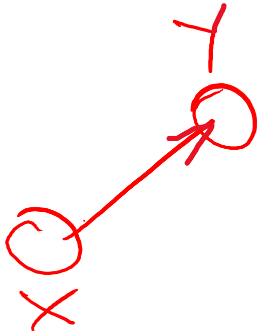
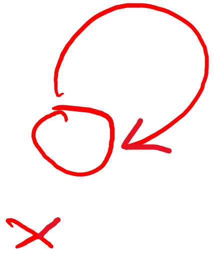


E

s	t
1	2

E

s	t
1	1



A more abstract (general) view on homomorphisms

Homomorphisms on Binary Structures

- **Definition (Binary algebraic structure):** A binary algebraic structure is a set together with a binary operation on it. This is denoted by an ordered pair $(S,\star)$ in which S is a set and $\star$ is a binary operation on S.

- **Definition (homomorphism of binary structures):** Let $(S,\star)$ and $(S',\circ)$ be binary structures. A homomorphism from $(S,\star)$ to $(S',\circ)$ is a map $h: S \longrightarrow S'$ that satisfies, for all x, y in S:

  $$h(x \star y) = h(x) \circ h(y)$$

- We can denote it by $h: (S,\star) \longrightarrow (S',\circ)$.

Example: from addition to multiplication

- Let $h(x) = e^x$. Is h a homomorphism b/w two binary structures?

?

Example: from addition to multiplication

- Let $h(x) = e^x$. Is h a homomorphism b/w two binary structures?
  - Yes, from the real numbers with addition $(\mathbb{R},+)$ to
  - the positive real numbers with multiplication $(\mathbb{R}^+,\cdot)$
  - It is even an isomorphism!

$h(x+y) = h(x) \cdot h(y)$

$h: (\mathbb{R},+) \longrightarrow (\mathbb{R}^+,\cdot)$

The exponential map $\exp : \mathbb{R} \to \mathbb{R}^+$ defined by $\exp(x) = e^x$, where e is the base of the natural logarithm, is an isomorphism from $(\mathbb{R}, +)$ to $(\mathbb{R}^+, \times)$. Exp is a bijection since it has an inverse function (namely $\log_e$) and exp preserves the group operations since $e^{x+y} = e^x e^y$. In this example both the elements and the operations are different yet the two groups are isomorphic, that is, as groups they have identical structures.

- Let $g(x) = e^{ix}$. Is g also a homomorphism?

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Example: from addition to multiplication

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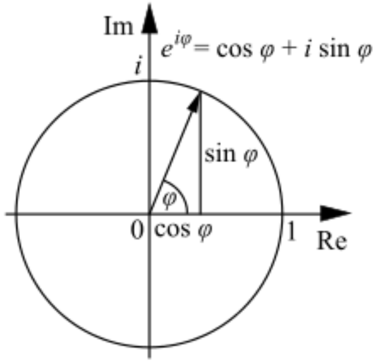
- Let $g(x) = e^{ix}$. Is g also a homomorphism?
  - Yes, from the real numbers with addition ($\mathbb{R}$,+) to
  - the unit circle in the complex plane with rotation



Example: from addition to multiplication

$G = \mathbb{R}$ under $+$

$H = \{ z \in \mathbb{C} : |z| = 1 \}$

$= $ Group under $\times$

Hint:

Every $z \in \mathbb{C}$ with $|z| = 1$ can be written as $z = e^{i\theta}$.

$$f : G \longrightarrow H$$
$$x \longmapsto e^{ix}$$

Show $f(x + y) = f(x) \times f(y)$

$$e^{i(x+y)} = e^{ix} \times e^{iy}$$

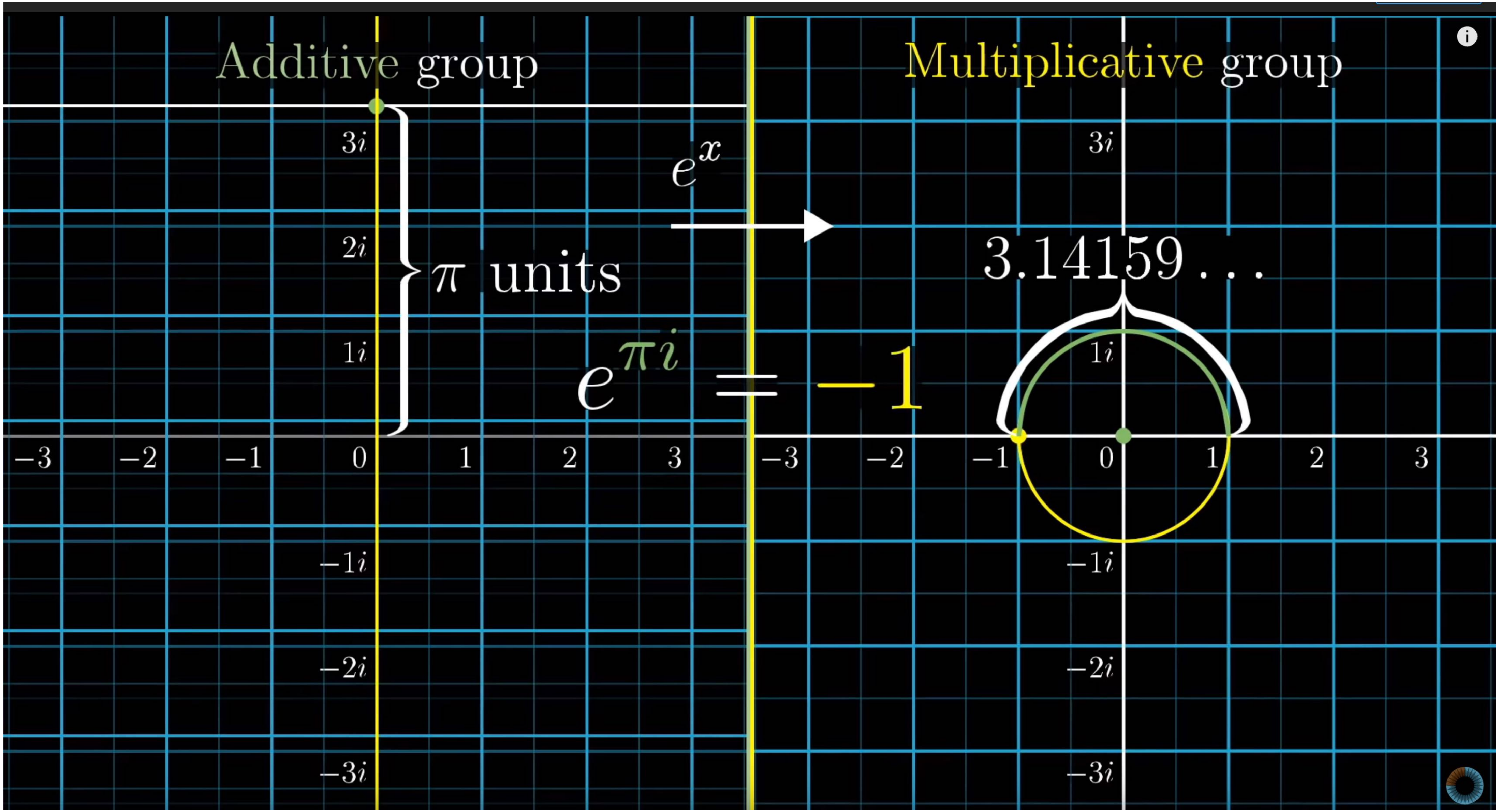
$$e^{ix+iy} = e^{ix} \times e^{iy}$$

$$e^{ix} \times e^{iy} = e^{ix} \times e^{iy}$$

$f(0) = f(2\pi) = 1, \quad f(2\pi n) = 1$

f is not 1-1

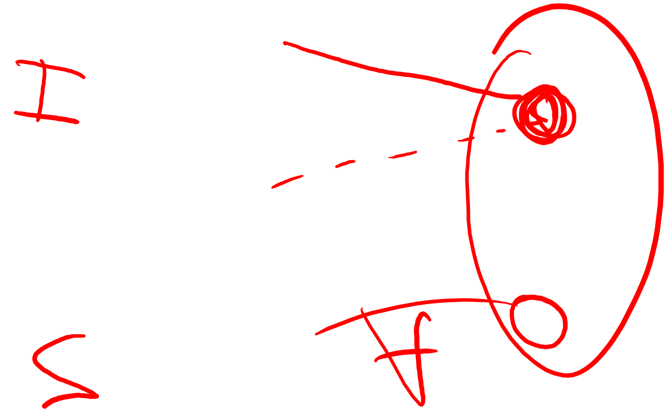
Example: from addition to multiplication



Source: 3blue1brown. Euler's formula with introductory group theory, 2017: https://www.youtube.com/watch?v=mvmuCPvRoWQ

Isomorphism

- **Definition**: A homomorphism of binary structures is called an isomorphism iff the corresponding map of sets is:
  - one-to-one (injective) and
  - onto (surjective).



Some homomorphisms

Binary structure (S,⋆) recall that ⋆ is closed

Change to Binary operator that is not closed and instead maps to $\mathbb{B}$ = {True, False}

Restriction to operations that closed, associative, with identify element, and inverse

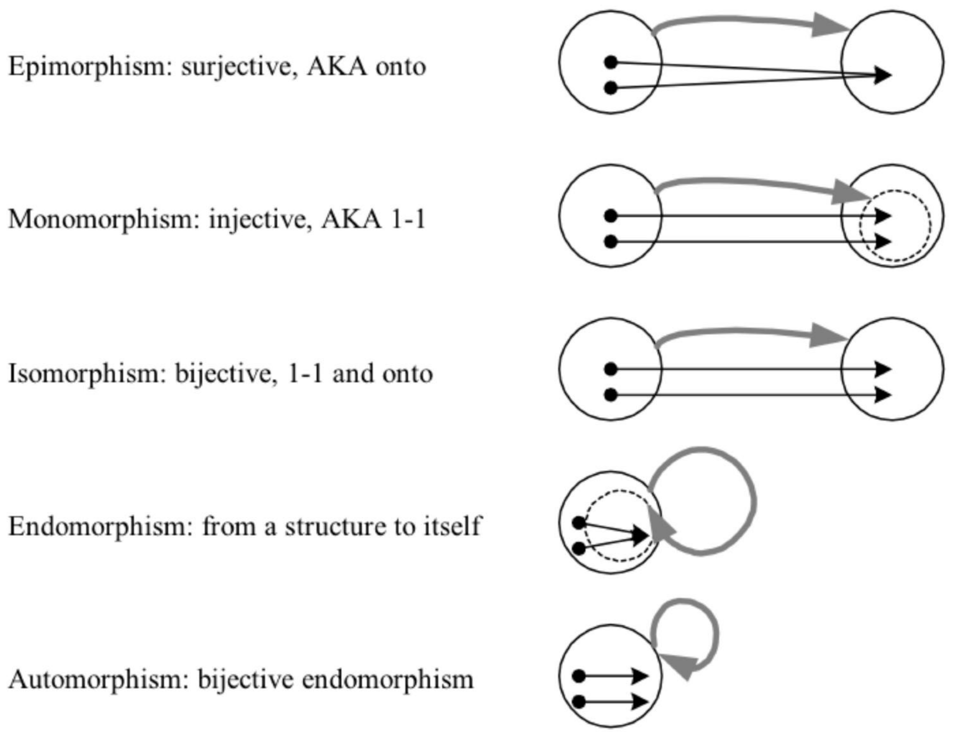
Graph (V, E(x,y))

Group (G,⋆) like ($\mathbb{R}$,+)

Extension to multiple d-ary relations

CQs (Conjunctive Queries)
(Var U Constants, Relations {R_i(x,y,z), ...})

- **Homomorphism**: preserves the structure (e.g. a homomorphism φ on $\mathbb{Z}_2$ satisfies $\varphi(g+h) = \varphi(g) + \varphi(h)$)
- **Epimorphism**: a homomorphism that is **surjective** (AKA onto)
- **Monomorphism**: a homomorphism that is **injective** (AKA one-to-one, 1-1, or univalent)
- **Isomorphism**: a homomorphism that is **bijective** (AKA 1-1 and onto); isomorphic objects are equivalent, but perhaps defined in different ways
- **Endomorphism**: a homomorphism from an object to itself
- **Automorphism**: a bijective endomorphism (an isomorphism from an object onto itself, essentially just a re-labeling of elements)



Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Query Containment

Two queries q_1, q_2 are equivalent, denoted $q_1 \equiv q_2$, if for every database instance D, we have $q_1(D) = q_2(D)$.

the answer (set of tuples) returned by one is guaranteed to be identical to the other answer

Query q_1 is contained in query q_2 , denoted $q_1 \subseteq q_2$, if for every database instance D, we have $q_1(D) \subseteq q_2(D)$

Corollary

$q_1 \equiv q_2$ is equivalent to ($q_1 \subseteq q_2$ and $q_1 \supseteq q_2$)

If queries are Boolean, then query containment = logical implication:

$q_1 \Leftrightarrow q_2$ is equivalent to ?

Query Containment

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<u>Corollary</u>

$q_1 \equiv q_2$ is equivalent to ($q_1 \subseteq q_2$ and $q_1 \supseteq q_2$)

If queries are Boolean, then query containment = logical implication:

$q_1 \Leftrightarrow q_2$ is equivalent to ($q_1 \Rightarrow q_2$ and $q_1 \Leftarrow q_2$)

Query homomorphisms

A homomorphism h from Boolean CQs q_1 to q_2 is a function
h: var(q_1) → var(q_2) ∪ const(q_2) such that:

for every atom $R(x_1,x_2,\ldots)$ in q_1, there is an atom $R(h(x_1), h(x_2), \ldots)$ in q_2

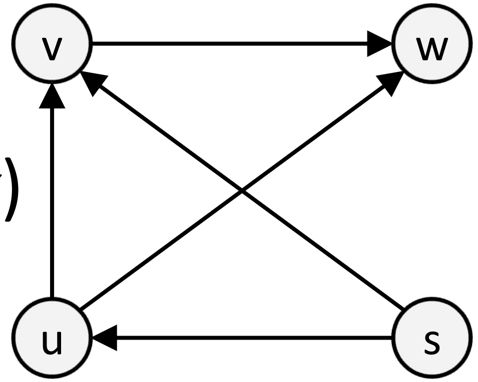
need to be same relation!

Example

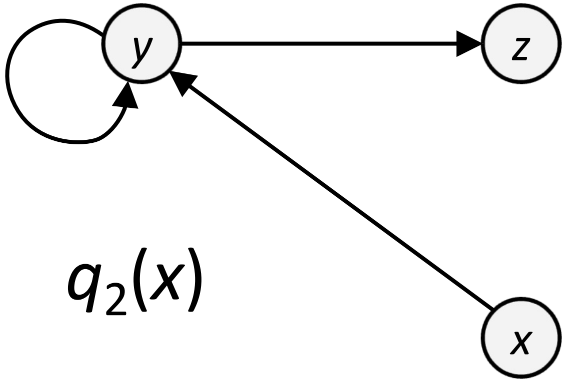
q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$

q_2 :- $R(x,y), R(y,y), R(y,z)$

$q_1(x)$



$h_{1\to 2}=$?



$q_2(x)$

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<u>Example</u>

q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$
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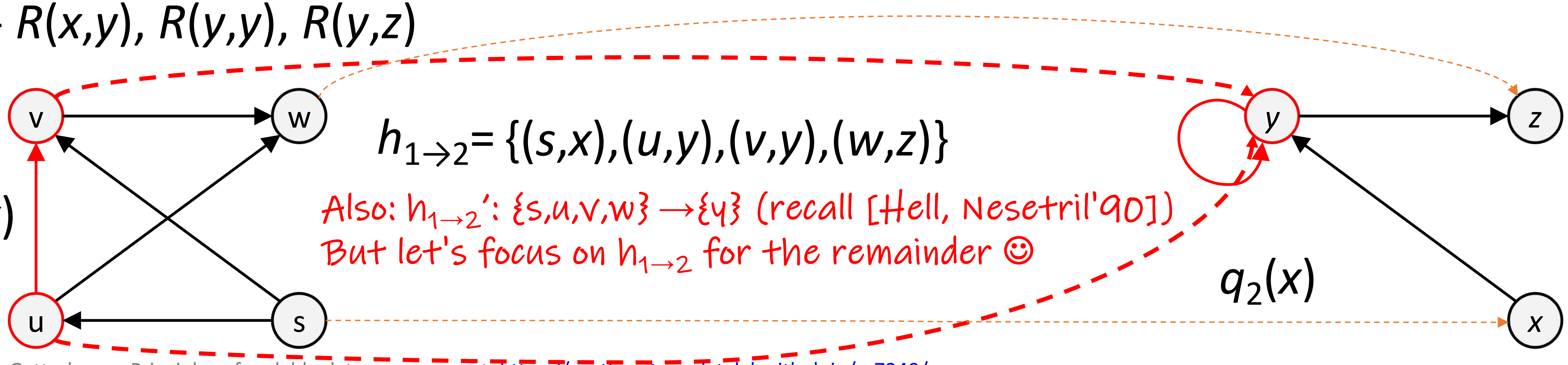

$h_{1\to2}$= {(s,x),(u,y),(v,y),(w,z)}

Also: $h_{1\to2}'$: {s,u,v,w} →{y} (recall [Hell, Nesetril'90])
But let's focus on $h_{1\to2}$ for the remainder ☺

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Example

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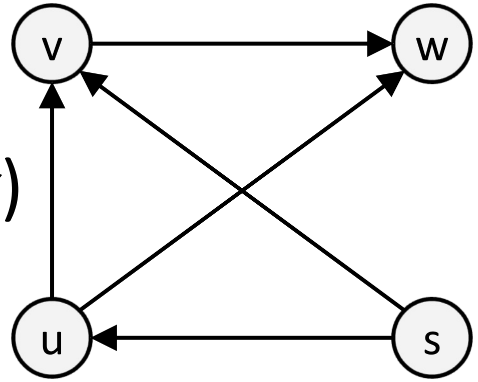
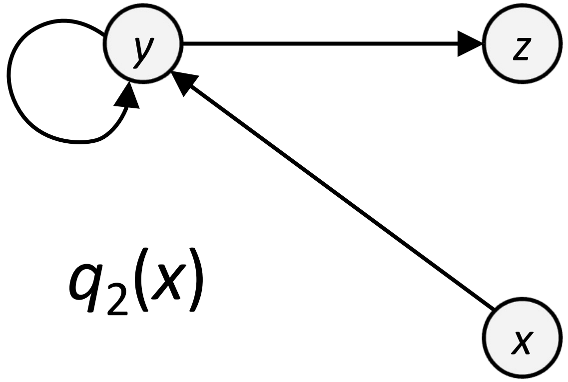


$h_{1\rightarrow 2} = \{(s,x),(u,y),(v,y),(w,z)\}$

$h_{2\rightarrow 1}$: ?



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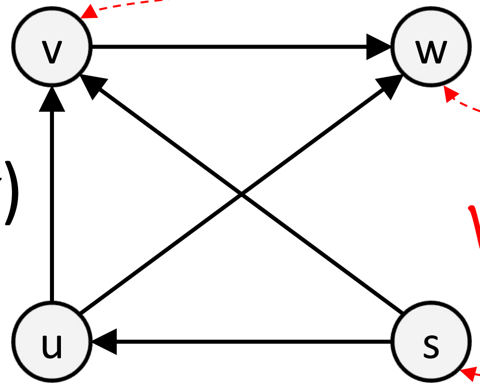
Example

q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$

q_2 :- $R(x,y), R(y,y), R(y,z)$

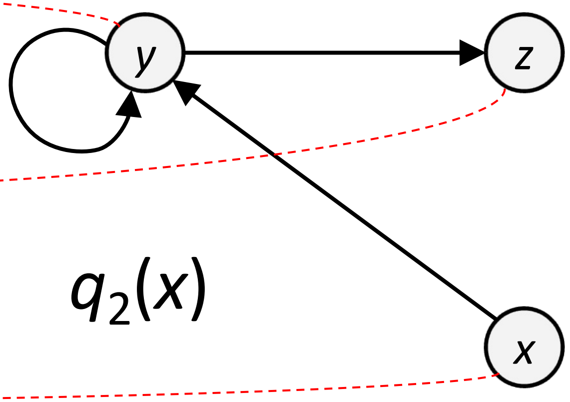
$q_1(x)$



$h_{1\rightarrow 2}$= {(s,x),(u,y),(v,y),(w,z)}

What about:

$h_{2\rightarrow 1}$: {(x,s),(y,v),(z,w)} ?



$q_2(x)$

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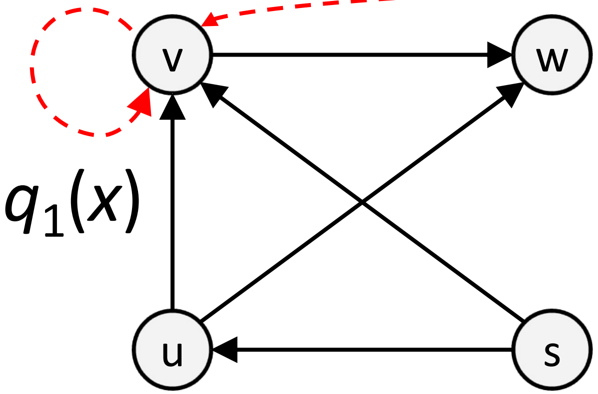
<u>Example</u>

q_1 :- $R(s,u)$, $R(u,w)$, $R(s,v)$, $R(v,w)$, $R(u,v)$, ~~R(v,v)~~

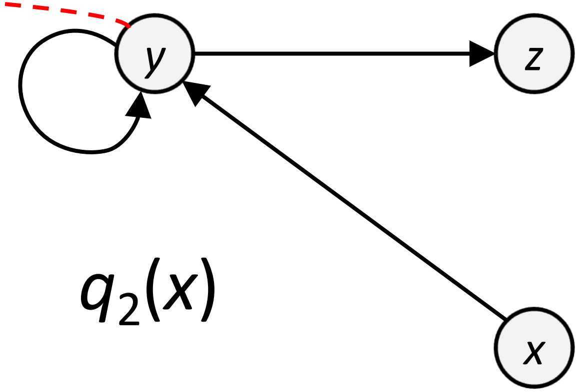
q_2 :- $R(x,y)$, $R(y,y)$, $R(y,z)$



$q_1(x)$

$h_{1\to2}$= {(s,x),(u,y),(v,y),(w,z)}

$h_{2\to1}$: ~~{(x,s),(y,v),(z,w)}~~



$q_2(x)$

Query homomorphisms and containment

A homomorphism *h* from Boolean CQs q_1 to q_2 is a function
h: var(q_1) → var(q_2) ∪ const(q_2) such that:
for every atom $R(x_1,x_2,\ldots)$ in q_1, there is an atom $R(h(x_1), h(x_2), \ldots)$ in q_2

E(1,2)

Compare to our earlier example:
$\exists x.\exists y.\ E(x,y) \rightleftarrows \exists x.\ E(x,x)$

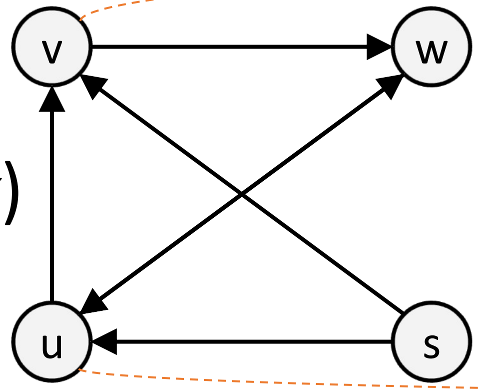
E(1,1)

?

Example

q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$

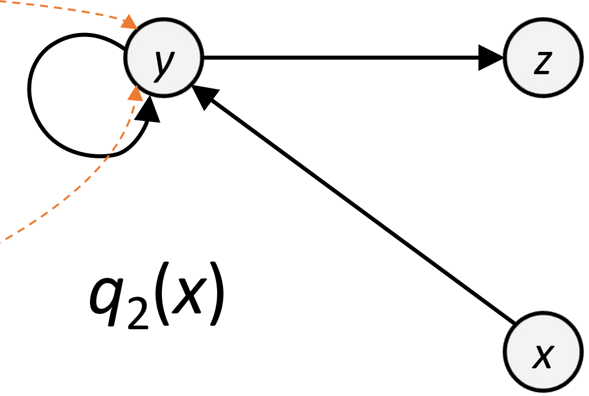
q_2 :- $R(x,y), R(y,y), R(y,z)$



$q_1(x)$

$h_{1\to 2}= \{(s,x),(u,y),(v,y),(w,z)\}$

~~$h_{2\to 1}$: {(x,s),(y,v),(z,w)}~~



$q_2(x)$

Query homomorphisms and containment

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for every atom $R(x_1,x_2,\ldots)$ in q_1, there is an atom $R(h(x_1), h(x_2), \ldots)$ in q_2

E(1,2)
True

Compare to our earlier example:

$\exists x.\exists y.\, E(x,y) \Leftarrow \exists x.\, E(x,x)$

E(1,1)
False

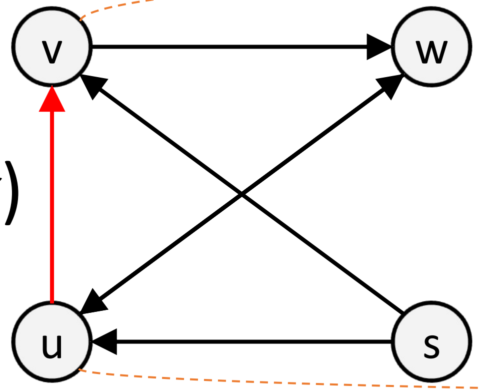
<u>Example</u>

q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$

q_2 :- $R(x,y), R(y,y), R(y,z)$

We will use homomorphisms to reason about query containment. We try to understand the direction

$q_1(x)$

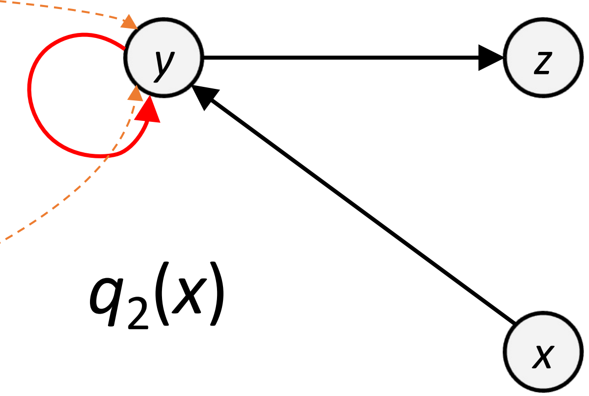


$h_{1\to2}$ = {(s,x),(u,y),(v,y),(w,z)}

$q_1 \Leftarrow q_2$

$q_1 \not\Rightarrow q_2$

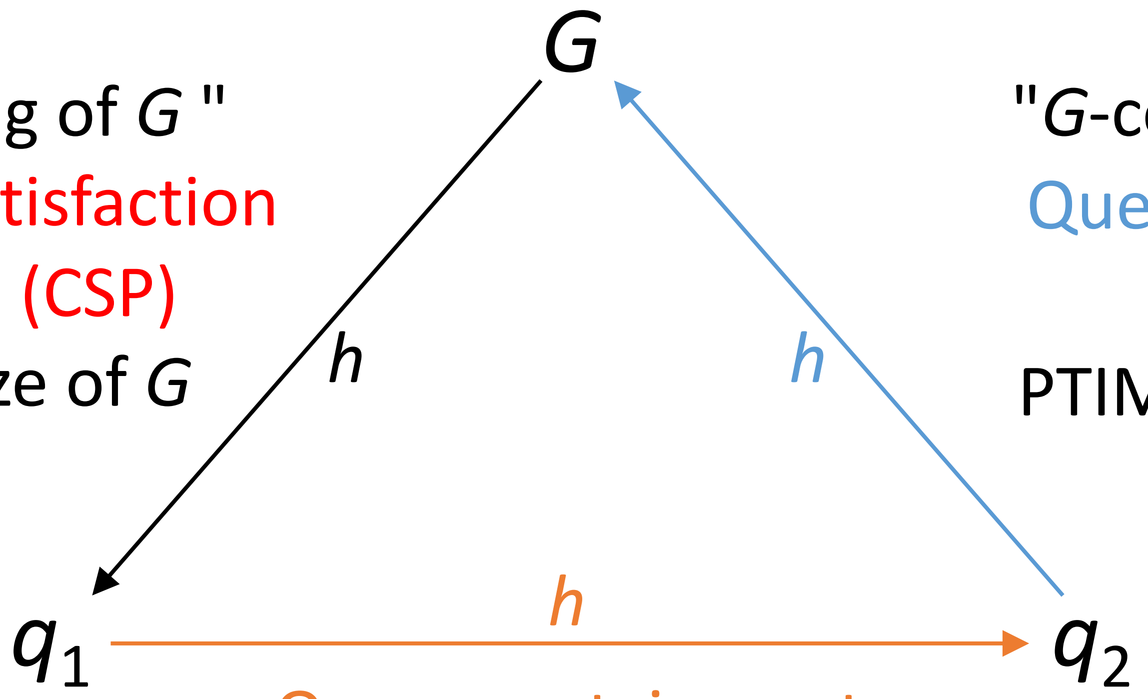
$h_{2\to1}$: ~~{(x,s),(y,v),(z,w)}~~



$q_2(x)$

Overview: "All homomorphisms" in one slide

"q_1-coloring of G"
Constraint Satisfaction Problems (CSP)
NP-C in size of G



"G-coloring of q_2"
Query evaluation
$G \models q_2$
PTIME in size of G

Query containment
$q_1 \supseteq q_2$
$q_1 \Leftarrow q_2$
"q_2-coloring of q_1" ← thus NP-C in size of q_1

Islands of Tractability of CQ Evaluation

- Major Research Program: Identify tractable cases of the combined complexity of conjunctive query evaluation.
- Over the years, this program has been pursued by two different research communities:
  - The Database Theory community
  - The Constraint Satisfaction community
- Explanation: Problems in those community are closely related:

Constraint Satisfaction Problem ≡ Homomorphism Problem ≡ CQ evaluation

[Feder, Vardi 1993] [Chandra, Merlin 1977]

[Kolaitis, Vardi 2000]

Feder, Vardi: Monotone monadic SNP and constraint satisfaction, STOC 1993 https://doi.org/10.1145/167088.167245 / Kolaitis, Vardi: Conjunctive-Query Containment and Constraint Satisfaction, JCSS 2000 https://doi.org/10.1006/jcss.2000.1713 / Chandra, Merlin. "Optimal implementation of conjunctive queries in relational data bases", STOC 1977. https://doi.org/10.1145/800105.803397
Based on Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

Topic 2: Complexity of Query Evaluation
Unit 1: Conjunctive Queries
Lecture 15

Wolfgang Gatterbauer

CS7240 Principles of scalable data management (sp24)

https://northeastern-datalab.github.io/cs7240/sp24/

3/12/2024

Pre-class conversations

- Last class summary
- Scribes & Projects: I hope you find the comments useful
  - If you ever have questions, please ask me after class -> discussion
  - 50% of class is over, <15% of scribes submitted

- Today:
  - Homomorphisms, Query Containment
  - Equivalence beyond CQs

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Canonical database

DEFINITION Canonical database

Given a conjunctive query q, the canonical database $D_c[q]$ is the database instance where each atom in q becomes a fact in the database instance.

<u>Example</u>

$q_2(x)$:- $R(x,y)$, $R(y,y)$, $R(y,z)$

$D_c[q_2]$ = ?

Canonical database

DEFINITION Canonical database

Given a conjunctive query q, the canonical database $D_c[q]$ is the database instance where each atom in q becomes a fact in the database instance.

<u>Example</u>

$q_2(x)$:- $R(x,y)$, $R(y,y)$, $R(y,z)$

$D_c[q_2]$ = {R('x','y'), R('y','y'), R('y','z')}

≡ {R(1,2), R(2,2), R(2,3)}

≡ {R(a,b), R(b,b), R(b,c)}

Var		Const
x	→	a
y	→	b
z	→	c

R

A	B
a	b
b	b
b	c

Just treat each variable as different constant ☺

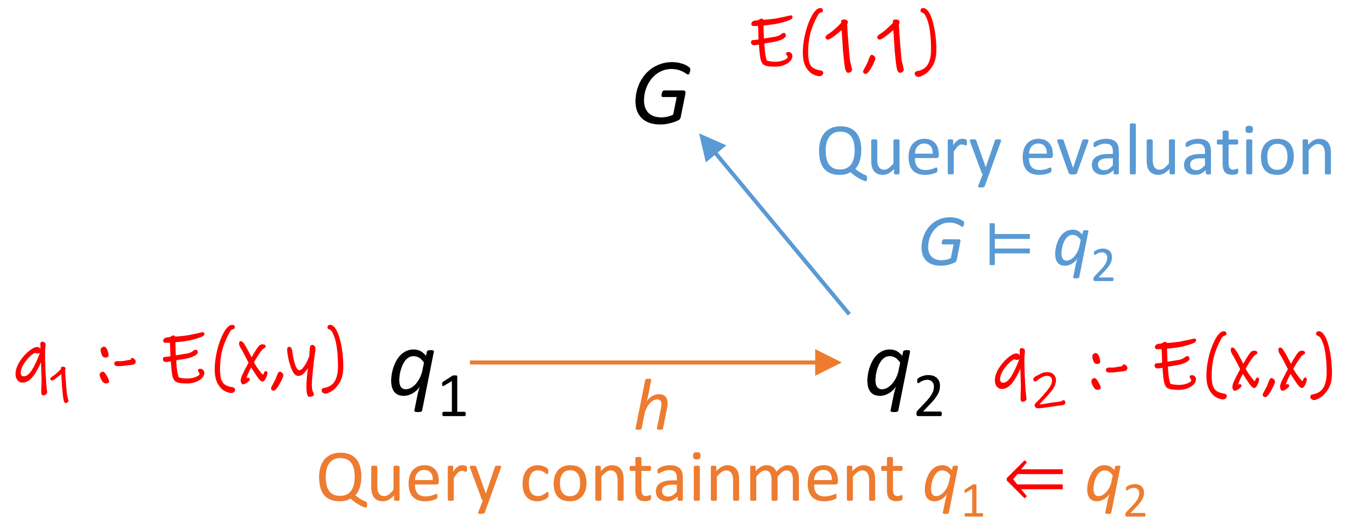
[Chandra and Merlin 1977]

THEOREM (Query Containment)

Given two Boolean CQs q_1, q_2, the following statements are equivalent:

1) $q_1 \Leftarrow q_2$ $\quad$ $(q_1 \supseteq q_2)$ $\quad$ (q_2 is contained in q_1)
2) There is a homomorphism $h_{1\rightarrow 2}$ from q_1 to q_2
3) $q_1(D_C[q_2])$ is true

We will look at 2) ⇒ 1),
and it is similar to 2) ⇒ 3)



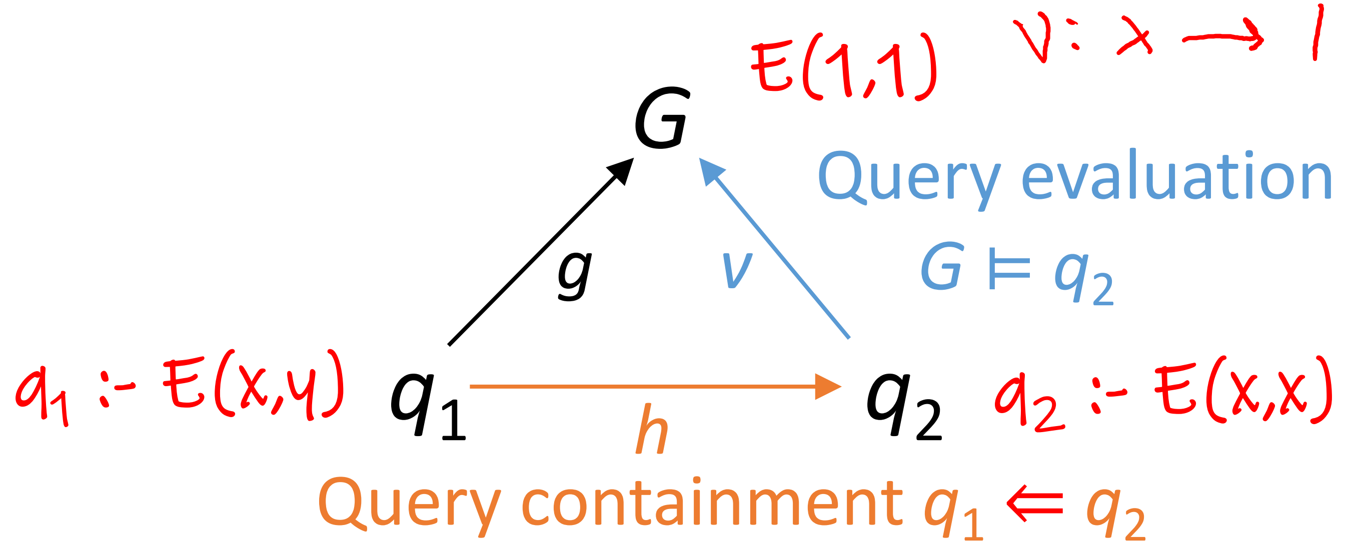
[Chandra and Merlin 1977]

We show: If there is a homomorphism $h_{1\to 2}$, then for any D: $q_1(D) \Leftarrow q_2(D)$

1. For $q_2(D)$ to hold, there is a valuation v s.t. $v(q_2) \in D$
2. We will show that the composition $g = v \circ h$ is a valuation for q_1

$g = v \circ h$

$g(x) = v(h(x))$



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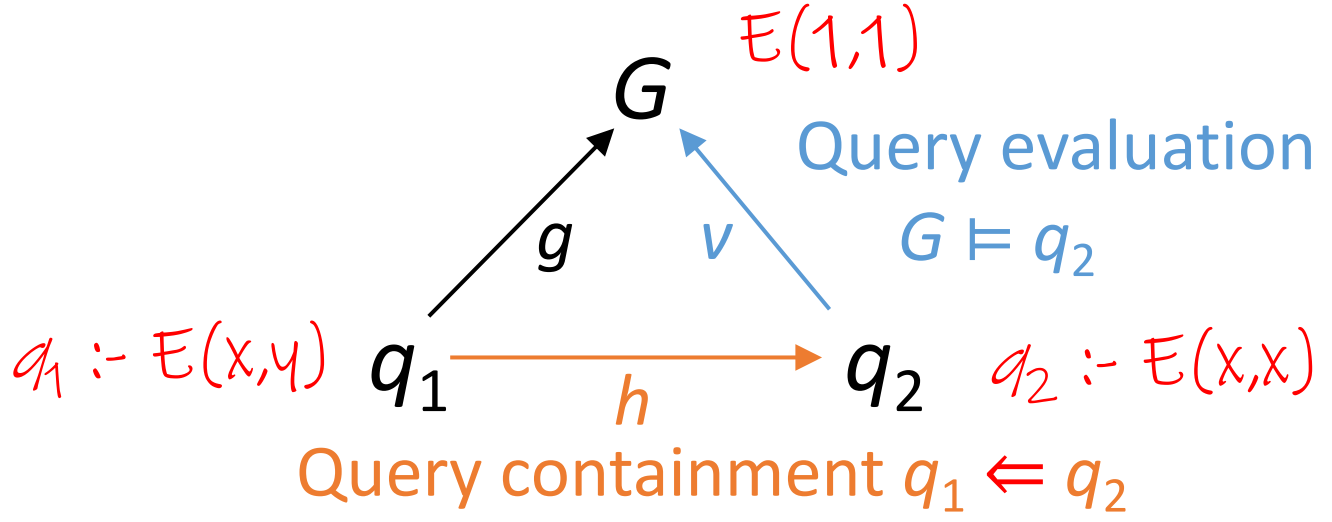
    2a. By definition of h, for every $R(x_1,x_2,\ldots)$ in q_1, $R(h(x_1),h(x_2),\ldots)$ in q_2
    2b. By definition of v, for every $R(x_1,x_2,\ldots)$ in q_1, $R(v(h(x_1)),v(h(x_2)),\ldots)$ in D

$g=v \circ h$
$g(x)=v(h(x))$

QED ☺



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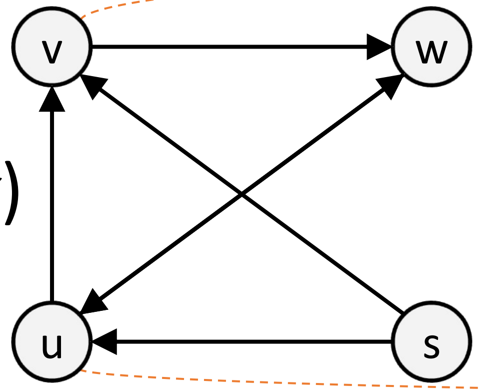
$g = v \circ h$
$g(x) = v(h(x))$

<u>Example</u>

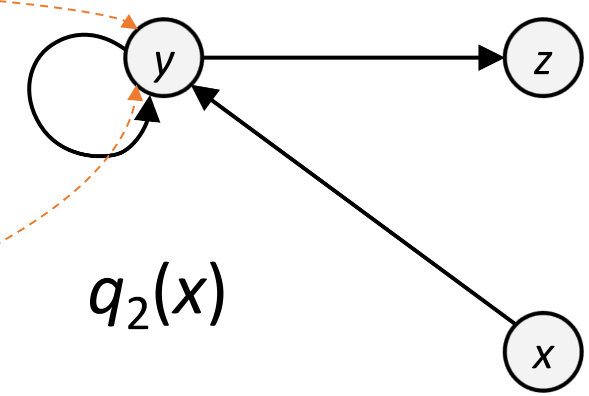
q_1 :- $R(s,u)$, $R(u,w)$, $R(s,v)$, $R(v,w)$, $R(u,v)$

q_2 :- $R(x,y)$, $R(y,y)$, $R(y,z)$

$q_1(x)$



$h_{1 \to 2} = \{(s,x),(u,y),(v,y),(w,z)\}$



$q_2(x)$

[Chandra and Merlin 1977]

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$g=v \circ h$
$g(x)=v(h(x))$

Example

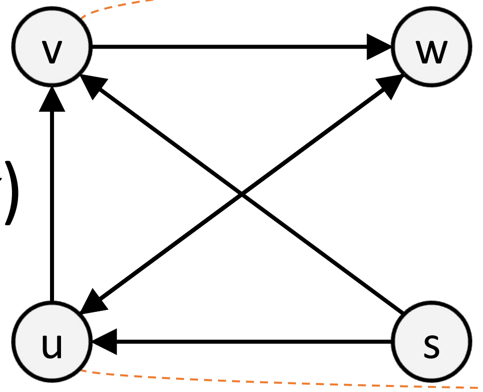
q_1 :- $R(s,u), R(u,w), R(s,v), R(v,w), R(u,v)$

q_2 :- $R(x,y), R(y,y), R(y,z)$

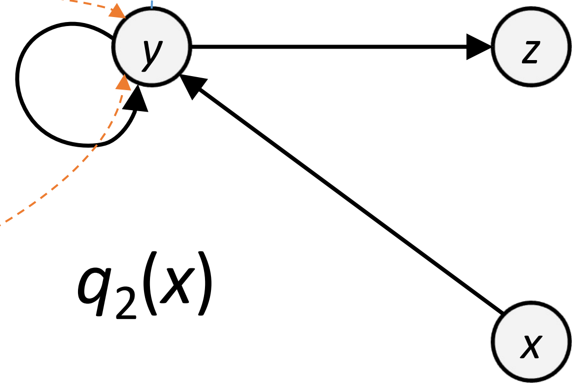
$v=\{(x,a),(y,b),(z,c)\}$

R: A	B
a	b
b	b
b	c

$h_{1\to 2}=\{(s,x),(u,y),(v,y),(w,z)\}$



$q_1(x)$



$q_2(x)$

[Chandra and Merlin 1977]

We show: If there is a homomorphism $h_{1\to 2}$, then for any D: q_1(D) ⇐ q_2(D)

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Example

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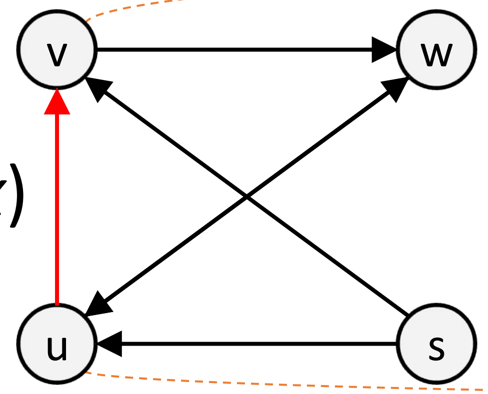
q_2 :- $R(x,y)$, $R(y,y)$, $R(y,z)$

v={(x,a),(y,b),(z,c)}

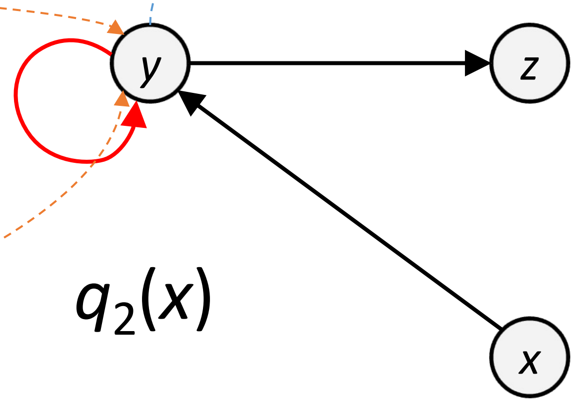
R A	B
a	b
b	b
b	c



$q_1(x)$

$h_{1\to 2}$= {(s,x),(u,y),(v,y),(w,z)}

g= {(s,a),(u,b),(v,b),(w,c)}



$q_2(x)$

Combined complexity of CQC and CQE

Corollary:

The following problems are NP-complete (in the size of Q or Q'):

1) Given two (Boolean) conjunctive queries Q and Q', is $Q \subseteq Q'$?

2) Given a Boolean conjunctive query Q and an instance D, does $D \models Q$?

Proof:

(a) Membership in NP follows from the Homomophism Theorem:

$Q \subseteq Q'$ if and only if there is a homomorphism $h: Q' \rightarrow Q$

(b) NP-hardness follows from 3-Colorability:

G is 3-colorable if and only if $Q^{K_3} \subseteq Q^{G}$.

The Complexity of Database Query Languages

	Relational Calculus	CQs
Query Eval.: Data Complexity	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)
Query Eval.: Combined Compl.	PSPACE-complete	NP-complete
Query Equivalence & Containment	Undecidable	NP-complete

Based on Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

Exercise: Find Homomorphisms

q_1: {E(x,y),E(y,z),E(z,w)}

Order of subgoals in the query does not matter (thus written here as sets)

q_2: {E(x,y),E(y,z),E(z,x)}

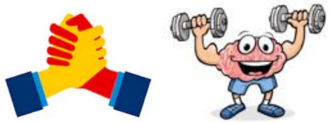
q_3: {E(x,y),E(y,x)}

What is the containment relation between these queries?

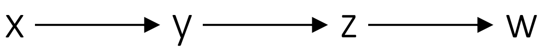
q_4: {E(x,y),E(y,x),E(y,y)}

q_5: {E(x,x)}

Exercise: Find the Homomorphisms



q_1: {E(x,y),E(y,z),E(z,w)}



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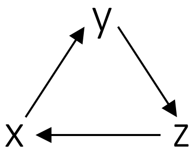
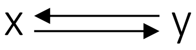
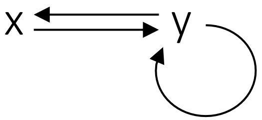


q_3: {E(x,y),E(y,x)}

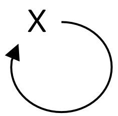


What is the containment relation between these queries?

q_4: {E(x,y),E(y,x),E(y,y)}



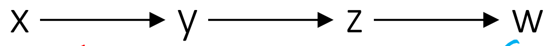
q_5: {E(x,x)}



Exercise: Find the Homomorphisms

Q

q_1: {E(x,y),E(y,z),E(z,w)}

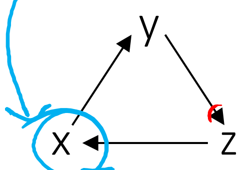


{x⟶x, y⟶y, z⟶z, w⟶x}

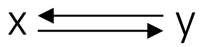
or {x⟶y, y⟶z, z⟶x, w⟶y}, etc.

q_2: {E(x,y),E(y,z),E(z,x)}

D

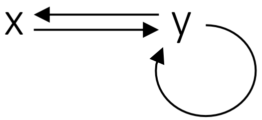


q_3: {E(x,y),E(y,x)}

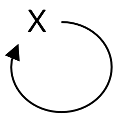


What is the containment relation between these queries?

q_4: {E(x,y),E(y,x),E(y,y)}



q_5: {E(x,x)}



Exercise: Find the Homomorphisms

Q

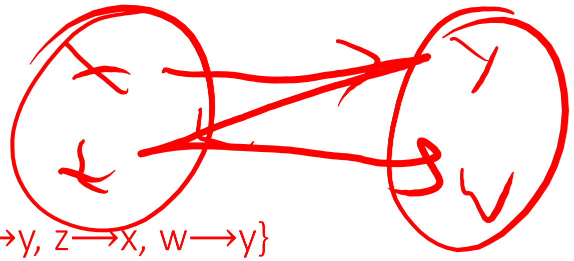
q_1: {E(x,y),E(y,z),E(z,w)}

x → y → z → w

{x⟶x, y⟶y, z⟶z, w⟶x}

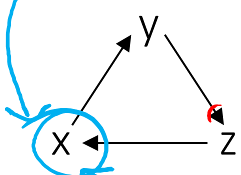
or {x⟶y, y⟶z, z⟶x, w⟶y}, etc.

{x⟶x, y⟶y, z⟶x, w⟶y}

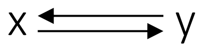


q_2: {E(x,y),E(y,z),E(z,x)}

D



q_3: {E(x,y),E(y,x)}



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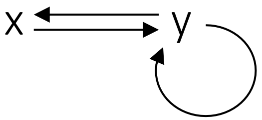
q_4: {E(x,y),E(y,x),E(y,y)}



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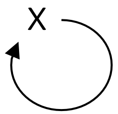
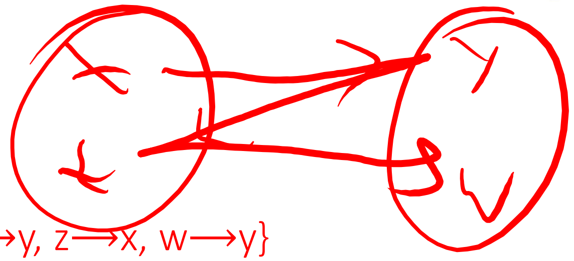


Exercise: Find the Homomorphisms

Q

q_1: {E(x,y),E(y,z),E(z,w)}

x ⟶ y ⟶ z ⟶ w



{x⟶x, y⟶y, z⟶z, w⟶x}

or {x⟶y, y⟶z, z⟶x, w⟶y}, etc.

⊆ ⇒ ⊇ ⇐

{x⟶x, y⟶y, z⟶x, w⟶y}

q_2: {E(x,y),E(y,z),E(z,x)}

D

x ⟶ y ⟶ z ⟶ x

q_3: {E(x,y),E(y,x)}

x ⇄ y

What is the containment relation between these queries?

q_4: {E(x,y),E(y,x),E(y,y)}

x ⇄ y, y ⟶ y

{x⟶y}

{x⟶x, y⟶x}

≡

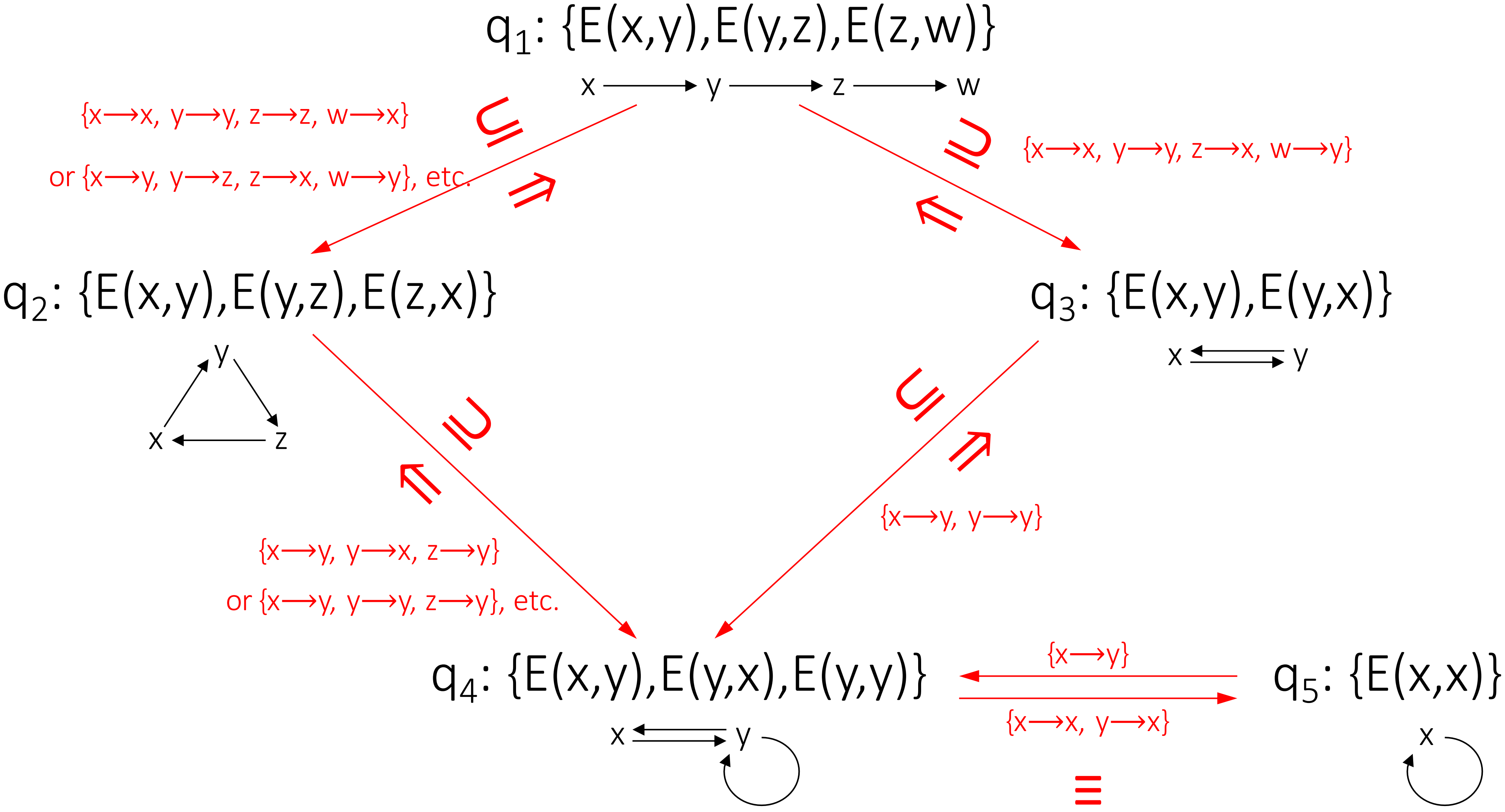
q_5: {E(x,x)}

x ⟶ x

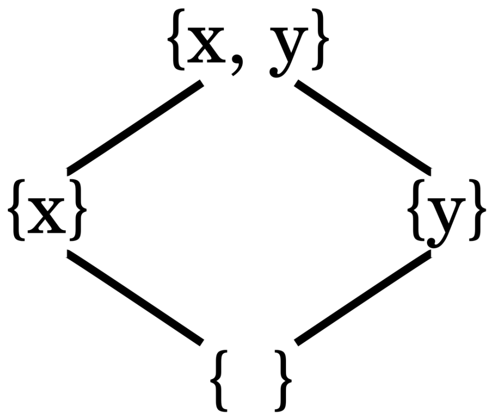
Example by Andreas Pieris: https://homepages.inf.ed.ac.uk/apieris/courses/atfd2020/index.html

Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

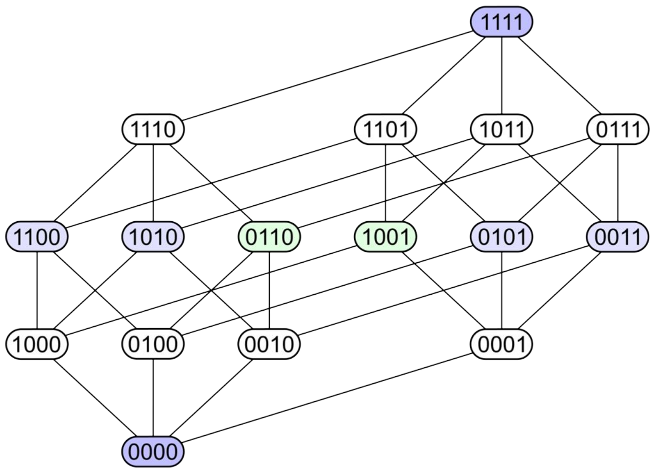
Exercise: Find the Homomorphisms



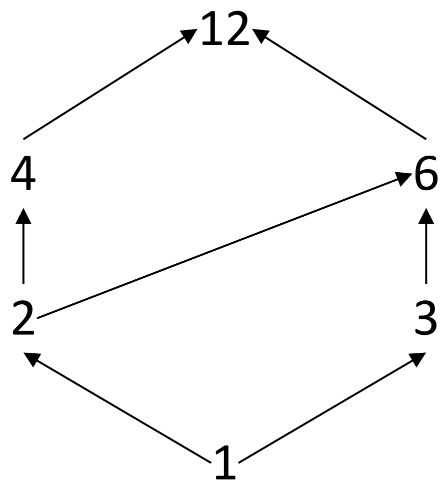
Side-topic: Hasse diagram



The power set of a 2-element set ordered by inclusion



Power set of a 4-element set ordered by inclusion ⊆



Positive integers divisors of 12 ordered by divisibility

Sources of two left figures: https://en.wikipedia.org/wiki/Hasse_diagram

Query Homomorphism Practice

$q_1(x,y)$:- $R(x,u),R(v,u),R(v,y)$ $\quad$ $var(q_1) = \{x, u, v, y\}$

$q_2(x,y)$:- $R(x,u),R(v,u),R(v,w),R(t,w),R(t,y)$ $\quad$ $var(q_2) = \{x, u, v, w, t, y\}$

Are these queries equivalent?

Query Homomorphism Practice


$q_1(x,y)$:- $R(x,u), R(v,u), R(v,y)$

$q_2(x,y)$:- $R(x,u), R(v,u), R(v,w), R(t,w), R(t,y)$

$var(q_1) = \{x, u, v, y\}$

$var(q_2) = \{x, u, v, w, t, y\}$

$q_1 \longrightarrow q_2$ Thus **?**

Which query contains the other?

Query Homomorphism Practice


$q_1(x,y)$:- $R(x,u), R(v,u), R(v,y)$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$q_2(x,y)$:- $R(x,u), R(v,u), R(v,w), R(t,w), R(t,y)$

$var(q_1) = \{x, u, v, y\}$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$var(q_2) = \{x, u, v, w, t, y\}$

$q_1 \longrightarrow q_2$ Thus $q_1 \supseteq q_2$!

Query Homomorphism Practice


$q_1(x,y)$:- $R(x,u),R(v,u),R(v,y)$ $\quad$ $var(q_1) = \{x, u, v, y\}$

$q_2(x,y)$:- $R(x,u),R(v,u),R(v,w),R(t,w),R(t,y)$ $\quad$ $var(q_2) = \{x, u, v, w, t, y\}$

Is there any homomorphism $\quad q_1 \longleftarrow q_2$? $\quad$ Thus $q_1 \subseteq q_2$

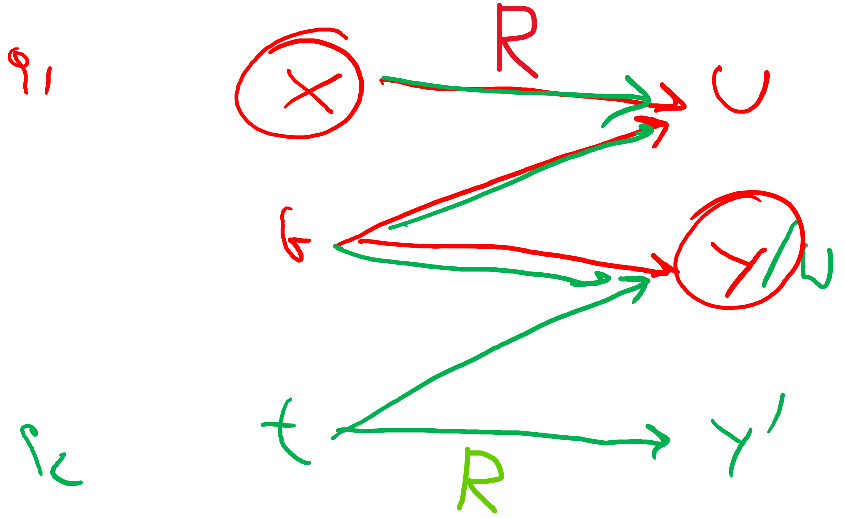
Query Homomorphism Practice

$q_1(x,y)$:- $R(x,u), R(v,u), R(v,y)$

$q_2(x,y)$:- $R(x,u), R(v,u), R(v,w), R(t,w), R(t,y)$

$var(q_1) = \{x, u, v, y\}$

$var(q_2) = \{x, u, v, w, t, y\}$



$q_1 \longleftarrow q_2$ Thus $q_1 \subseteq q_2$

Query Homomorphism Practice

$q_1(x,y)$:- $R(x,u), R(v,u), R(v,y)$

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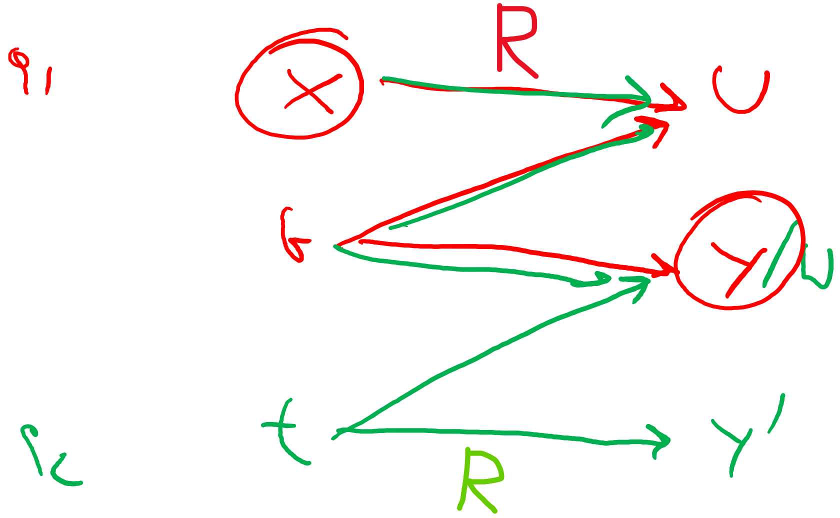


$q_1 \longrightarrow q_2$ Thus $q_1 \supseteq q_2$

$q_1 \longleftarrow q_2$ Thus $q_1 \subseteq q_2$

Thus equivalent!

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

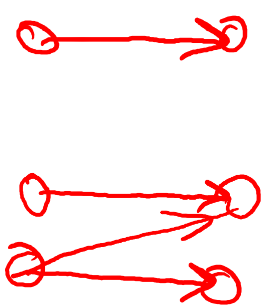
Minimizing Conjunctive Queries

- Goal: minimize the number of joins in a query
- Definition: A conjunctive query Q is minimal if...

?

Minimizing Conjunctive Queries

- Goal: minimize the number of joins in a query
- Definition: A conjunctive query Q is minimal if there is no other conjunctive query Q' such that:
    1. Q ≡ Q'
    2. Q' has fewer atoms than Q



- The task of CQ minimization is, given a conjunctive query Q, to compute a minimal one that is equivalent to Q

Minimizing Conjunctive Queries (CQs) by Deletion

THEOREM: Given a CQ $Q_1(\mathbf{x})$:- $body_1$ that is logically equivalent to a CQ $Q_2(\mathbf{x})$:- $body_2$ where $|body_1| > |body_2|$ (in number of atoms). Then Q_1 is equivalent to a CQ $Q_3(\mathbf{x})$:- $body_3$ s.t. $body_1 \supseteq body_3$

Intuitively, the above theorem states that to minimize a CQ, we simply need to remove some atoms from its body

Conjunctive query minimization algorithm

Minimize(Q(**x**) :- body)

Repeat {

- Choose an atom α ∈ body; let Q' be the new query after removing α from Q

until no atom can be removed}

Q :-E(x,y), E(z,y)

Q':-E(x,y)

1. We trivially know Q⟵Q' (Thus: Q⊆Q')

Conjunctive query minimization algorithm

Notice: the order in which we inspect subgoals doesn't matter

Minimize(Q(**x**) :- body)

Repeat {
- Choose an atom α ∈ body; let Q' be the new query after removing α from Q
- If there is a homomorphism from Q to Q', then body := body \ {α}

until no atom can be removed}

Q :-E(x,y), E(z,y)
Q':-E(x,y)

1. We trivially know Q⟵Q' (Thus: Q⊆Q')

2. This forward direction is non-trivial: Q⟶ Q' (Thus: Q⊇Q')

Minimization Procedure: Example

a,b,c,d are constants

Q(x) :- R(x,y), R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

Minimization Procedure: Example

a,b,c,d are constants

Q(x) :- R(x,y), R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

↑ ↑ ↑ ↑ ↑ ↑ trivial direction

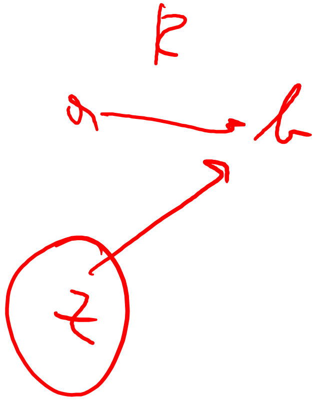
Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

Minimization Procedure: Example

a,b,c,d are constants

Q(x) :- R(x,y), R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

{y⟶'b'}



Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

Is this query minimal ?

Minimization Procedure: Example

a,b,c,d are constants

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{y⟶'b'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

{v⟶'c'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), S('a','c','d')

Is this query minimal ?

Based upon an example by Andreas Pieris: https://homepages.inf.ed.ac.uk/apieris/courses/atfd2020/index.html

Minimization Procedure: Example

a,b,c,d are constants

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{y⟶'b'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

{v⟶'c'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), S('a','c','d')

{x⟶'a'}

Q('a') :- R('a','b'), R(u,'c'), S('a','c','d')

Is this query minimal ?

Based upon an example by Andreas Pieris: https://homepages.inf.ed.ac.uk/apieris/courses/atfd2020/index.html
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Minimization Procedure: Example

a,b,c,d are constants

Q(x) :- R(x,y), R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

{y⟶'b'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), R(u,v), S('a','c','d')

{v⟶'c'}

Q(x) :- R(x,'b'), R('a','b'), R(u,'c'), S('a','c','d') *Minimal query*

{x⟶'a'}

~~Q('a') :- R('a','b'), R(u,'c'), S('a','c','d')~~

Actually, we went too far: Mapping x⟶'a' is not valid since x is a head variable!

Uniqueness of Minimal Queries

Natural question: does the order in which we remove atoms from the body of the conjunctive query during minimization matter?

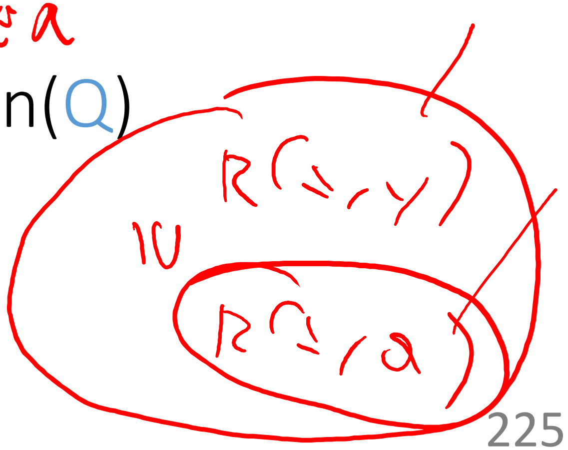
?

Uniqueness of Minimal Queries

Natural question: does the order in which we remove atoms from the body of the conjunctive query during minimization matter?

THEOREM: Consider a conjunctive query Q. Let Q_1 and Q_2 be minimal conjunctive queries such that $Q_1 \equiv Q$ and $Q_2 \equiv Q$. Then, Q_1 and Q_2 are isomorphic (i.e., they are the same up to variable renaming)

Therefore, given a conjunctive query Q, the result of Minimization(Q) is unique (up to variable renaming) and is called the core of Q.



Query Minimization for Views

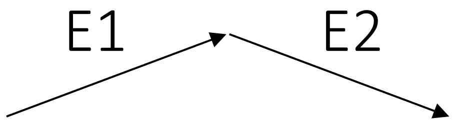
Employee(name, university, manager)

611

NEU employees managed by NEU emp.:

```
CREATE VIEW NeuMentors AS
SELECT DISTINCT E1.name,E1.manager
FROM Employee E1, Employee E2
WHERE E1.manager = E2.name
AND E1.university = 'Northeastern'
AND E2.university= 'Northeastern'
```

←This query / view is minimal



name	university	manager
Alice	Northeastern	Bob
Bob	Northeastern	Cecile
Cecile	Northeastern	
...	...	...

NEU emp. managed by NEU emp. managed by NEU emp.:

```
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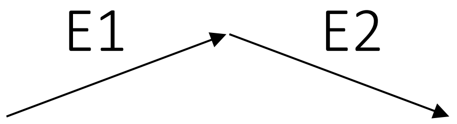

<u>name</u>	university	manager
Alice	Northeastern	Bob
Bob	Northeastern	Cecile
Cecile	Northeastern	
...	...	...

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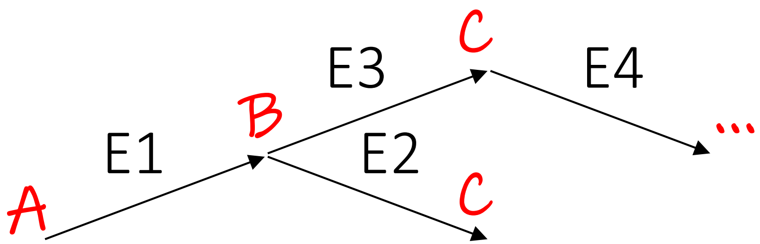

View expansion (when you run a SQL query on a view)

```
SELECT DISTINCT E1.name
FROM Employee E1, Employee E2, Employee E3, Employee E4
WHERE E1.manager = E2.name AND E1.manager = E3.name AND E3.manager = E4.name
AND E1.university = 'Northeastern' AND E2.university = 'Northeastern'
AND E3.university = 'Northeastern' AND E4.university = 'Northeastern'
```

Is this query still minimal? ?

Example adopted from Dan Suciu

Query Minimization for Views

Employee(name, university, manager)

611

NEU employees managed by NEU emp.:

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SELECT DISTINCT E1.name,E1.manager
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WHERE E1.manager = E2.name
AND E1.university = 'Northeastern'
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```
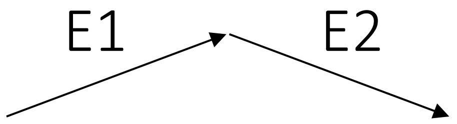

←This query / view is minimal



name	university	manager
Alice	Northeastern	Bob
Bob	Northeastern	Cecile
Cecile	Northeastern	...
...	...	...

NEU emp. managed by NEU emp. managed by NEU emp.:

```
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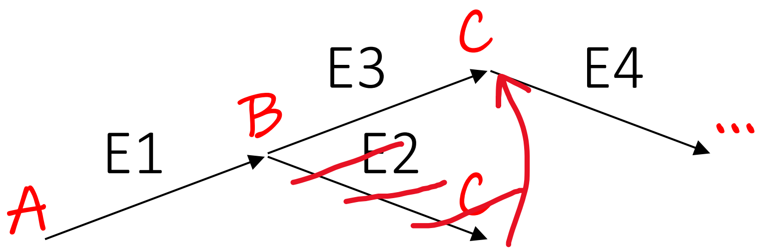

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FROM Employee E1, Employee E2, Employee E3, Employee E4
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AND E1.university = 'Northeastern' AND E2.university = 'Northeastern'
AND E3.university = 'Northeastern' AND E4.university = 'Northeastern'
```

(Struck through: "Employee E2,", "E1.manager = E2.name AND", "AND E2.university = 'Northeastern'")

E2 is redundant!

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  - CQ containment
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Beyond Conjunctive Queries

- What can we say about query languages of intermediate expressive power between conjunctive queries and the full relational calculus?

- Conjunctive queries form the sublanguage of relational algebra obtained by using only cartesian product, projection, and selection with equality conditions.

- The next step would be to consider relational algebra expressions that also involve union.

Based on Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

Beyond Conjunctive Queries

- Definition:
  - A Union of Conjunctive Queries (UCQ) is a query expressible by an expression of the form $q_1 \cup q_2 \cup \ldots \cup q_m$, where each q_i is a conjunctive query.
  - A monotone query is a query expressible by a relational algebra expression which uses only union, cartesian product, projection, and selection (with equality condition only).
- Fact:
  - Monotone queries are precisely the queries expressible by relational calculus expressions using ∧, ∨, and ∃ only (also assuming restriction to equality here).
  - Every UCQ is a monotone query.
  - Every monotone query is equivalent to a UCQ
    - but this normal form may have exponentially many disjuncts

$(a+b+c)(d+e+f)(g+h+j) = \ldots$ *how big as sum of products?*

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  - Every monotone query is equivalent to a UCQ
    - but this normal form may have exponentially many disjuncts

$$(a+b+c)(d+e+f)(g+h+j) = adg + adh + adj + aeg + aeh + \ldots + cfj$$

27 products

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

RA ?

(unnamed RA)

DRC ?

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

RA $E \cup \pi_{\$1,\$4}(\sigma_{\$2=\$3}(E \times E))$ (unnamed RA)

DRC ?

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

RA $E \cup \pi_{\$1,\$4}(\sigma_{\$2=\$3}(E \times E))$

DRC $\{(x,y) | E(x,y) \lor \exists z[E(x,z) \land E(z,y)]\}$

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

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Monotone Query

Assume schema R(A,B), S(A,B), T(B,C), V(B,C)

Is following query monotone ? $(R \cup S) \bowtie (T \cup V)$

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

RA $E \cup \pi_{\$1,\$4}(\sigma_{\$2=\$3}(E \times E))$

DRC $\{(x, y) | E(x, y) \vee \exists z[E(x, z) \wedge E(z, y)]\}$

Monotone Query

Assume schema R(A,B), S(A,B), T(B,C), V(B,C)

Following query is monotone: $(R \cup S) \bowtie (T \cup V)$

Equal to a UCQ? ?

Unions of CQs and Monotone Queries

Union of Conjunctive Queries (UCQ)

Given edge relation *E*(*A*,*B*), find paths of length 1 or 2

RA $E \cup \pi_{\$1,\$4}(\sigma_{\$2=\$3}(E \times E))$

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Monotone Query

Assume schema R(A,B), S(A,B), T(B,C), V(B,C)

Following query is monotone: $(R \cup S) \bowtie (T \cup V)$

Equal to following UCQ: $(R \bowtie T) \cup (R \bowtie V) \cup (S \bowtie T) \cup (S \bowtie V)$

q_1 q_2

The Containment Problem for Unions of CQs

THEOREM [Sagiv, Yannakakis 1980]

Let $q_1 \cup q_2 \cup \cdots \cup q_{\mathrm{m}}$ *and* $q'_1 \cup q'_2 \cup \cdots \cup q'_n$ *be two UCQs.*

Then the following are equivalent:

1) $q_1 \cup q_2 \cup \cdots \cup q_{\mathrm{m}} \subseteq q'_1 \cup q'_2 \cup \cdots \cup q'_n$

2) For every $i \leq m$, there is $j \leq n$ such that $q_i \subseteq q'_j$

Proof:

2. ⇒ 1. This direction is obvious.

1. ⇒ 2. Since $D_C[q_i] \vDash q_i$, we have that $D_C[q_i] \vDash q_1 \cup q_2 \cup \ldots \cup q_m$.

Because of containment, $D_C[q_i] \vDash q'_1 \cup q'_2 \cup \ldots \cup q'_n$.

Thus there is some $j \leq n$ with $D_C[q_i] \vDash q'_j$.

Thus from the CQ homomorphism Theorem $q_i \subseteq q'_j$.

Sagiv, Yannakakis. Equivalences Among Relational Expressions with the Union and Difference Operators, JACM 1980. https://doi.org/10.1145/322217.322221

The Complexity of Database Query Languages

	Relational Calculus	CQs	UCQs
Query Evaluation: Data Complexity	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)
Query Evaluation: Combined Compl.	PSPACE-complete	NP-complete	NP-complete
Query Equivalence & Containment	Undecidable	NP-complete	NP-complete

Source: Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

Monotone Queries

- Even though monotone queries have the same expressive power as unions of conjunctive queries, the containment problem for monotone queries has higher complexity than the containment problem for unions of conjunctive queries (syntax/complexity tradeoff)

- Theorem: Sagiv and Yannakakis – 1982

  The containment problem for monotone queries is Π_2^p-complete.

- Note: The prototypical Π_2^p-complete problem is $\forall\exists$-SAT, i.e., the restriction of QBF to formulas of the form

$$\forall x_1 \ldots \forall x_m \exists y_1 \ldots \exists y_n \, \phi.$$

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The Complexity of Database Query Languages

	Relational Calculus	CQs	UCQs	Monotone queries
Query Evaluation: Data Complexity	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)
Query Evaluation: Combined Compl.	PSPACE-complete	NP-complete	NP-complete	NP-complete
Query Equivalence & Containment	Undecidable	NP-complete	NP-complete	Π_2^p-complete

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Conjunctive Queries with Inequalities

- Definition: Conjunctive queries with inequalities form the sublanguage of relational algebra obtained by using only cartesian product, projection, and selection with equality and inequality ($\neq$, $<$, $\leq$) conditions.

- Example: Q(x,y):-- E(x,z), E(z,w),E(w,y), $z \neq w$, $z < y$.

- Theorem: (Klug – 1988, van der Meyden – 1992)
  - The query containment problem for conjunctive queries with inequalities is Π_2^p-complete.
  - The query evaluation problem for conjunctive queries with inequalities in NP-complete.

Source: Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

The Complexity of Database Query Languages

	Relational Calculus	CQs	UCQs	Monotone queries / CQs with inequalities
Query Evaluation: Data Complexity	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)	In LOGSPACE (hence, in P)
Query Evaluation: Combined Compl.	PSPACE-complete	NP-complete	NP-complete	NP-complete
Query Equivalence & Containment	Undecidable	NP-complete	NP-complete	Π_2^p-complete

Source: Phokion Kolaitis' "Logic and Databases" series at Simons Institute, 2016. https://simons.berkeley.edu/talks/logic-and-databases

Outline: T2-1/2: Query Evaluation & Query Equivalence

- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

Following slides are literally from Phokion Kolaitis's talk on "Logic and databases" at "Logical structures in Computation Boot Camp", Berkeley 2016:
https://simons.berkeley.edu/talks/logic-and-databases

Liar Paradox

Pinocchios Nase wächst bekanntlich genau dann, wenn er lügt. Was passiert aber, wenn er sagt „Meine Nase wächst gerade“?

In philosophy and logic, the classical liar paradox or liar's paradox or antinomy of the liar is the statement of a liar that they are lying: for instance, declaring that "I am lying". If the liar is indeed lying, then the liar is telling the truth, which means the liar just lied. In "this sentence is a lie" the paradox is strengthened in order to make it amenable to more rigorous logical analysis.

Source: https://de.wikipedia.org/wiki/L%C3%BCgner-Paradox , https://en.wikipedia.org/wiki/Liar_paradox

Logic and Databases

Phokion G. Kolaitis

UC Santa Cruz & IBM Research – Almaden

Lecture 4 – Part 1

1

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Thematic Roadmap

- ✓ Logic and Database Query Languages
  - Relational Algebra and Relational Calculus
  - Conjunctive queries and their variants
  - Datalog
- ✓ Query Evaluation, Query Containment, Query Equivalence
  - Decidability and Complexity
- ✓ Other Aspects of Conjunctive Query Evaluation
- Alternative Semantics of Queries
  - Bag Databases: Semantics and Conjunctive Query Containment
  - Probabilistic Databases: Semantics and Dichotomy Theorems for Conjunctive Query Evaluation
  - Inconsistent Databases: Semantics and Dichotomy Theorems

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Alternative Semantics

- So far, we have examined logic and databases under classical semantics:
  - The database relations are sets.
  - Tarskian semantics are used to interpret queries definable be first-order formulas.
- Over the years, several different alternative semantics of queries have been investigated. We will discuss three such scenarios:
  - The database relations can be bags (multisets).
  - The databases may be probabilistic.
  - The databases may be inconsistent.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Sets vs. Multisets

Relation EMPLOYEE(name, dept, salary)

- Relational Algebra Expression:

  $\pi_{\text{salary}} (\sigma_{\text{dept = CS}} (\text{EMPLOYEE}))$
- SQL query:

  ```
  SELECT   salary
  FROM     EMPLOYEE
  WHERE    dpt = ‘CS’
  ```
- SQL returns a bag (multiset) of numbers in which a number may appear several times, provided different faculty had the same salary.
- SQL does not eliminate duplicates, in general, because:
  - Duplicates are important for aggregate queries (e.g., average)
  - Duplicate elimination takes nlogn time.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Relational Algebra Under Bag Semantics

Operation	Multiplicity
Union $R_1 \cup R_2$	$m_1 + m_2$
Intersection $R_1 \cap R_2$	$\min(m_1, m_2)$
Product $R_1 \times R_2$	$m_1 \times m_2$
Projection and Selection	Duplicates are not eliminated

- R_1

A	B
1	2
1	2
2	3

- R_2

B	C
2	4
2	5

- $(R_1 \bowtie R_2)$

A	B	C
1	2	4
1	2	4
1	2	5
1	2	5

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Conjunctive Queries Under Bag Semantics

Chaudhuri & Vardi – 1993

Optimization of ***Real*** Conjunctive Queries

- Called for a re-examination of conjunctive-query optimization under bag semantics.
- In particular, they initiated the study of the containment problem for conjunctive queries under bag semantics.
- This problem has turned out to be *much more challenging* than originally perceived.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

PROBLEMS

Problems worthy
of attack
prove their worth
by hitting back.

in: *Grooks* by Piet Hein (1905-1996)

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Query Containment Under Set Semantics

Class of Queries	**Complexity of Query Containment**
Conjunctive Queries	NP-complete Chandra & Merlin – 1977
Unions of Conjunctive Queries	NP-complete Sagiv & Yannakakis - 1980
Conjunctive Queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete Klug 1988, van der Meyden -1992
First-Order (SQL) queries	Undecidable Trakhtenbrot - 1949

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Bag Semantics vs. Set Semantics

- For bags R_1, R_2:
  $R_1 \subseteq_{BAG} R_2$ if $m(\mathbf{a},R_1) \leq m(\mathbf{a},R_2)$, for every tuple **a**.
- $Q^{BAG}(D)$: Result of evaluating Q on (bag) database D.
- $Q_1 \subseteq_{BAG} Q_2$ if for every (bag) database D, we have that

$$Q_1^{BAG}(D) \subseteq_{BAG} Q_2^{BAG}(D).$$

Fact:

- $Q_1 \subseteq_{BAG} Q_2$ implies $Q_1 \subseteq Q_2$.
- The converse does **not** always hold.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Bag Semantics vs. Set Semantics

Fact: $Q_1 \subseteq Q_2$ does not imply that $Q_1 \subseteq_{BAG} Q_2$.

Example:

- $Q_1(x)$:- $P(x), T(x)$
- $Q_2(x)$:- $P(x)$

- $Q_1 \subseteq Q_2$ (obvious from the definitions)
- $Q_1 \not\subseteq_{BAG} Q_2$
- Consider the (bag) instance D = {P(a), T(a), T(a)}. Then:
  - $Q_1(D) = \{a,a\}$
  - $Q_2(D) = \{a\}$, so $Q_1(D) \not\subseteq Q_2(D)$.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Query Containment under Bag Semantics

- Chaudhuri & Vardi - 1993 stated that:

  Under bag semantics, the containment problem for conjunctive queries is Π_2^p-hard.

- Problem:
  - What is the exact complexity of the containment problem for conjunctive queries under bag semantics?
  - Is this problem decidable?

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Query Containment Under Bag Semantics

- 23 years have passed since the containment problem for conjunctive queries under bag semantics was raised.
- Several attacks to solve this problem have failed.
- At least two technically flawed PhD theses on this problem have been produced.
- Chaudhuri and Vardi have withdrawn the claimed Π_2^p-hardness of this problem; no one has provided a proof.

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Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Query Containment Under Bag Semantics

- The containment problem for conjunctive queries under bag semantics remains **open** to date.

- However, progress has been made towards the containment problem under bag semantics for the two main extensions of conjunctive queries:
  - Unions of conjunctive queries
  - Conjunctive queries with ≠

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Unions of Conjunctive Queries

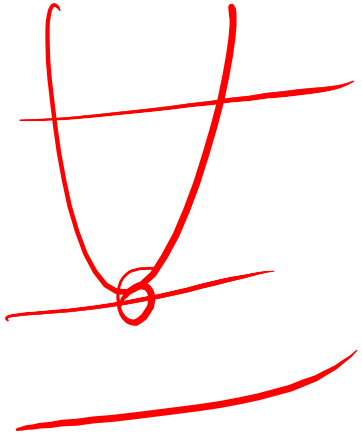
Theorem (Ioannidis & Ramakrishnan – 1995):
Under bag semantics, the containment problem for unions of conjunctive queries is **undecidable**.

Hint of Proof:
Reduction from Hilbert's 10^{th} Problem.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

$4x_1^7 - 18x_2^5 + 2 = 0 \quad x \in \mathbb{Z}$

INTEGER



Hilbert's 10th Problem

- Hilbert's 10th Problem – 1900
  (10th in Hilbert's list of 23 problems)

  Given a Diophantine equation with any number of unknown quantities and with rational integral numerical coefficients: To devise a process according to which it can be determined in a finite number of operations whether the equation is solvable in rational integers.

  In effect, Hilbert's 10th Problem is:
  Find an algorithm for the following problem:
  Given a polynomial $P(x_1,\dots,x_n)$ with integer coefficients, does it have an all-integer solution?

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Hilbert's 10th Problem

- Hilbert's 10th Problem – 1900
  (10th in Hilbert's list of 23 problems)
  Find an algorithm for the following problem:
  Given a polynomial $P(x_1,\ldots,x_n)$ with integer coefficients, does it have an all-integer solution?

- Y. Matiyasevich – 1971
  (building on M. Davis, H. Putnam, and J. Robinson)
  - Hilbert's 10th Problem is **undecidable**, hence **no** such algorithm exists.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Hilbert's 10th Problem

- Fact: The following variant of Hilbert's 10th Problem is undecidable:
  - Given two polynomials $p_1(x_1,\ldots x_n)$ and $p_2(x_1,\ldots x_n)$ with positive integer coefficients and no constant terms, is it true that $p_1 \leq p_2$?

    In other words, is it true that $p_1(a_1,\ldots,a_n) \leq p_2(a_1,\ldots a_n)$, for all positive integers $a_1,\ldots,a_n$?

- Thus, there is no algorithm for deciding questions like:
  - Is $3x_1^4x_2x_3 + 2x_2x_3 \leq x_1^6 + 5x_2x_3$?

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Unions of Conjunctive Queries

Theorem (Ioannidis & Ramakrishnan – 1995):
Under bag semantics, the containment problem for unions of conjunctive queries is **undecidable**.

Hint of Proof:

- Reduction from the previous variant of Hilbert's 10th Problem:
  - Use joins of unary relations to encode monomials (products of variables).
  - Use unions to encode sums of monomials.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Unions of Conjunctive Queries

Example: Consider the polynomial $3x_1^4x_2x_3 + 2x_2x_3$

- The monomial $x_1^4x_2x_3$ is encoded by the conjunctive query $P_1(w),P_1(w),P_1(w), P_1(w), P_2(w),P_3(w)$.

- The monomial x_2x_3 is encoded by the conjunctive query $P_2(w),P_3(w)$.

- The polynomial $3x_1^4x_2x_3 + 2x_2x_3$ is encoded by the union having:
  - three copies of $P_1(w),P_1(w),P_1(w), P_1(w), P_2(w),P_3(w)$ and
  - two copies of $P_2(w),P_3(w)$.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Complexity of Query Containment

Class of Queries	Complexity – Set Semantics	Complexity – Bag Semantics
Conjunctive queries	NP-complete CM – 1977	
Unions of conj. queries	NP-complete SY - 1980	Undecidable IR - 1995
Conj. queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete vdM - 1992	
First-order (SQL) queries	Undecidable Trakhtenbrot - 1949	Undecidable

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Conjunctive Queries with ≠

Theorem (Jayram, K …, Vee – 2006):
Under bag semantics, the containment problem for conjunctive queries with ≠ is **undecidable**.

In fact, this problem is **undecidable** even if

- the queries use only a single relation of arity 2;
- the number of inequalities in the queries is at most some fixed (albeit huge) constant.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Complexity of Query Containment

Class of Queries	Complexity – Set Semantics	Complexity – Bag Semantics
Conjunctive queries	NP-complete CM – 1977	**Open**
Unions of conj. queries	NP-complete SY - 1980	Undecidable IR - 1995
Conj. queries with $\neq$, $\leq$, $\geq$	Π_2^p-complete vdM - 1992	Undecidable JKV - 2006
First-order (SQL) queries	Undecidable Trakhtenbrot - 1949	Undecidable

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Subsequent Developments

- Some progress has been made towards identifying special classes of conjunctive queries for which the containment problem under bag semantics is decidable.

  – Afrati, Damigos, Gergatsoulis – 2010
    - Projection-free conjunctive queries.

  – Kopparty and Rossman – 2011
    - A large class of boolean conjunctive queries on graphs.

Source: Phokion Kolaitis: https://simons.berkeley.edu/talks/phokion-kolaitis-2016-09-01

Outline: T2-1/2: Query Evaluation & Query Equivalence

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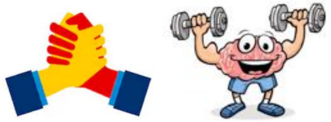
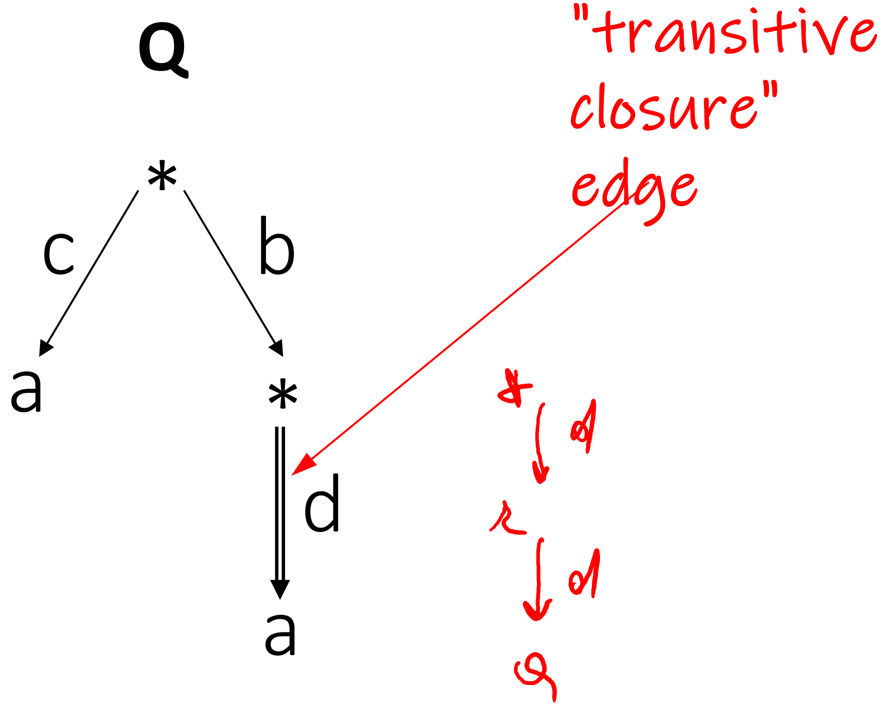
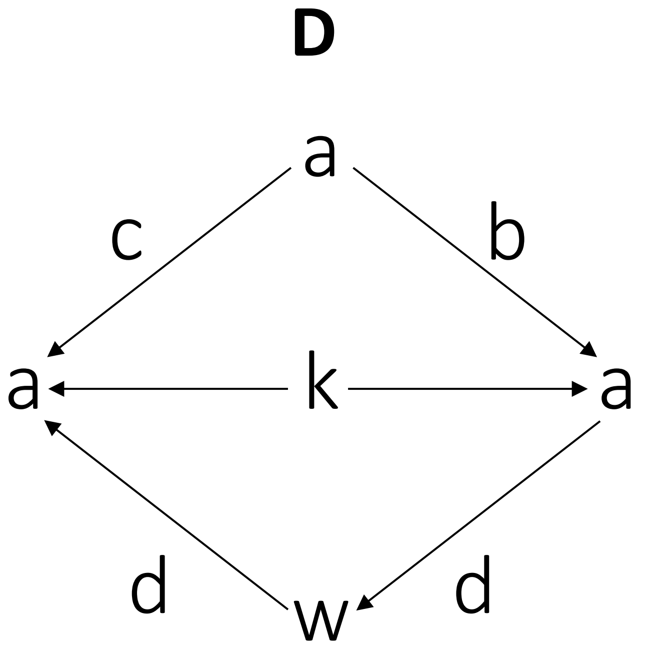
Tree pattern queries





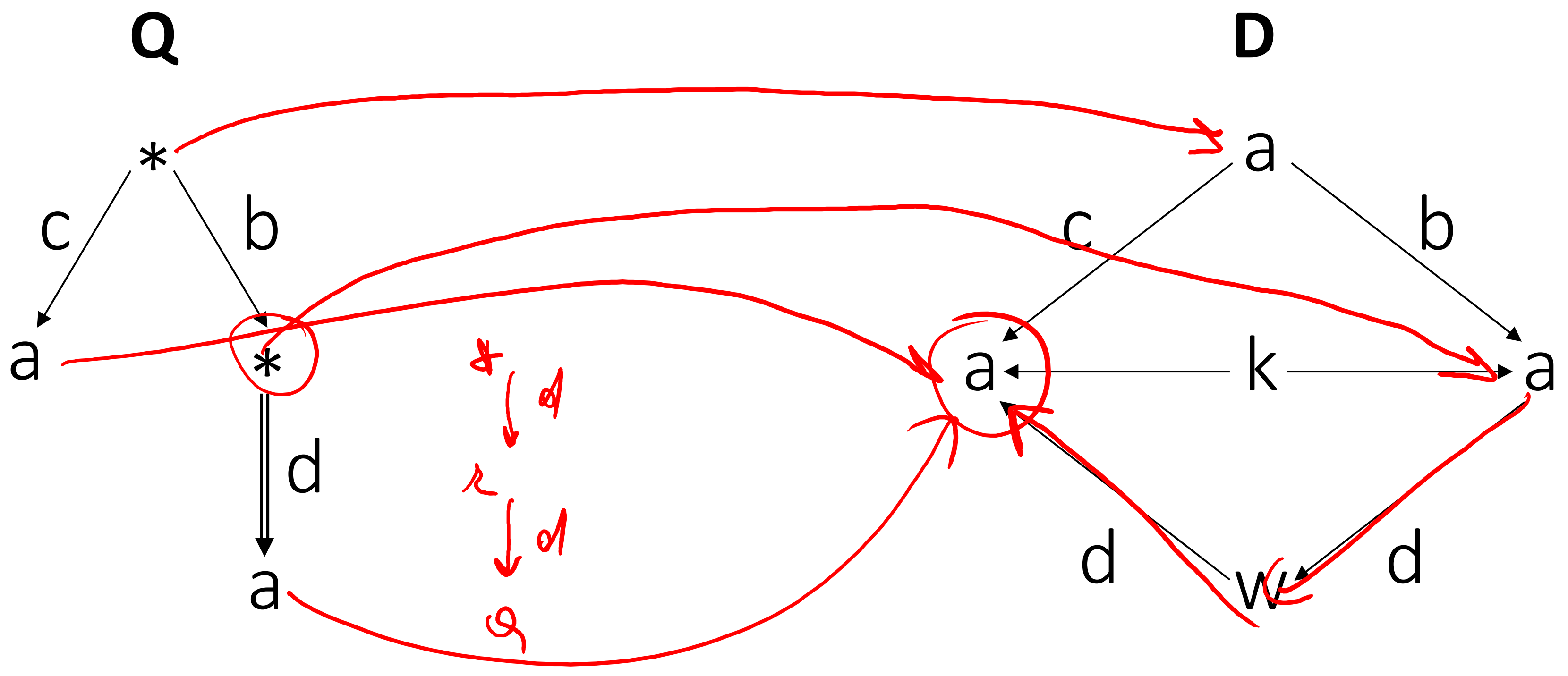


Does the query on the left have a match on in the data on the right (i.e. is there a homomorphism from left to right)?

?

Notice that "a", "b", "c" are labels (not node ids), thus like constants in a query, or like predicates (colored edges)

Tree pattern queries



Example from: "Optimizing Tree Patterns for Querying Graph- and Tree-Structured Data" by Czerwinski, Martens, Niewerth, Parys. SIGMOD record 2017. https://doi.org/10.1145/3093754.3093759

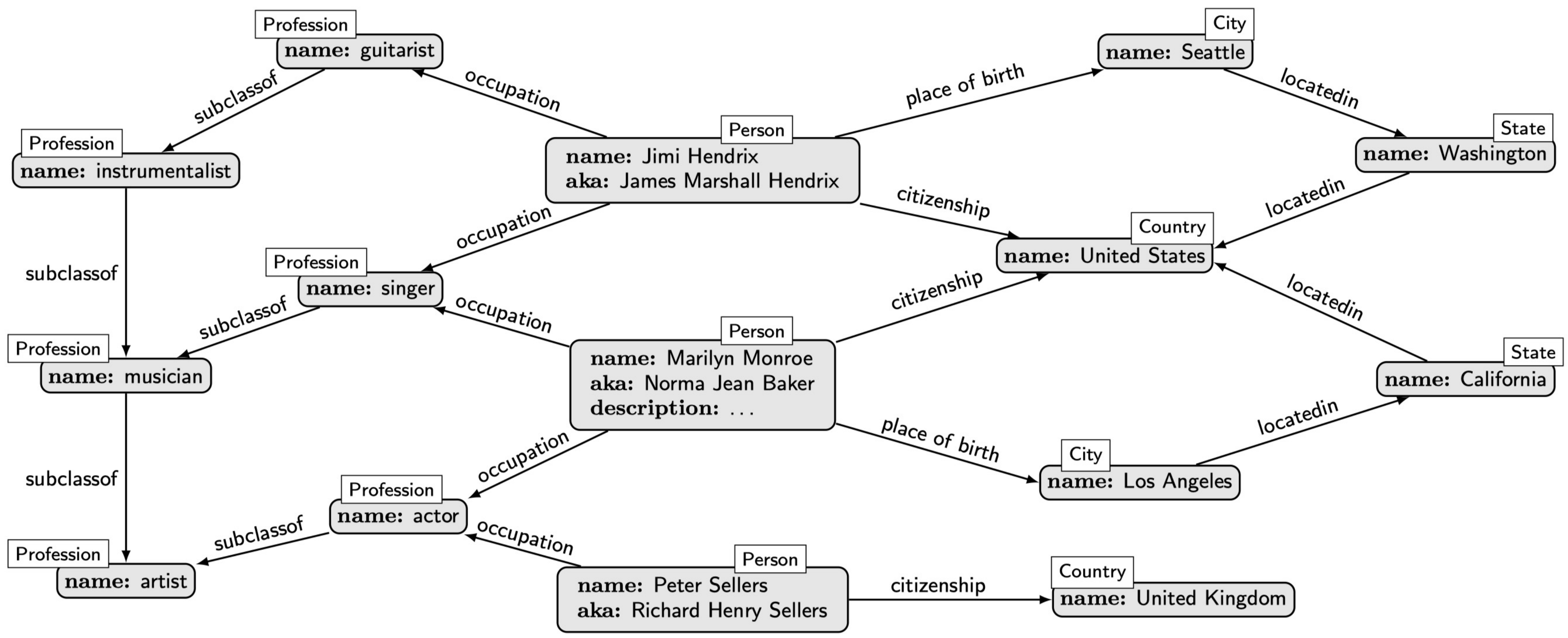


Figure 1: A graph database (as a *property graph*), inspired on a fragment of WikiData

?

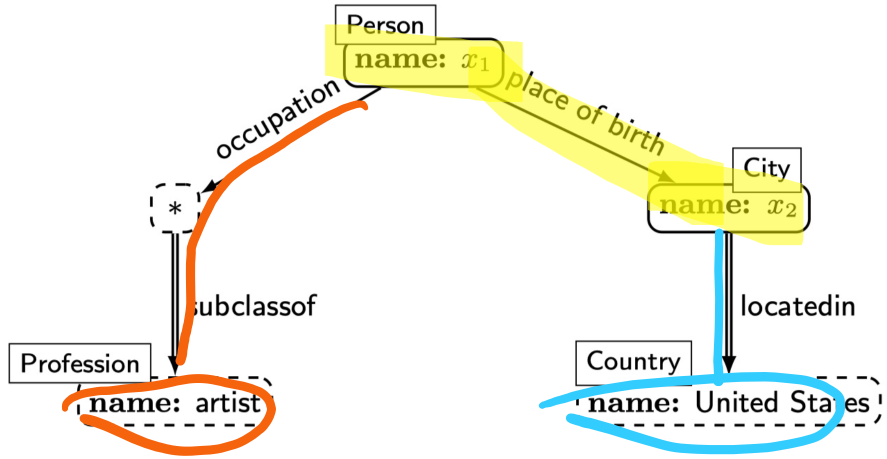
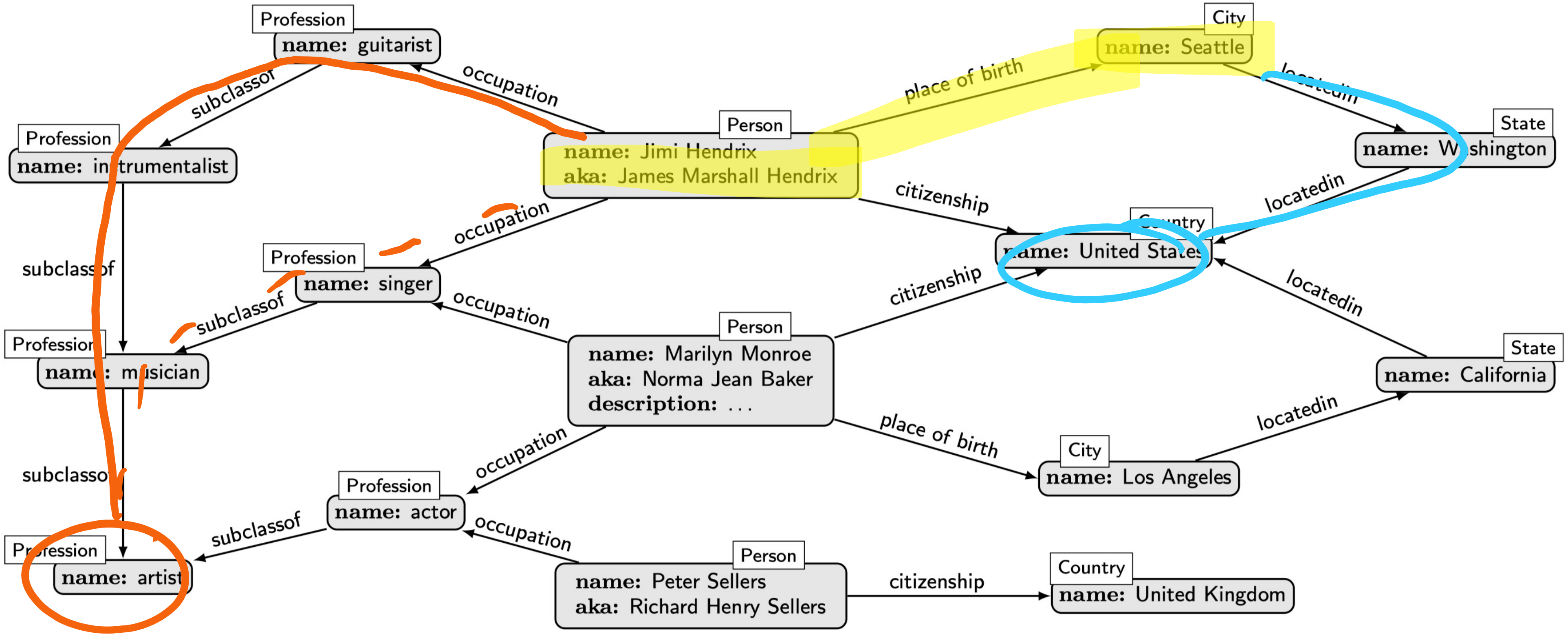


Figure 2: A tree pattern finding the artists who were born in the United States. The query returns the person names and the cities where they were born. (Fully circled nodes are return nodes.)



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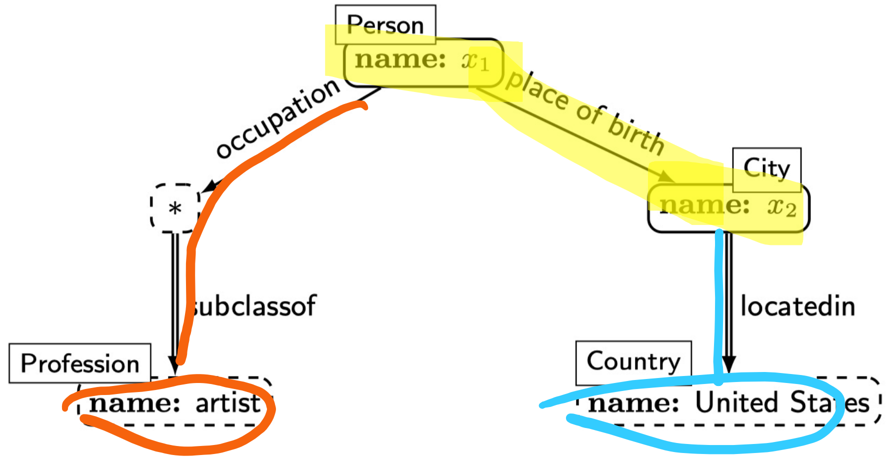

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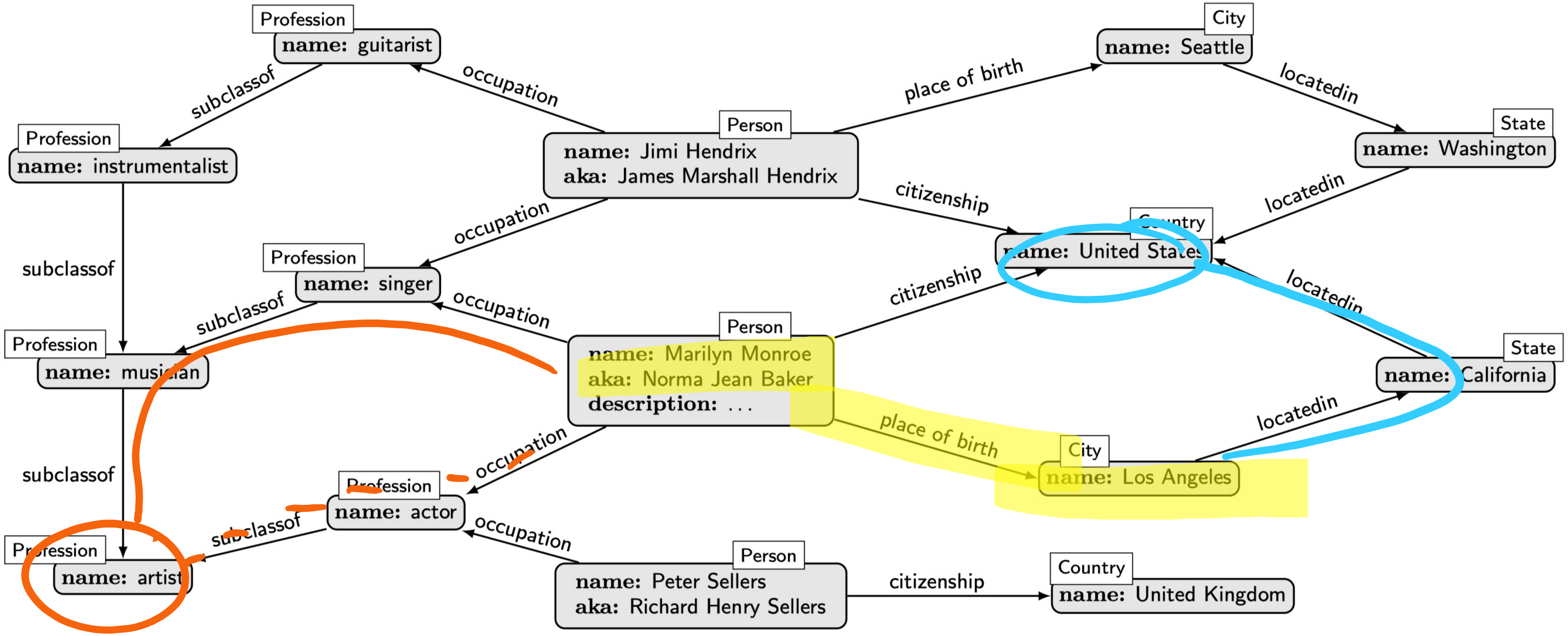

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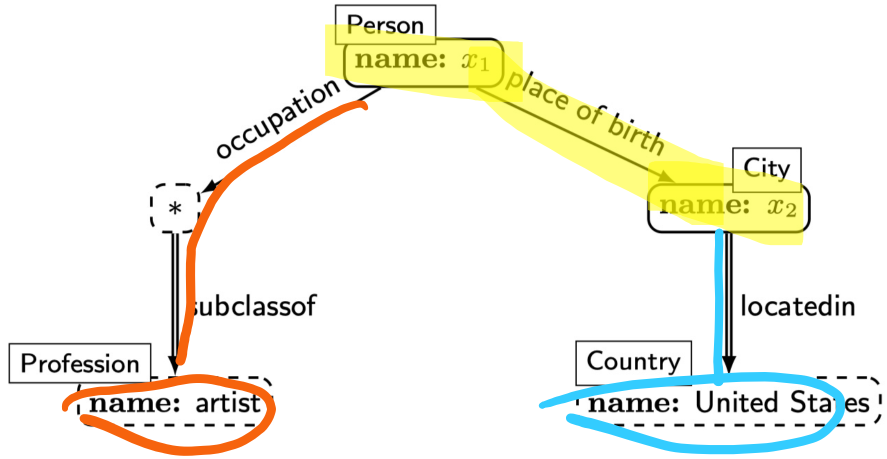

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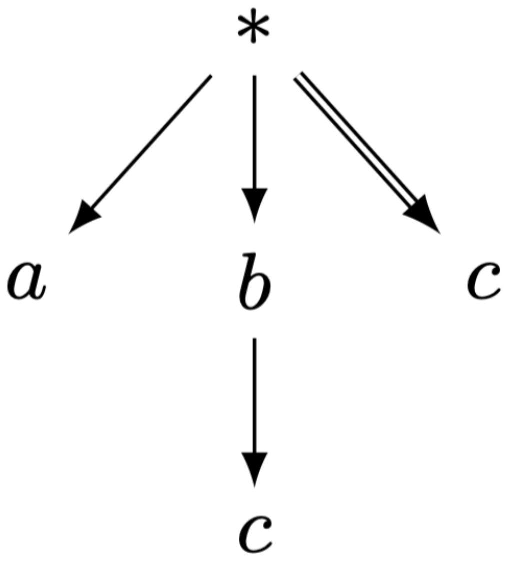
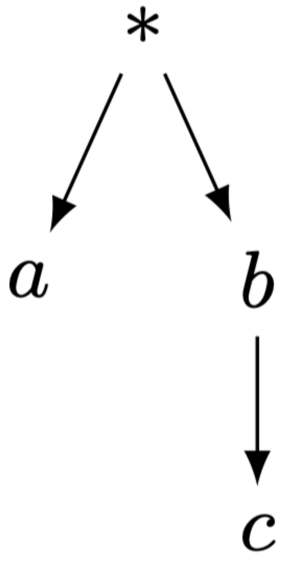
Optimizing tree patterns





How are those two tree patterns related to each other?

?

Example from: “Optimizing Tree Patterns for Querying Graph- and Tree-Structured Data” by Czerwinski, Martens, Niewerth, Parys. SIGMOD record 2017. https://doi.org/10.1145/3093754.3093759

Optimizing tree patterns

TREE PATTERN MINIMIZATION

Given:	A tree pattern p and $k \in \mathbb{N}$
Question:	Is there a tree pattern q, equivalent to p, such that its size is at most k?

Example from: "Optimizing Tree Patterns for Querying Graph- and Tree-Structured Data" by Czerwinski, Martens, Niewerth, Parys. SIGMOD record 2017. https://doi.org/10.1145/3093754.3093759

Minimality =? Nonredundancy

1.4 History of the Problem

Although the patterns we consider here have been widely studied [14, 24, 36, 15, 22, 1, 9, 4, 32], their minimization problem remained elusive for a long time. The most important previous work for their minimization was done by Kimelfeld and Sagiv [22] and by Flesca, Furfaro, and Masciari [14, 15].

The key challenge was understanding the relationship between *minimality* (M) and *nonredundancy* (NR). Here, a tree pattern is minimal if it has the smallest number of nodes among all equivalent tree patterns. It is nonredundant if none of its leaves (or branches[2]) can be deleted while remaining equivalent. The question was if minimality and nonredundancy are the same ([22, Section 7] and [15, p. 35]):

> $M \stackrel{?}{=} NR$ PROBLEM:
>
> Is a tree pattern minimal if and only if it is nonredundant?

Notice that a part of the $M \stackrel{?}{=} NR$ problem is easy to see: a minimal pattern is trivially also nonredundant (that is, $M \subseteq NR$). The opposite direction is much less clear.

If the problem would have a positive answer, it would mean that the simple algorithmic idea summarised in Algorithm 1 correctly minimizes tree patterns. Therefore, the $M \stackrel{?}{=} NR$ problem is a natural question about the design of minimization algorithms for tree patterns.

Algorithm 1 Computing a nonredundant subpattern

Input: A tree pattern p
Output: A nonredundant tree pattern q, equivalent to p

```
while a leaf of p can be removed
                    (remaining equivalent to p) do
    Remove the leaf
end while
return the resulting pattern
```

Example from: "Optimizing Tree Patterns for Querying Graph- and Tree-Structured Data" by Czerwinski, Martens, Niewerth, Parys. SIGMOD record 2017. https://doi.org/10.1145/3093754.3093759

The $\mathrm{M} \stackrel{?}{=} \mathrm{NR}$ problem is also a question about complexity. The main source of complexity of the nonredundancy algorithm lies in testing equivalence between a pattern p and a pattern p', which is generally coNP-complete [24]. If $\mathrm{M} \stackrel{?}{=} \mathrm{NR}$ has a positive answer, then TREE PATTERN MINIMIZATION would also be coNP-complete.

In fact, the problem was claimed to be coNP-complete in 2003 [14, Theorem 2], but the status of the minimization- and the $\mathrm{M} \stackrel{?}{=} \mathrm{NR}$ problems were re-opened by Kimelfeld and Sagiv [22], who found errors in the proofs. Flesca et al.'s journal paper then proved that $\mathrm{M} = \mathrm{NR}$ for a limited class of tree patterns, namely those where *every wildcard node has at most one child* [15]. Nevertheless, for tree patterns,

(a) the status of the $\mathrm{M} \stackrel{?}{=} \mathrm{NR}$ problem and

(b) the complexity of the minimization problem

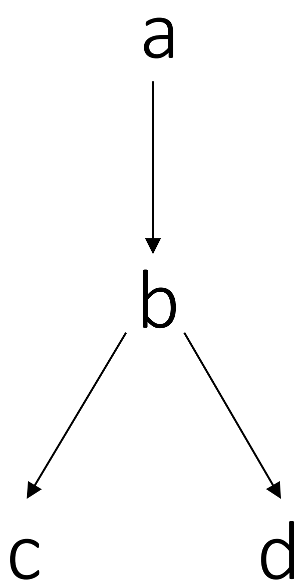
remained open.

Czerwinski, Martens, Niewerth, Parys [PODS 2016}

(a) There exists a tree pattern that is nonredundant but not minimal. Therefore, $\mathrm{M} \neq \mathrm{NR}$.

(b) TREE PATTERN MINIMIZATION is Σ_2^P-complete. This implies that even the main idea in Algorithm 1 cannot work unless $\mathrm{coNP} = \Sigma_2^P$.

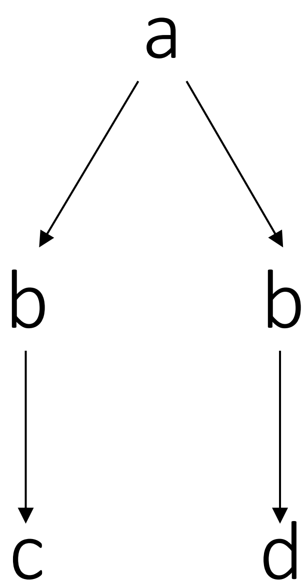
Tree pattern containment



$\subseteq$

or

$\supseteq$



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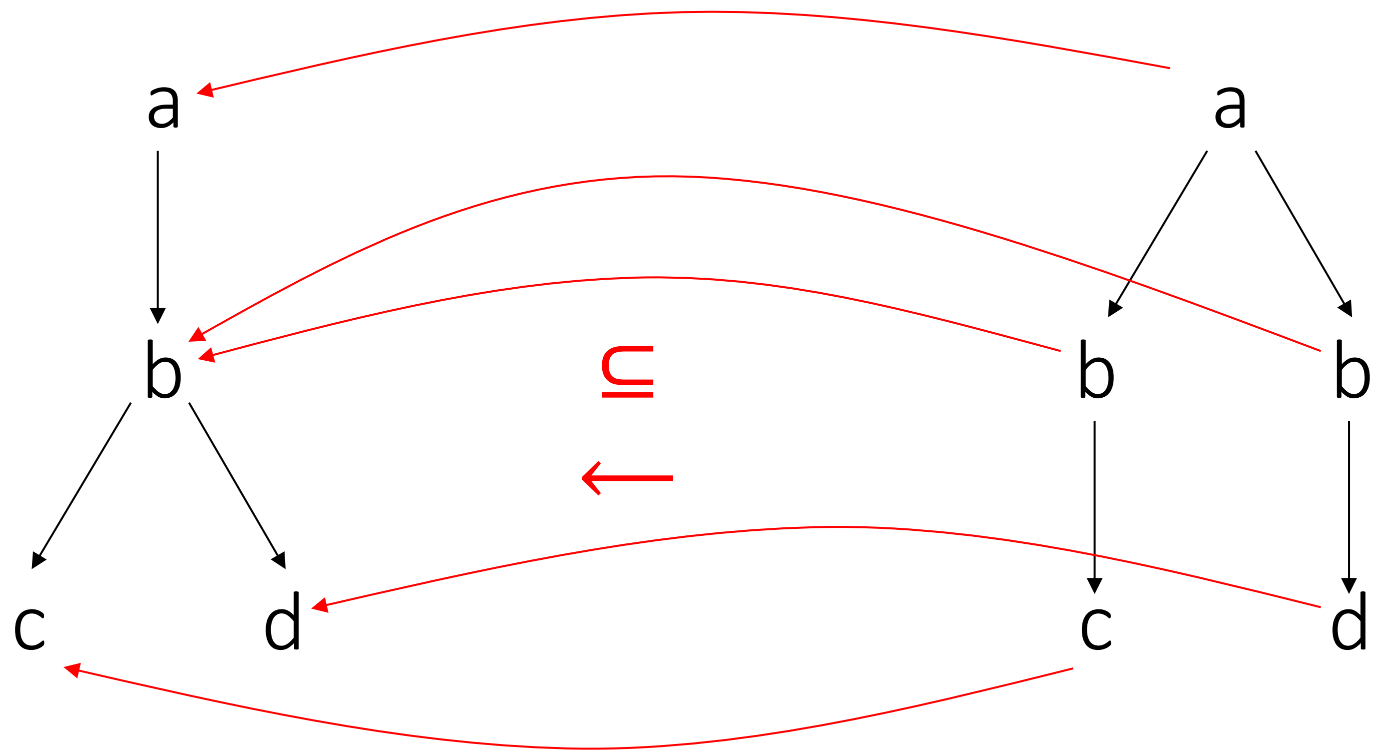
Tree pattern containment



but ⊉!

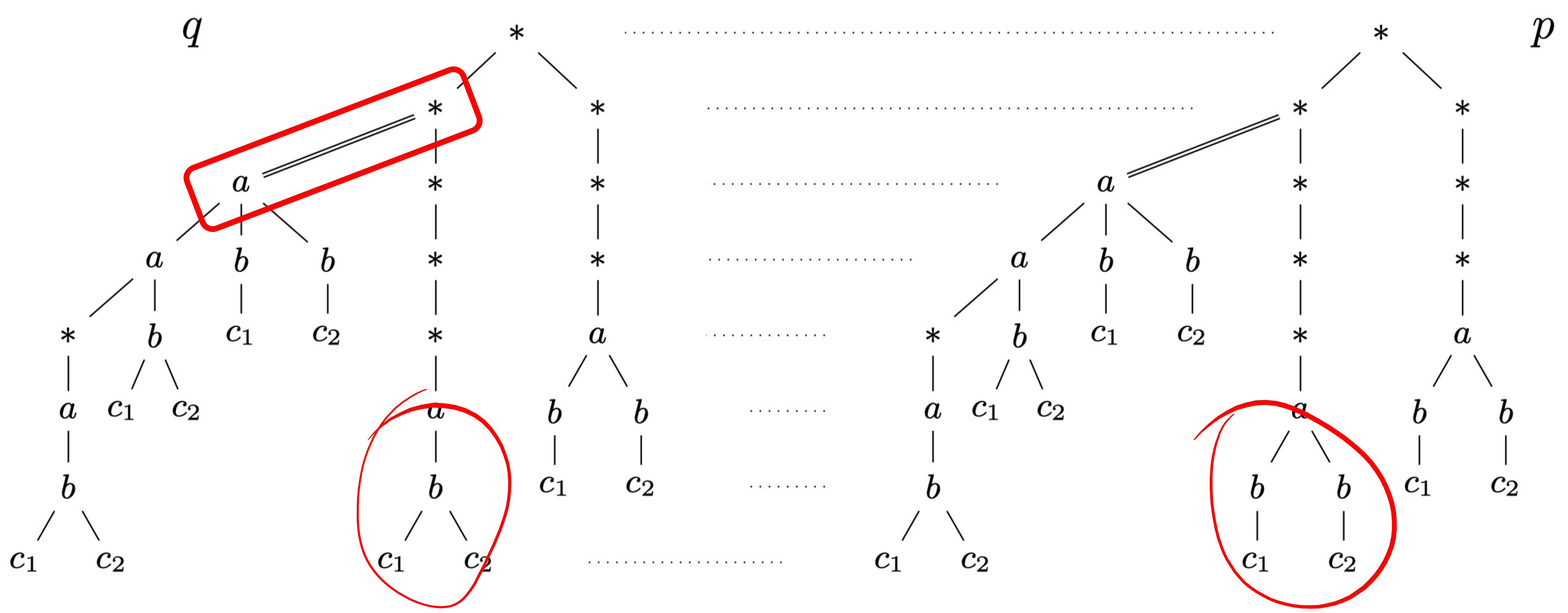
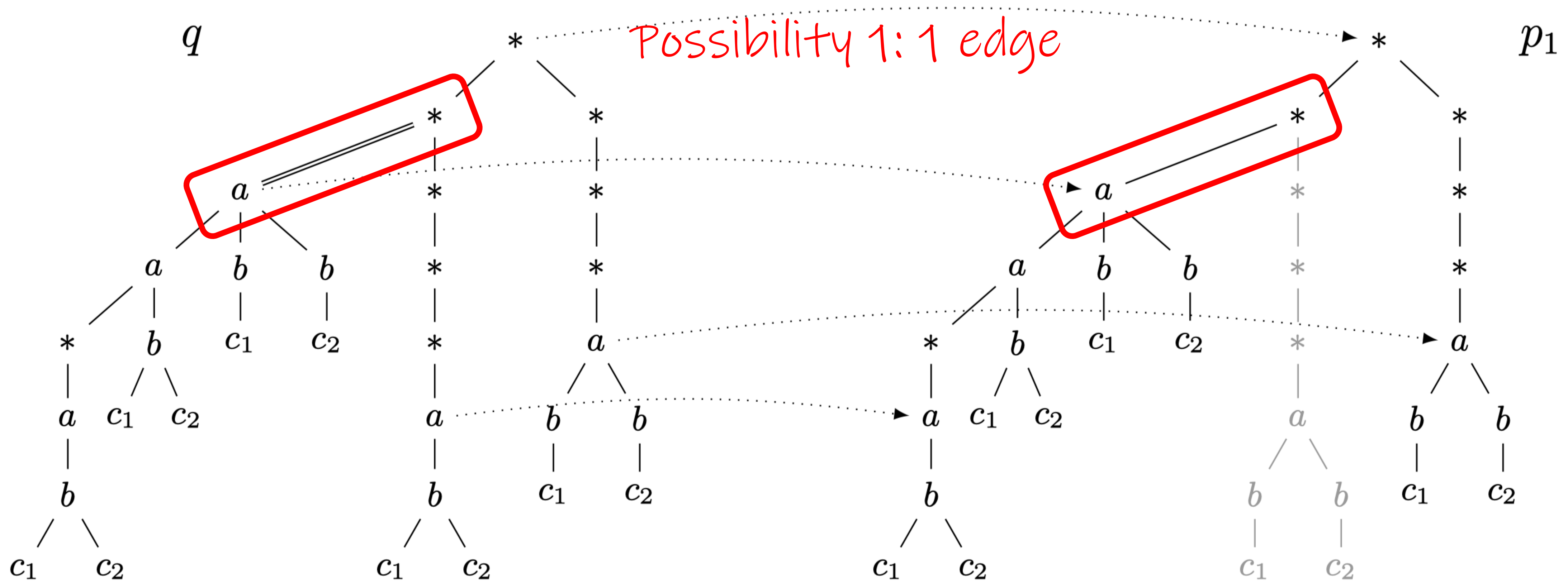


Figure 7: A non-redundant tree pattern p (right) and an equivalent tree pattern q that is smaller (left)

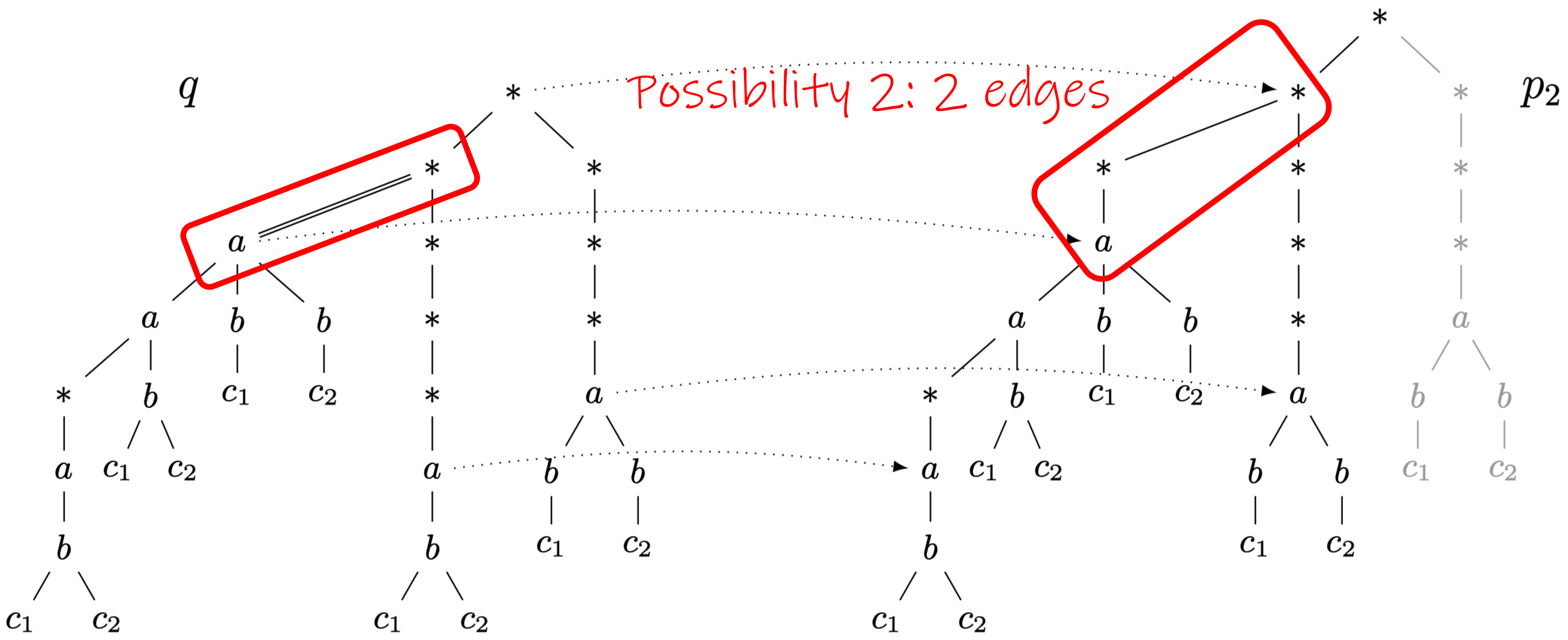
$q \subseteq p$ follows from argument on previous page.

To be shown $q \supseteq p$, then equivalent. Idea: whenever p matches, then also q.

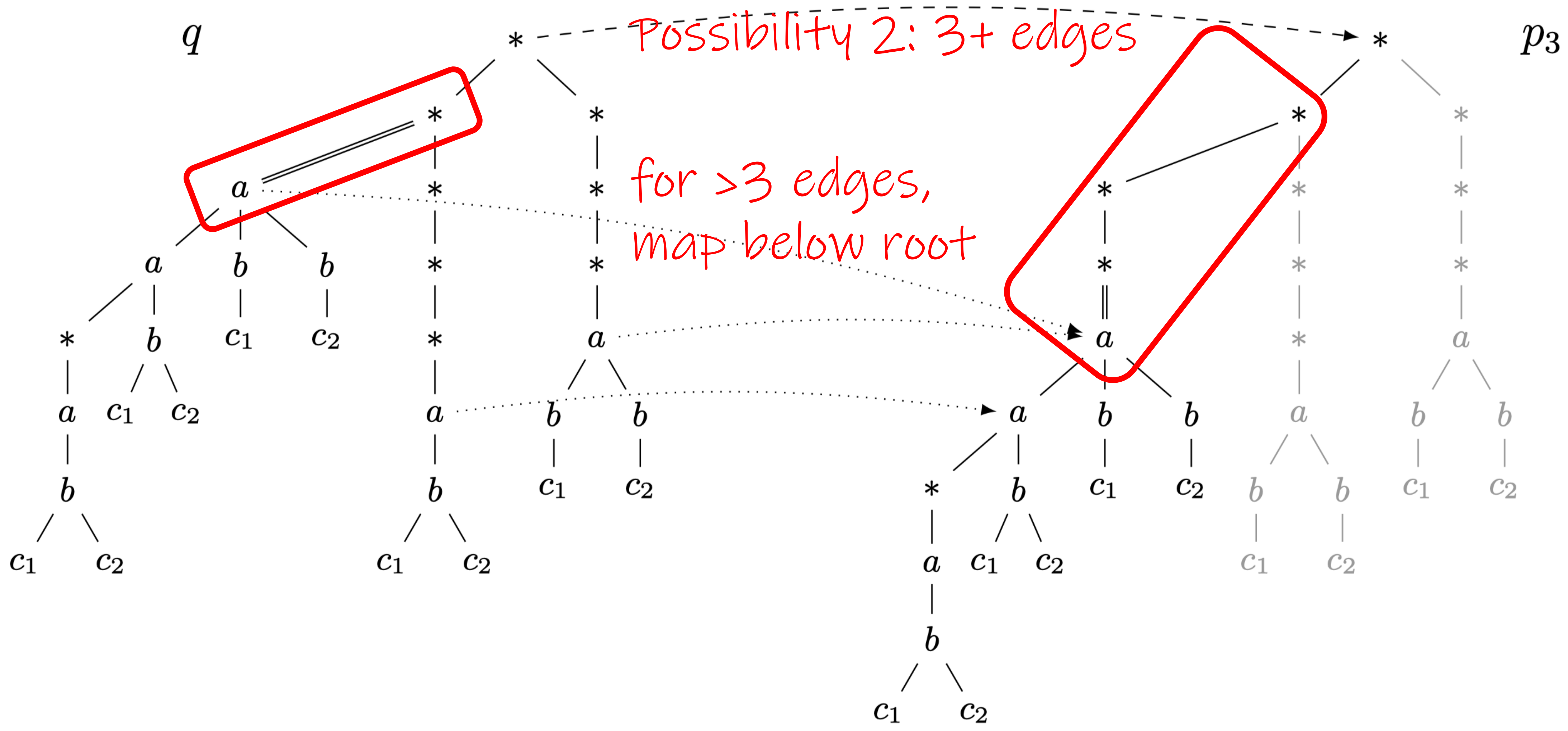
Idea: $a = \star$ can be matched in 3 ways in a graph



(a) How q can be matched if p_1 can be matched



(b) How q can be matched if p_2 can be matched



(c) How q can be matched if p_3 can be matched

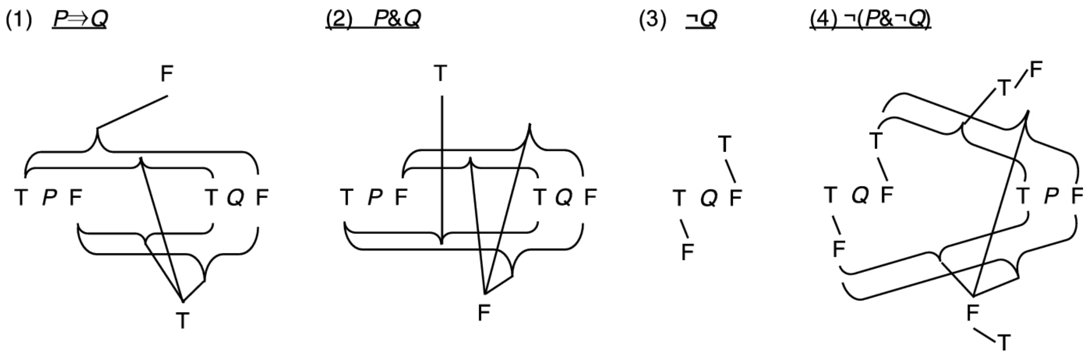
Outline: T2-1/2: Query Evaluation & Query Equivalence

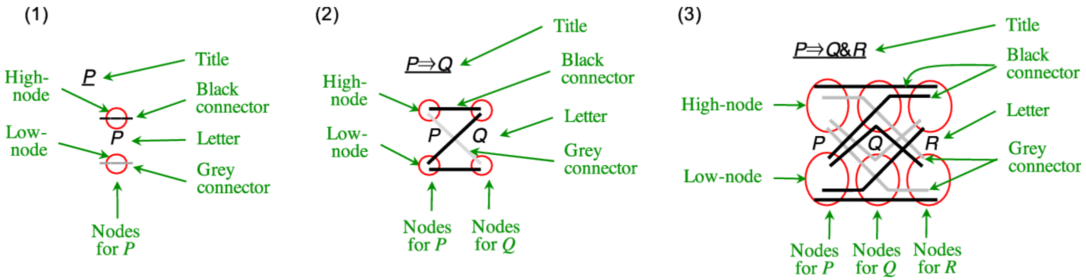
- T2-1: Conjunctive Queries (CQs)
  - Query equivalence and containment (& motivation of CQs)
  - Graph homomorphisms
  - Homomorphism beyond graphs
  - CQ containment
  - CQ minimization
- T2-2: Equivalence Beyond CQs
  - Union of CQs, and inequalities
  - Union of CQs equivalence under bag semantics
  - Tree pattern queries
  - Nested queries

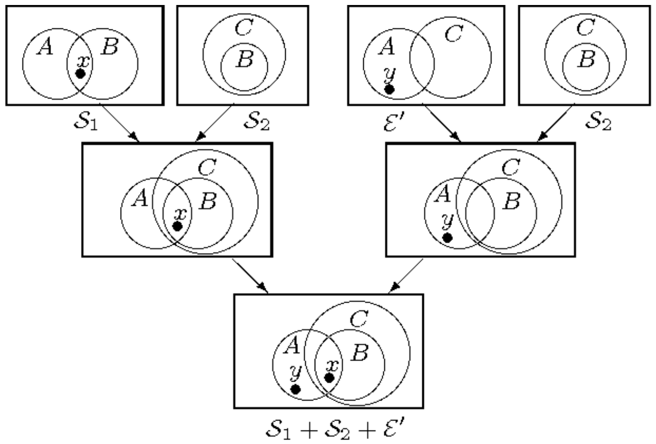
Equivalence of nested queries

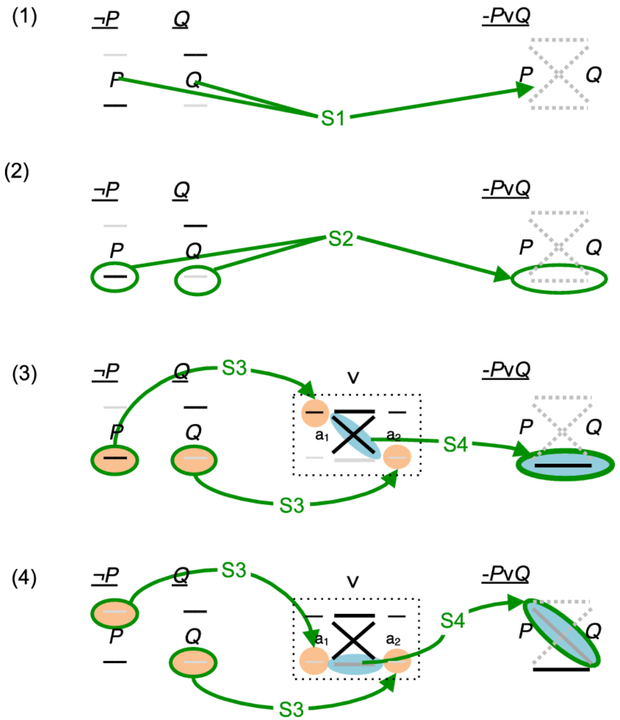
- Query equivalence is one of the foundational questions in database theory (and practice?)
  - touches on logics and decidability
  - what modifications allow tractability
  - Lots of work (and open questions) on query equivalence
- But not so much work on nested queries!
- Related to **Relational Diagrams** (https://relationaldiagrams.com/) and **QueryVis** projects (https://queryvis.com/) and two foundational questions on visual formalism:
  1. When can visual formalism *unambiguously* express logical statements?
  2. When can equivalent logical statements be transformed to each other by a sequence of visual transformations? (*Query equivalence*)

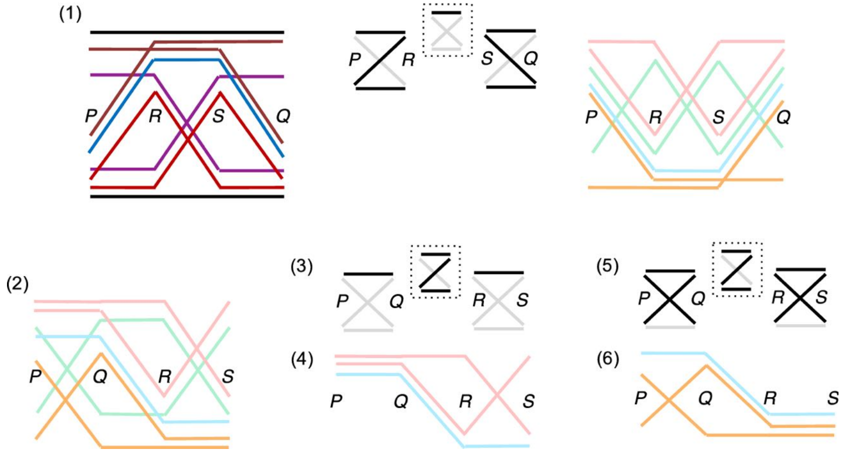
Diagrammatic reasoning systems and their expressiveness





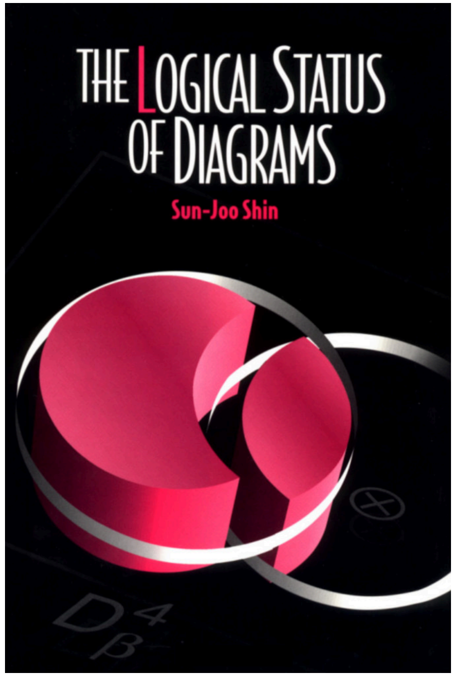






Figures from: “Truth Diagrams Versus Extant Notations for Propositional Logic” by Cheng. Journal of Logic, Language and Information, 2019. https://doi.org/10.1007/s10849-019-09299-y

Diagrammatic reasoning systems and their expressiveness



Diagrams are widely used in reasoning about problems in physics, mathematics, and logic, but have traditionally been considered to be only heuristic tools and not valid elements of mathematical proofs. This book challenges this prejudice against visualization in the history of logic and mathematics and provides a formal foundation for work on natural reasoning in a visual mode.

The author presents Venn diagrams as a formal system of representation equipped with its own syntax and semantics and specifies rules of transformation that make this system sound and complete. The system is then extended to the equivalent of a first-order monadic language. The soundness of these diagrammatic systems refutes the contention that graphical representation is misleading in reasoning. The validity of the transformation rules ensures that the correct application of the rules will not lead to fallacies. The book concludes with a discussion of some fundamental differences between graphical systems and linguistic systems.

This groundbreaking work will have important influence on research in logic, philosophy, and knowledge representation.

objects. Conjunctive information is more naturally represented by diagrams than by linguistic formulæ. For example, a single Venn diagram can

Still, not all relations can be viewed as membership or inclusion. Shin has been careful throughout her book to restrict herself to monadic systems. Relations per se (polyadic predicates) are not considered. And while it may be true that the formation of a system (such as Venn-II) that is provably both sound and complete would help mitigate the prejudice

perception. In her discussion of perception she shows that disjunctive information is not representable in *any* system. In doing so she relies on

The logical status of diagrams, Sun-Joo Shin, Cambridge university press 1994. https://doi.org/10.1017/CBO9780511574696
Sun-Joo Shin at Yale: https://philosophy.yale.edu/people/sun-joo-shin

Relational Diagrams / QueryVis

- Motivation: Can we create an automatic diagramming system that:
  - unambiguously visualizes the logical intent of a relational query (thus no two different queries lead to an "identical" visualization; with "identical" to be formalized correctly)
  - for some important subset of nested queries (later extensions from SQL)
  - with visual diagrams that allow us to reason about logical SQL design patterns
- Related:
  - Lot's of interest on conjunctive queries equivalence. Now: For what fragment of nested queries is equivalence decidable (under set semantics)?
- Suggestion:
  - nested queries, with inequalities, without any disjunctions
  - Strict superset of conjunctive queries

Logical SQL Patterns

Logical patterns are the building blocks of most SQL queries.

Patterns are very hard to extract from the SQL text.

A pattern can appear across different database schemas.

Think of queries like:

- Find sailors who reserved all red boats
- Find students who took all art classes
- Find actors who played in all movies by Hitchcock

What does this query return ?

Likes(drinker,beer)

```
SELECT L1.drinker
FROM Likes L1
WHERE not exists
  (SELECT *
  FROM Likes L2
  WHERE L1.drinker <> L2.drinker
  AND not exists
    (SELECT  *
    FROM Likes L3
    WHERE L3.drinker = L2.drinker
    AND not exists
      (SELECT *
      FROM Likes L4
      WHERE L4.drinker = L1.drinker
      AND L4.beer = L3.beer))
  AND not exists
    (SELECT *
    FROM Likes L5
    WHERE L5. drinker = L1. drinker
    AND not exists
      (SELECT *
      FROM Likes L6
      WHERE L6.drinker = L2.drinker
      AND L6.beer= L5.beer)))
```

See: Gatterbauer, Dunne. On the Reasonable Effectiveness of Relational Diagrams, SIGMOD 2024, https://arxiv.org/pdf/2401.04758 ,

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    AND not exists
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      FROM Likes L4
      WHERE L4.drinker = L1.drinker
      AND L4.beer = L3.beer))
  AND not exists
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    WHERE L5. drinker = L1. drinker
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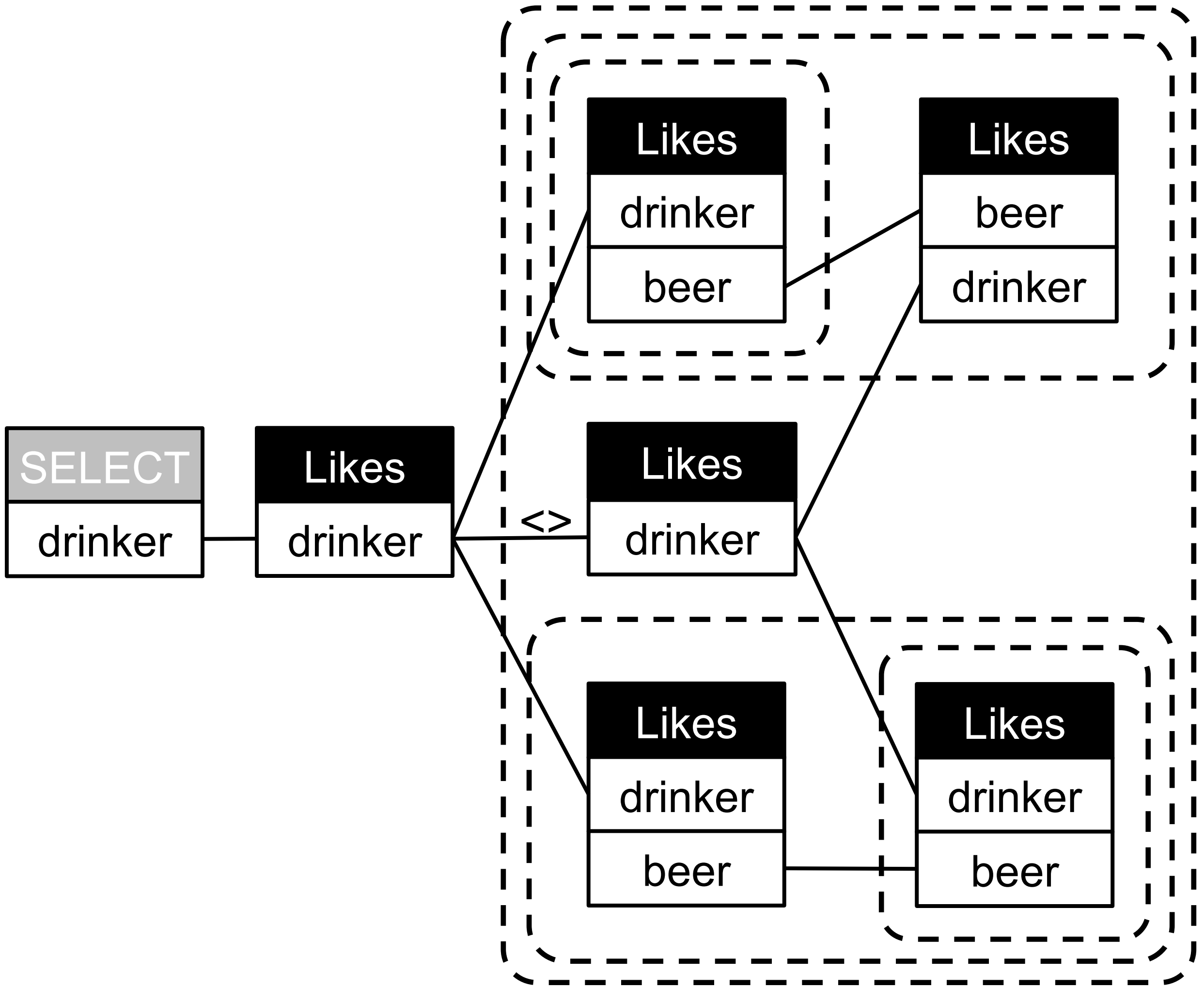
```



Relational Diagrams scoping

See: Gatterbauer, Dunne. On the Reasonable Effectiveness of Relational Diagrams, SIGMOD 2024, https://arxiv.org/pdf/2401.04758 ,

Q: Finder drinkers with a unique beer taste

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    FROM Likes L3
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    AND not exists
      (SELECT *
      FROM Likes L4
      WHERE L4.drinker = L1.drinker
      AND L4.beer = L3.beer))
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      FROM Likes L6
      WHERE L6.drinker = L2.drinker
      AND L6.beer= L5.beer)))
```

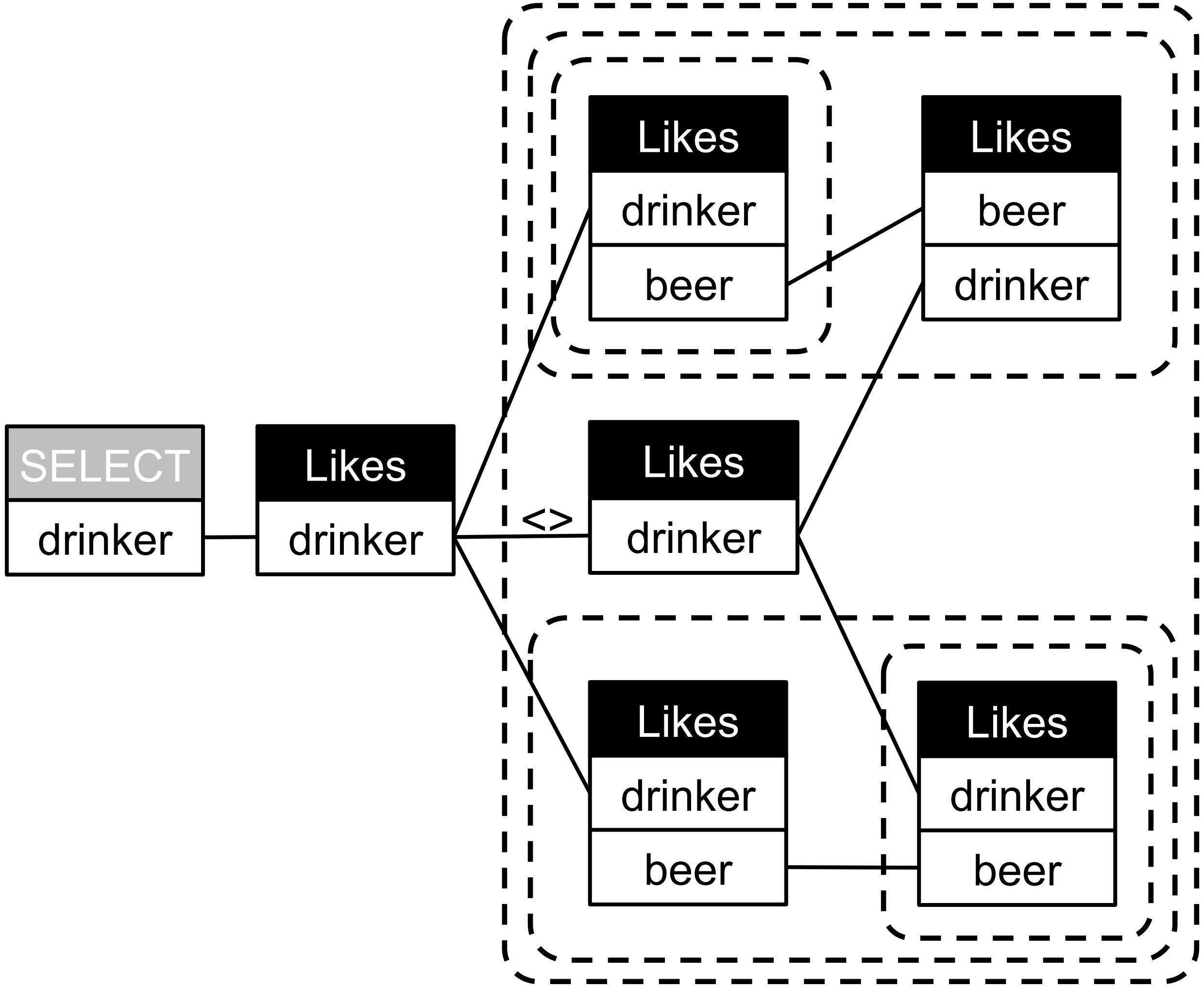

Relational Diagrams scoping

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Wolfgang Gatterbauer. Principles of scalable data management: https://northeastern-datalab.github.io/cs7240/

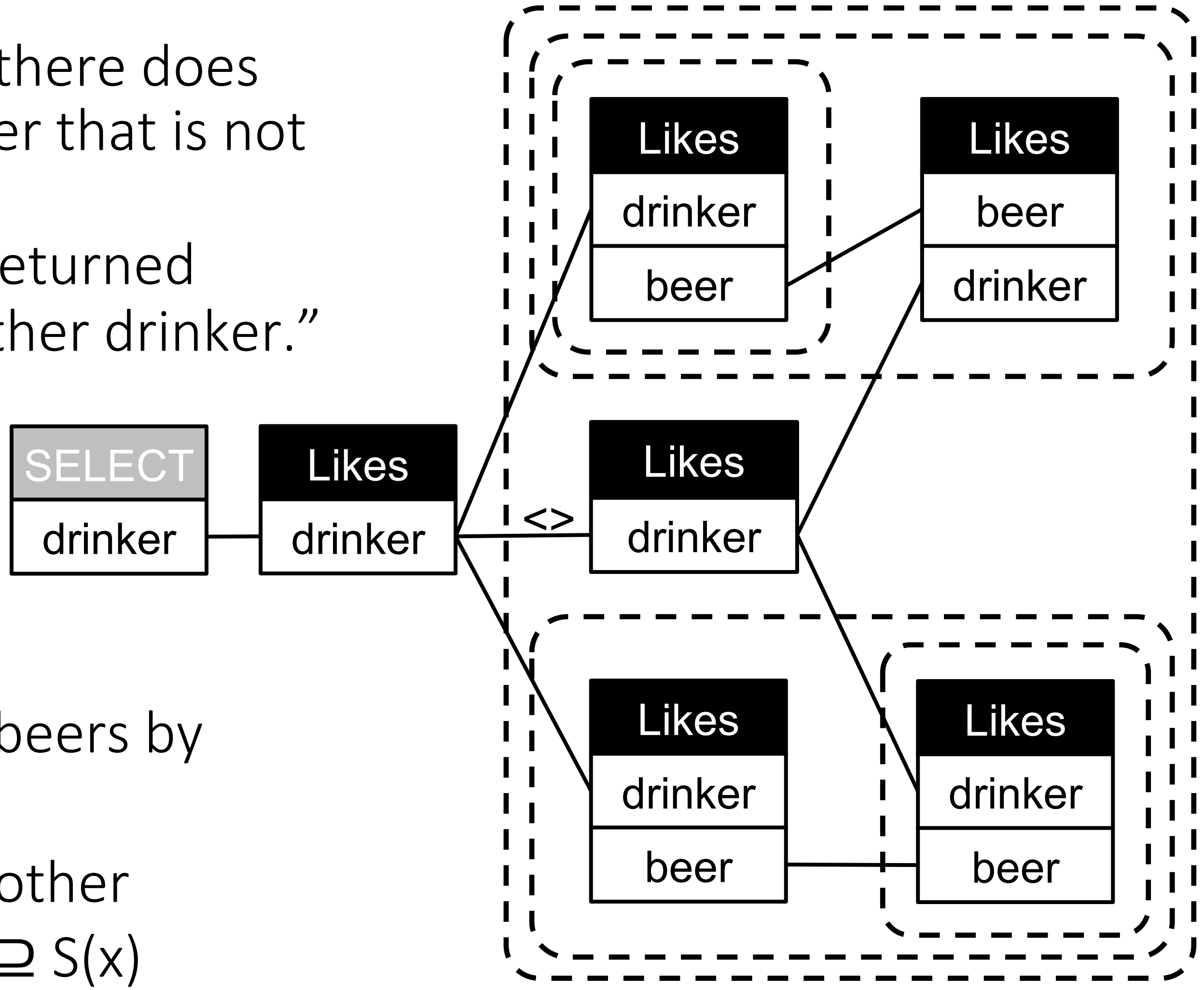
Q: Finder drinkers with a unique beer taste

Likes(drinker,beer)

"Return any drinker, s.t.

- there does not exist any other drinker, s.t. there does not exist any beer liked by that other drinker that is not also liked by the returned drinker, and
- there does not exist any beer liked by the returned drinker that is not also liked by the same other drinker."

"Unique set query"



Let x be a drinker, and S(x) be the set of liked beers by drinker x.
Find any drinker x, s.t. there does not exist another drinker x′, x for which: $S(x') \subseteq S(x)$ and $S(x') \supseteq S(x)$

See: Gatterbauer, Dunne. On the Reasonable Effectiveness of Relational Diagrams, SIGMOD 2024, https://arxiv.org/pdf/2401.04758 ,
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      WHERE L4.drinker = L1.drinker
      AND L4.beer = L3.beer))
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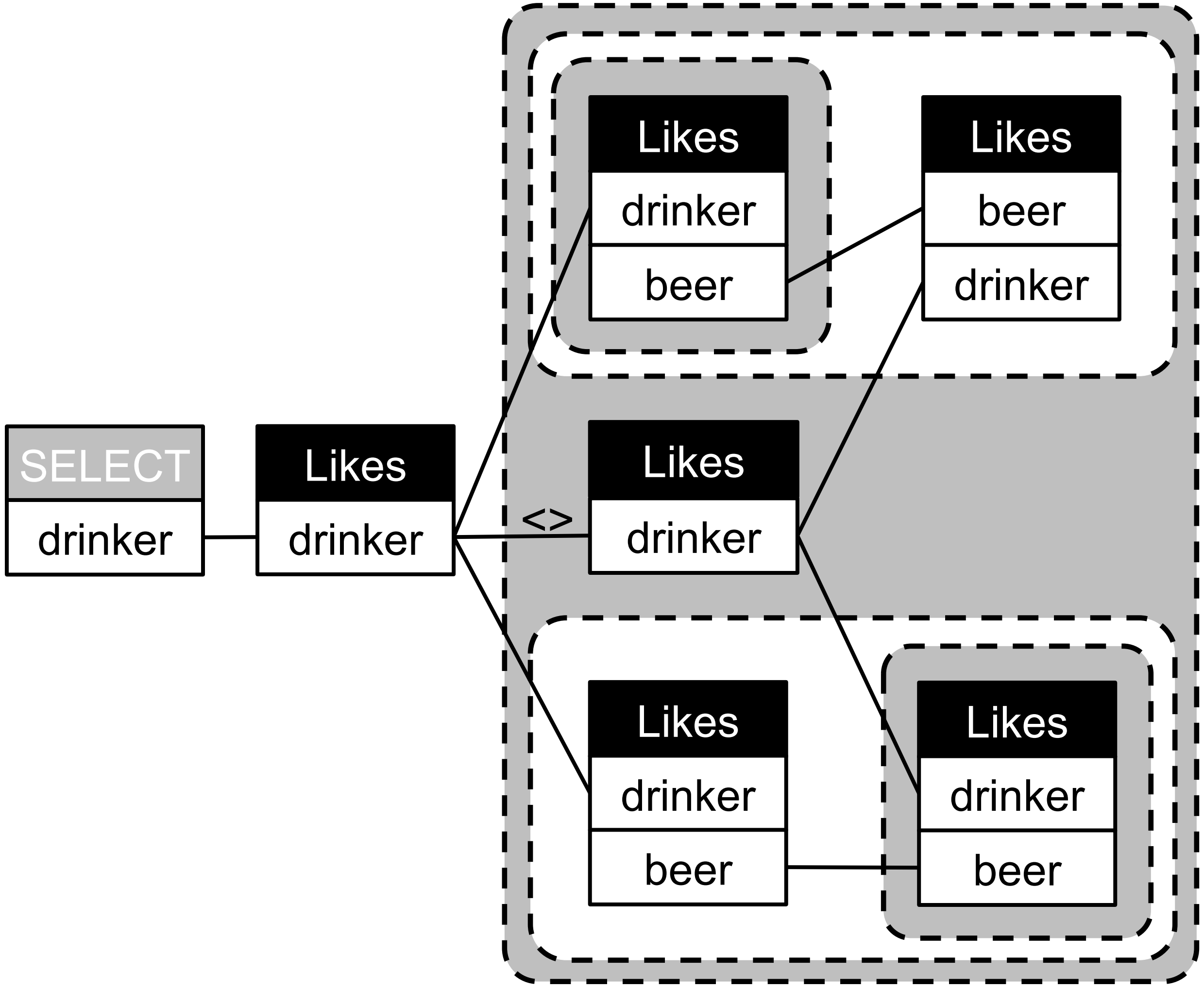
```



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    AND not exists
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      FROM Likes L4
      WHERE L4.drinker = L1.drinker
      AND L4.beer = L3.beer))
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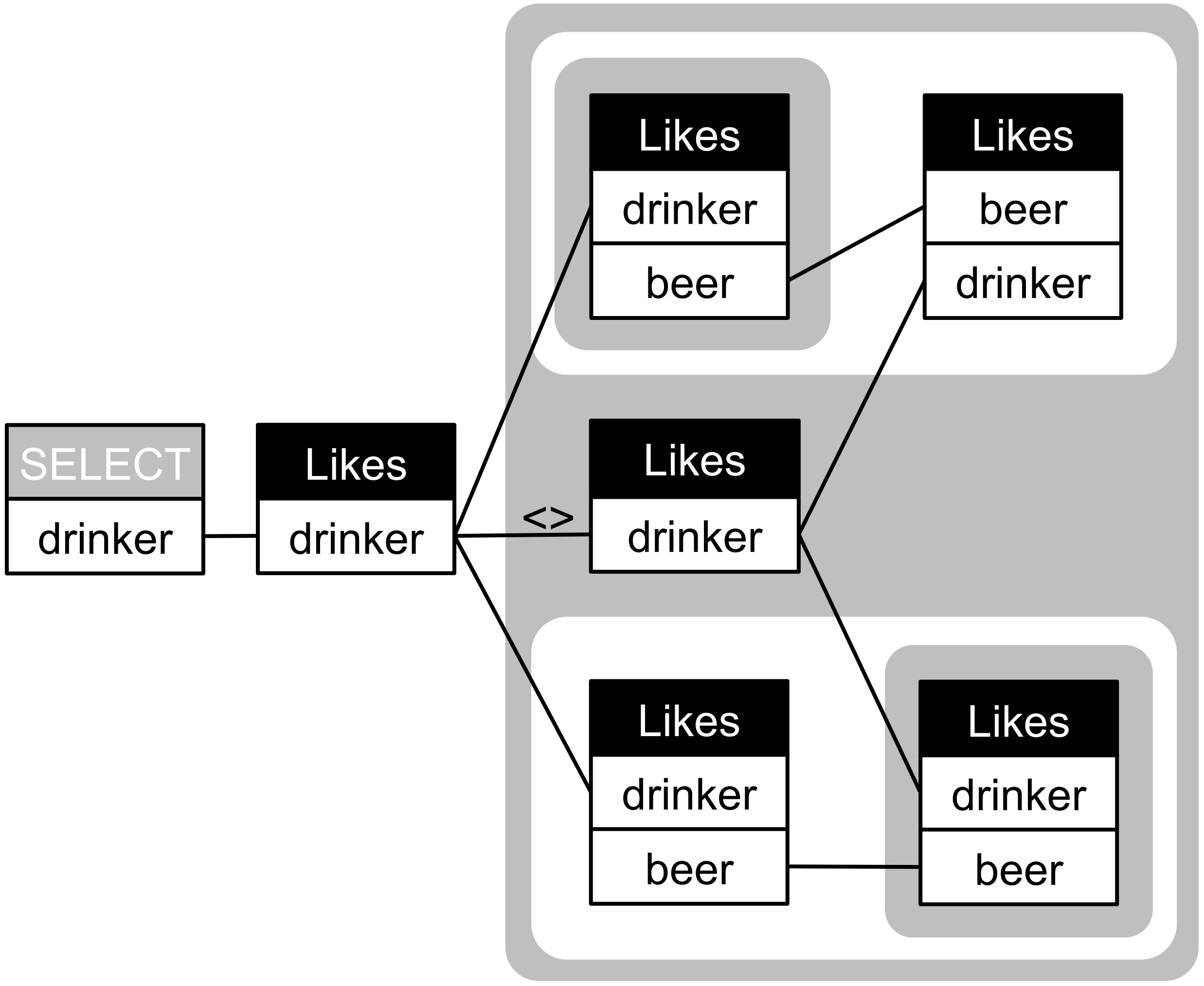

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    WHERE L3.drinker = L2.drinker
    AND not exists
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      FROM Likes L4
      WHERE L4.drinker = L1.drinker
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```
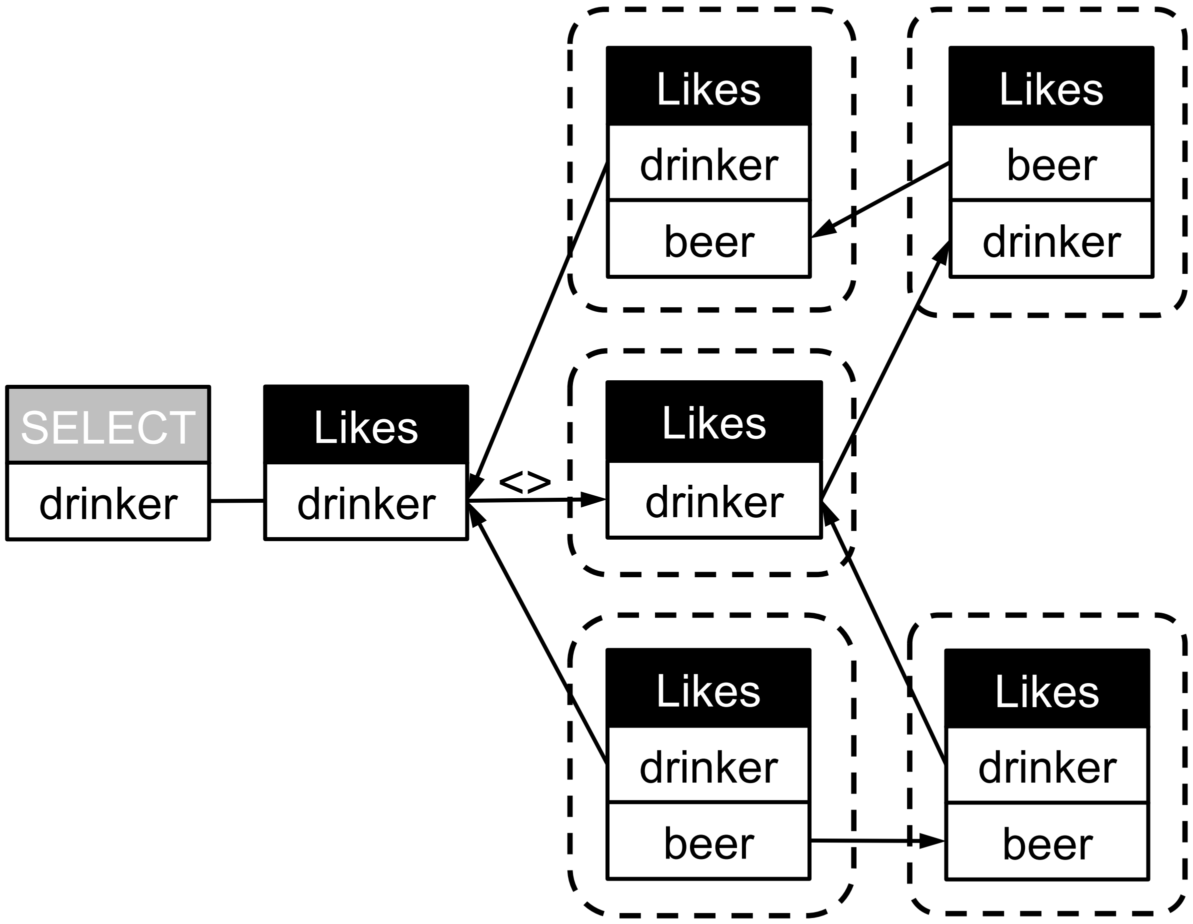

QueryVis scoping

Relational Diagrams scoping



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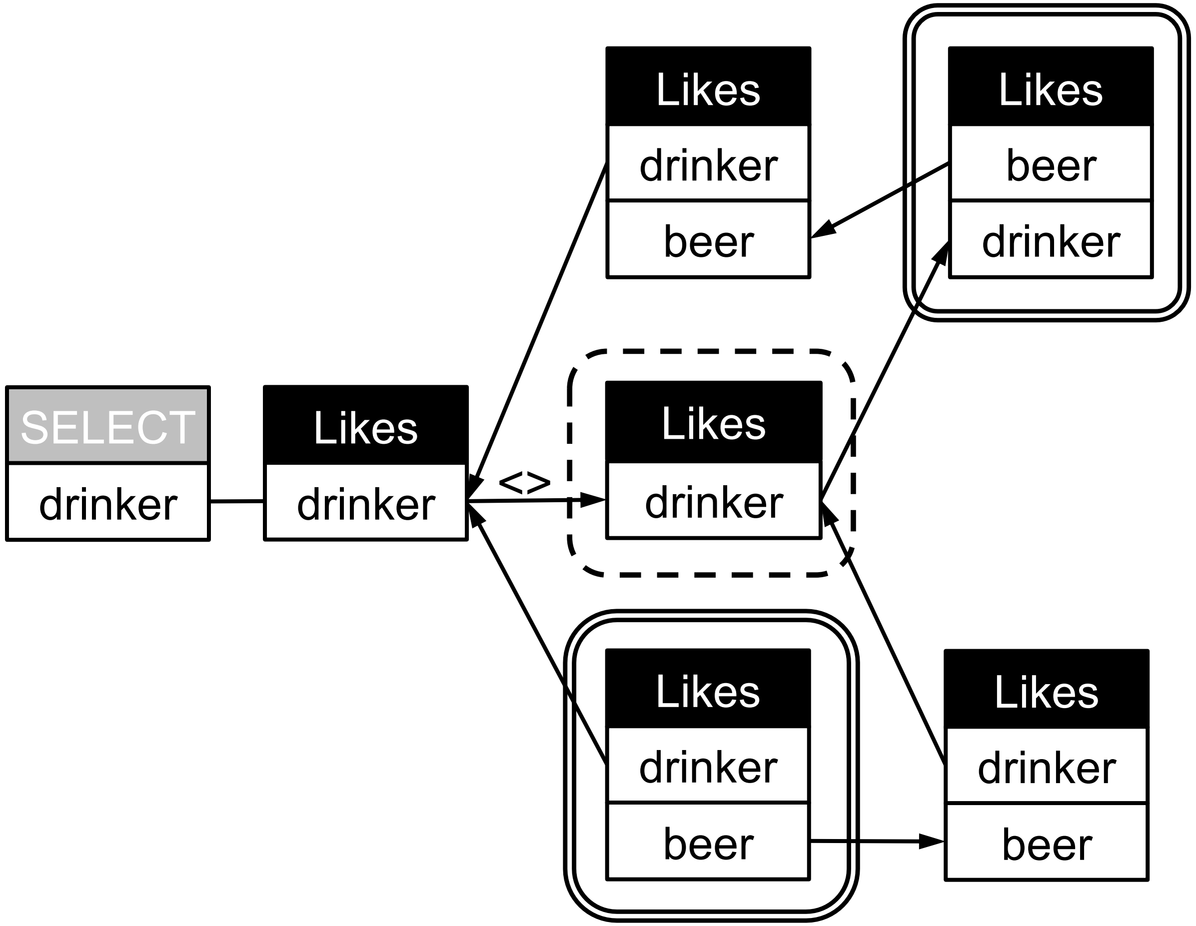

QueryVis scoping

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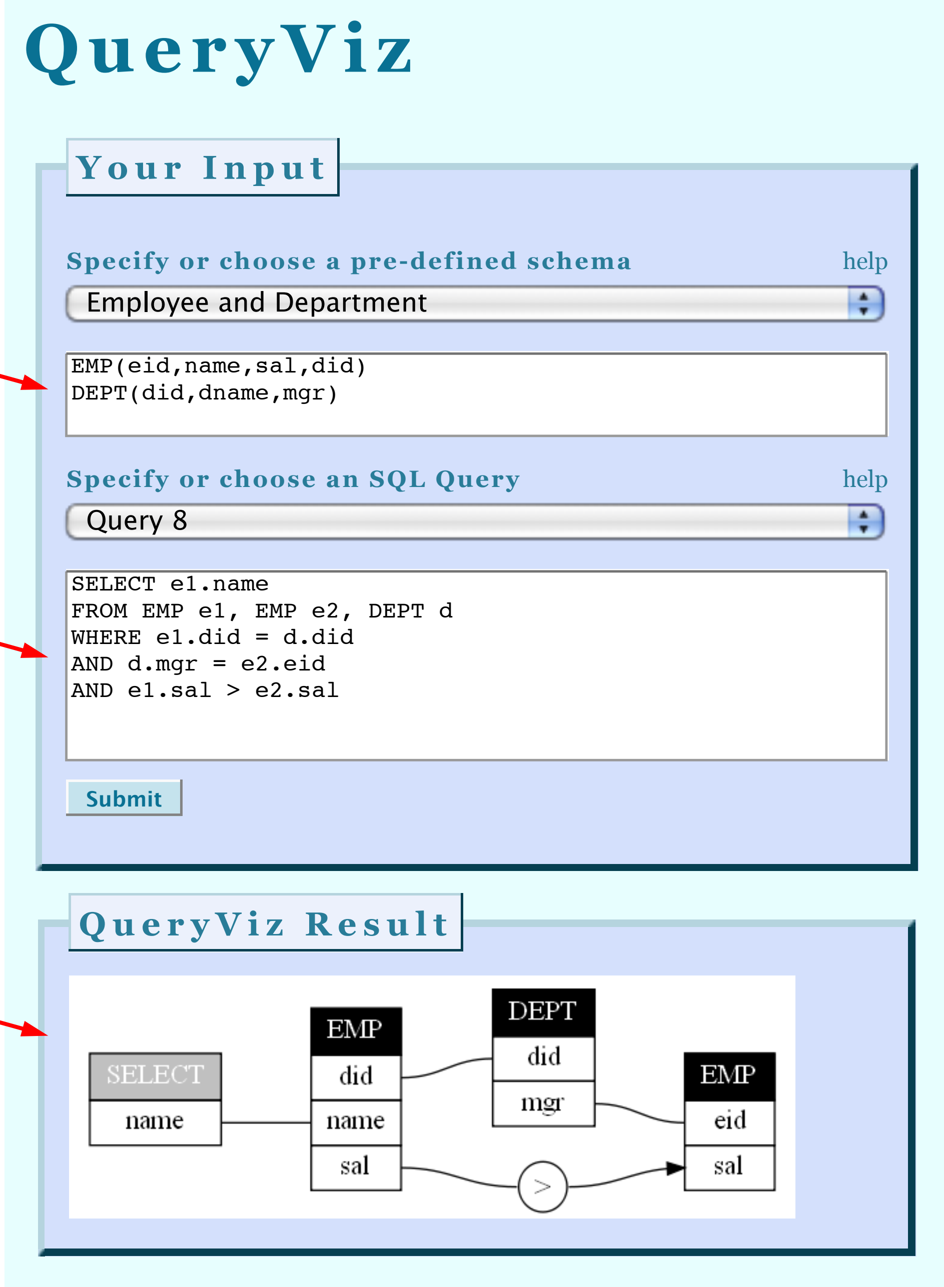
https://demo.queryvis.com

Input: Schema

Input Query

Output: Visualization



https://queryvis.com/

http://www.youtube.com/watch?v=kVFnQRGAQls

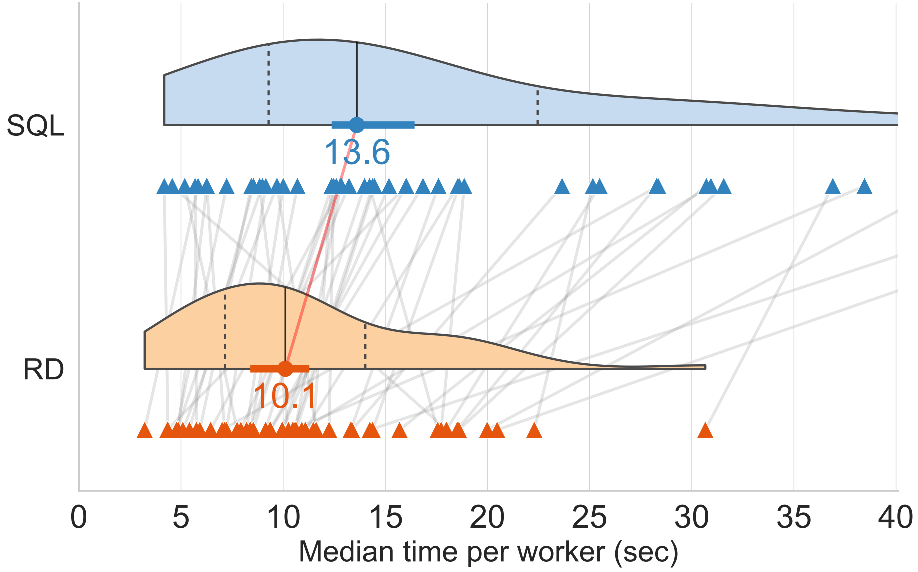
Source: Danaparamita, Gatterbauer: QueryViz: Helping users understand SQL queries and their patterns. EDBT 2011. https://doi.org/10.14778/3402755.3402805

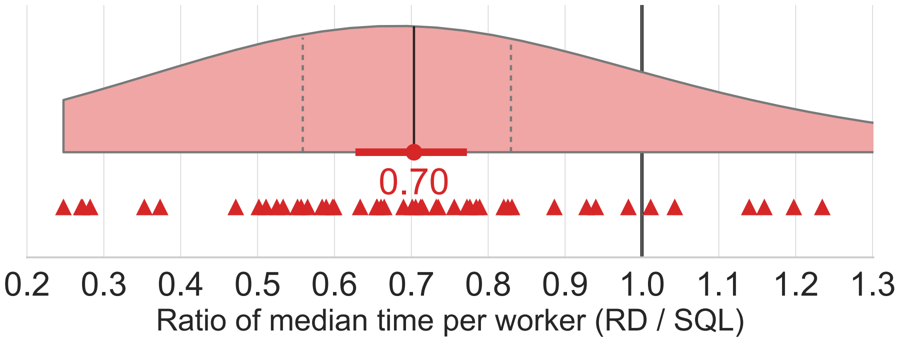
See also: Gatterbauer, Dunne, Jagadish, Riedewald: Principles of Query Visualization. IEEE Debull 2023. http://sites.computer.org/debull/A22sept/p47.pdf

Preregistered, randomized user study on AMT

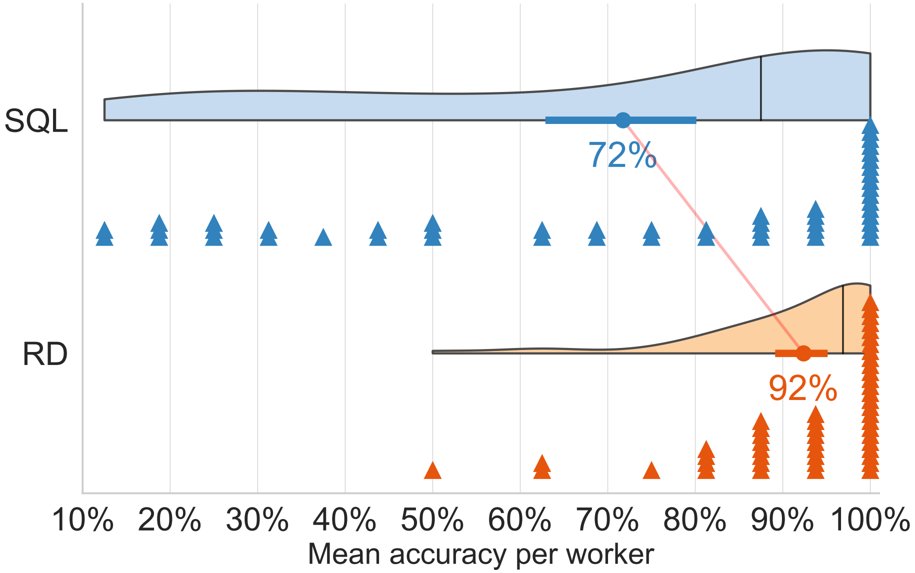
n = 50 participants, preregistration: https://osf.io/4zpsk

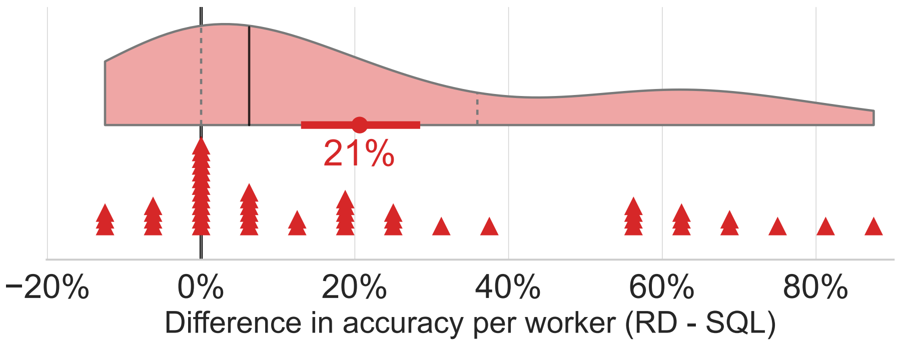
Speed





Accuracy





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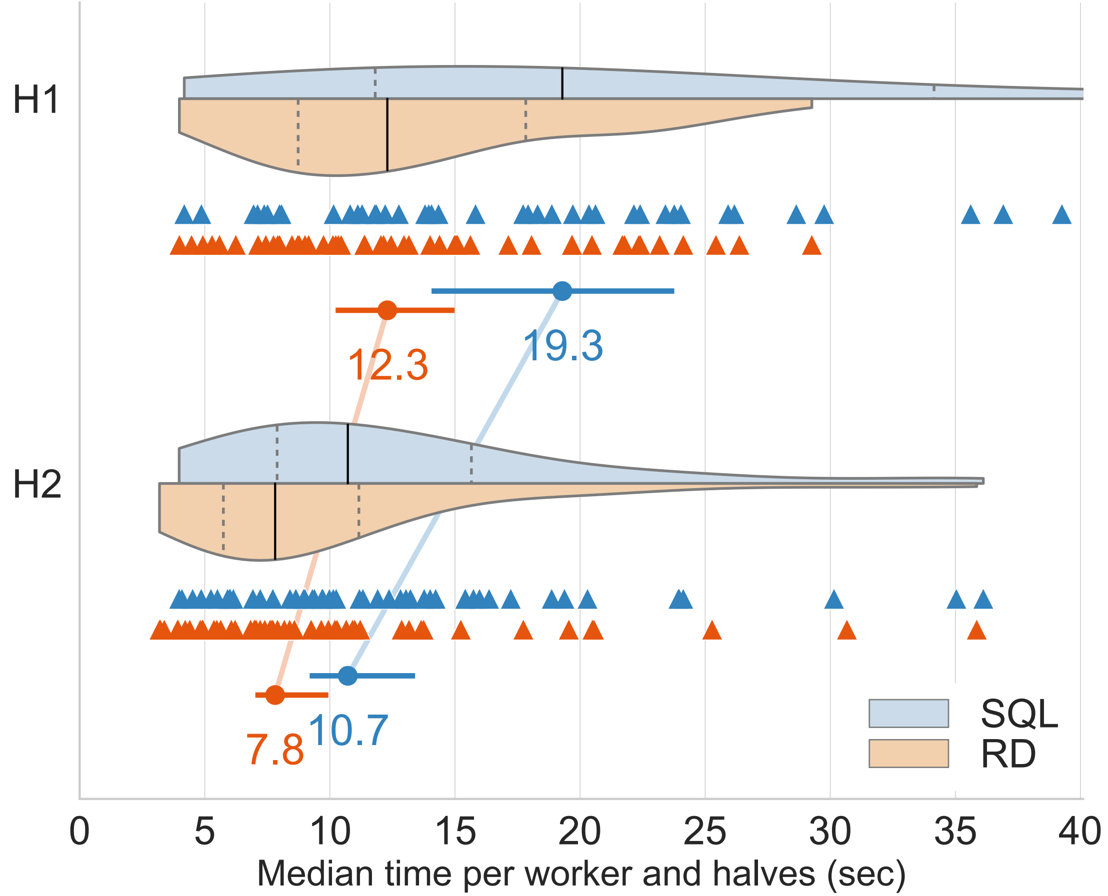
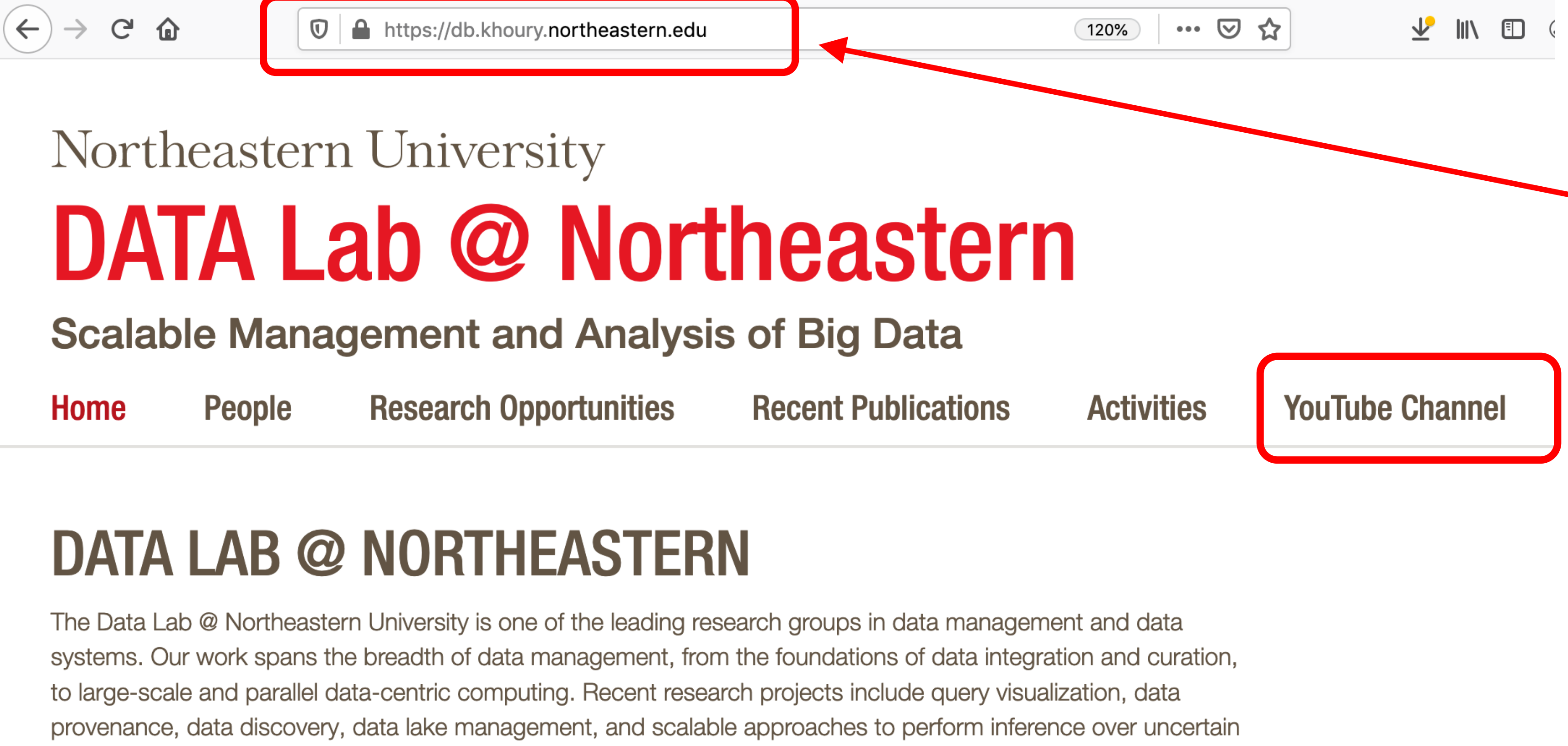
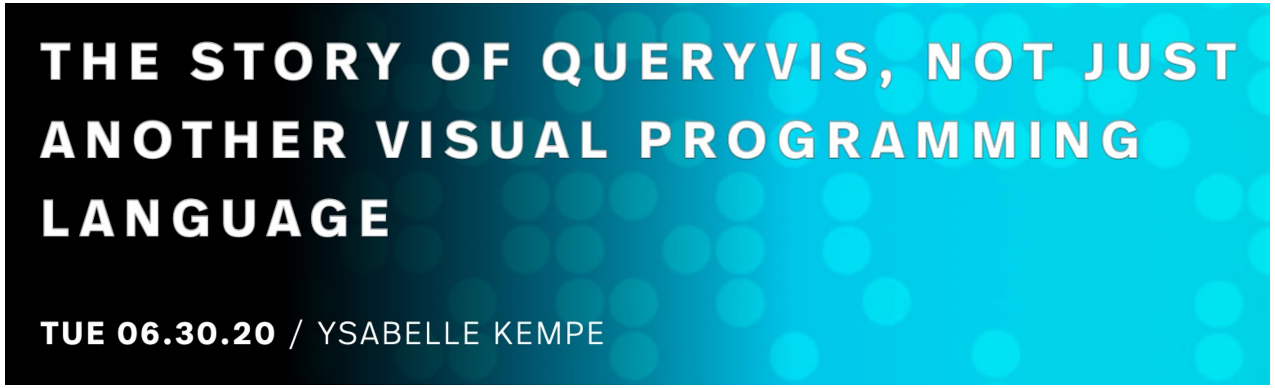
Learning

H1 = first 16 questions
H2 = second 16 questions





https://queryvis.com



https://www.khoury.northeastern.edu/the-story-of-queryvis-not-just-another-visual-programming-language/

Focus: one single nesting level

- We first restrict ourselves to
  - equi-joins (no inequalities like T.A < T.B)
  - paths (no siblings = every node can have only one nested child)
  - one single nesting level
  - Boolean queries
  - no foreign predicates
  - only binary relations (thus can be represented as graphs)
  - only one single relation R
  - (and as before only conjunctions)
- Given two such queries, what is a generalization of the homomorphism procedure that works for that fragment?

Simplifying notation

Schema: R(A,B)

What will become handy, is a short convenient notation for queries

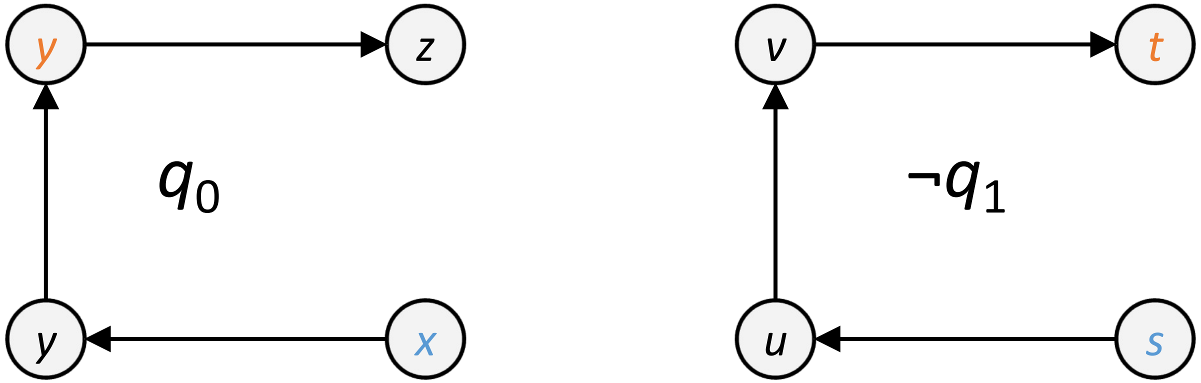
```
SELECT   TRUE
FROM     R R1, R R2, R R3
WHERE    R1.B = R2.A
AND      R2.B = R3.A
NOT EXISTS
         (SELECT *
         FROM     R R4, R R5, R R6
         WHERE    R4.B = R5.A
         AND      R5.B = R6.A
         AND      R4.A = R1.A
         AND      R6.A = R2.B)
```

∃ R1, R2, R3 ∈ R
(R1.B=R2.A ∧ R2.B=R3.A ∧
∄ R4, R5, R6 ∈ R
(R4.B=R5.A ∧ R5.B=R6.A ∧
R4.A=R1.A ∧ R6.A = R2.B)
)

q_0 :- $R(x,y), R(y,z), R(z,w)$

q_1(s,t):- $R(s,u), R(u,v), R(v,t)$, s=x, t=y

q :- $R(x,y), R(y,z), R(z,w), \neg q_1$(x,z)



s=x, t=y

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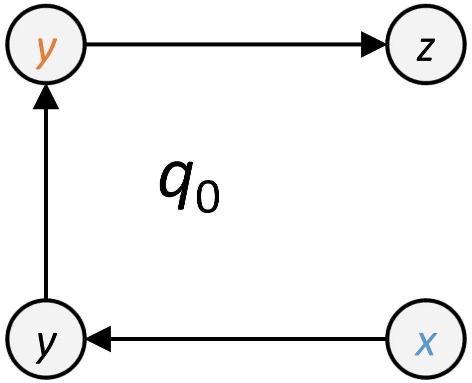
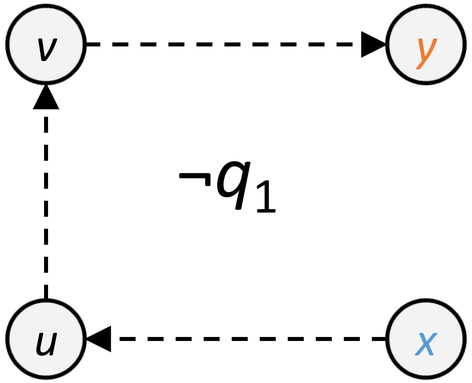
         WHERE    R4.B = R5.A
         AND      R5.B = R6.A
         AND      R4.A = R1.A
         AND      R6.A = R2.B)
```

q_0 :- $R(x,y), R(y,z), R(z,w)$

$\neg q_1$:- $R(x,u), R(u,v), R(v,y)$

$\exists$ R1, R2, R3 $\in$ R
  (R1.B=R2.A $\wedge$ R2.B=R3.A $\wedge$
    $\nexists$ R4, R5, R6 $\in$ R
      (R4.B=R5.A $\wedge$ R5.B=R6.A $\wedge$
      R4.A=R1.A $\wedge$ R6.A = R2.B)
)





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```
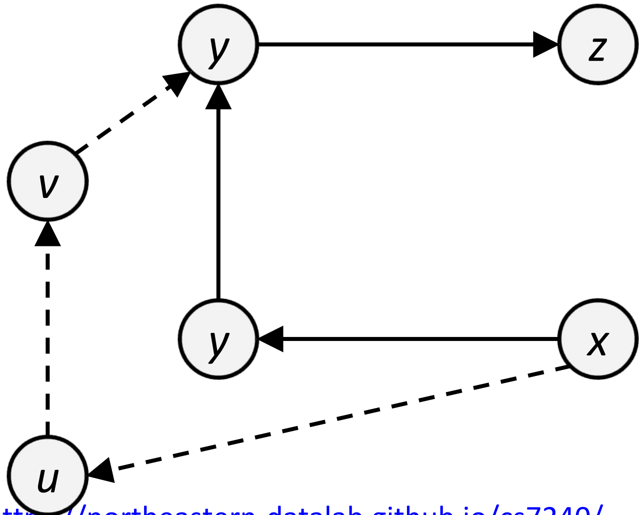

q_0 :- $R(x,y), R(y,z), R(z,w)$

$\neg q_1$:- $R(x,u), R(u,v), R(v,y)$

∃ R1, R2, R3 ∈ R
(R1.B=R2.A ∧ R2.B=R3.A ∧
∄ R4, R5, R6 ∈ R
(R4.B=R5.A ∧ R5.B=R6.A ∧
R4.A=R1.A ∧ R6.A = R2.B)
)



Cartesian product: R'(x,y,z,w)= R(x,y), R(y,z), R(z,w)?
can be expressed in guarded fragment of FOL (with negation)?
But single join already not guarded

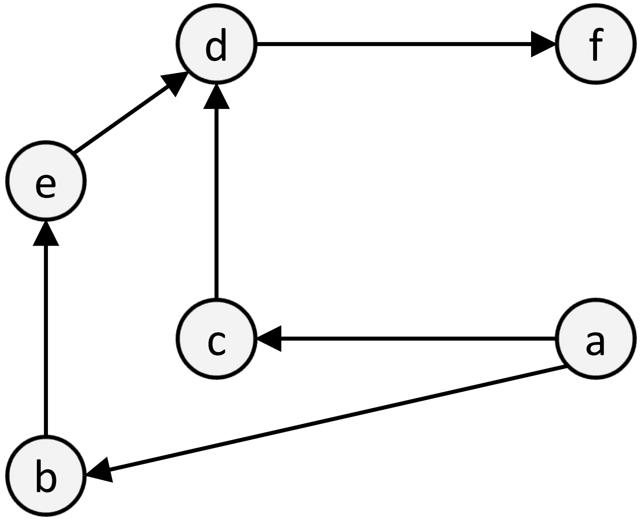
See Barany, Cate, Segoufin, "Guarded negatation", JACM 2015

guardedness

Exercise

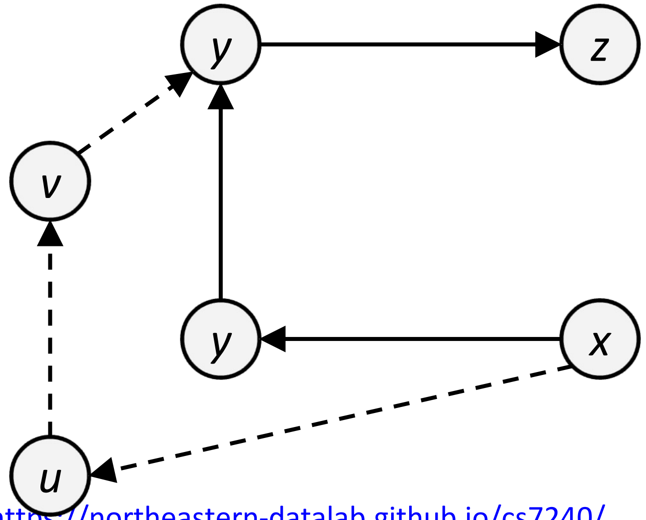
Schema: R(A,B)

Database *D*



Does the query below evaluate to true on above database?

Query *q*

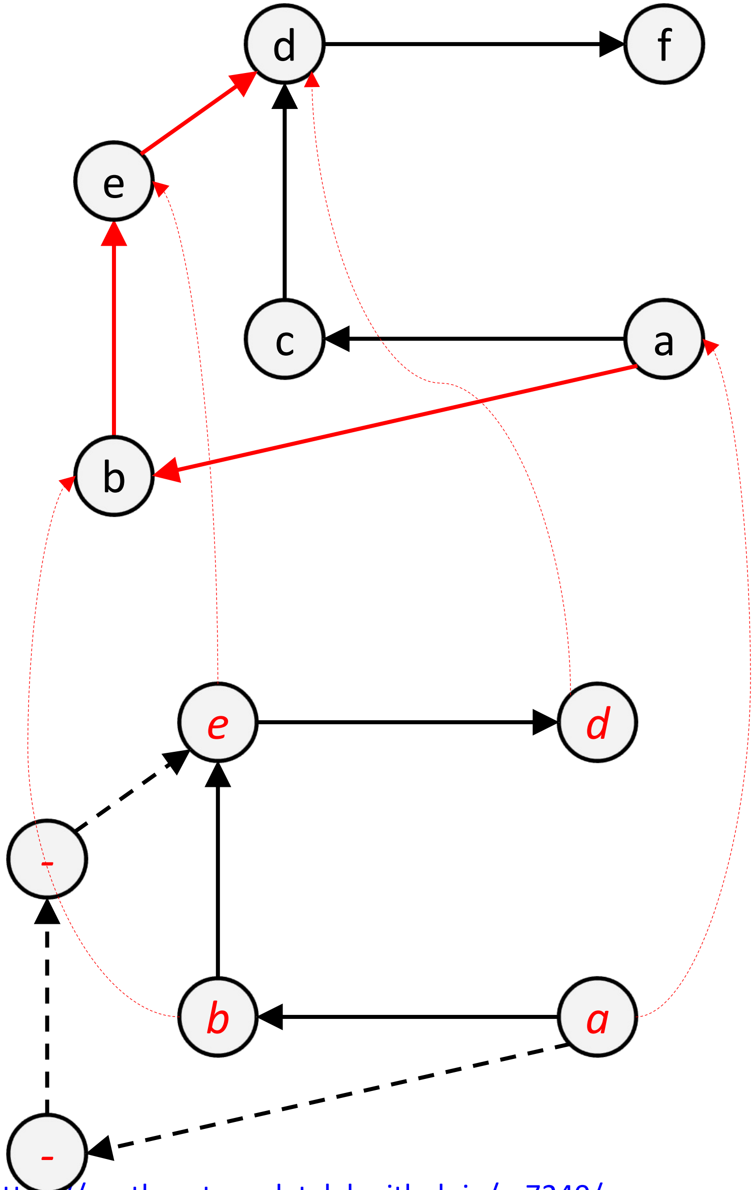


Exercise

Schema: R(A,B)

Database *D*

Query *q*



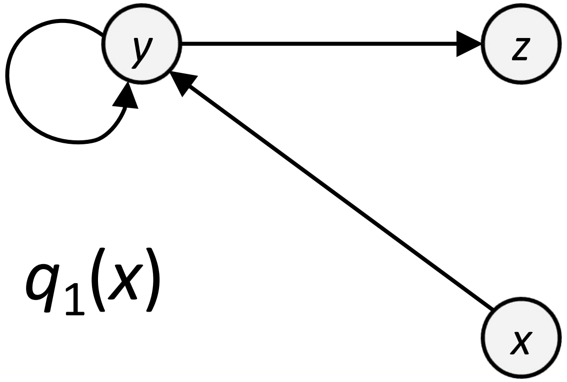
Question

- Find two such nested queries (somehow leveraging the example below) that are equivalent (based on some simple reasoning)
- What is then the *structured* procedure to prove equivalence?

<u>Example</u>

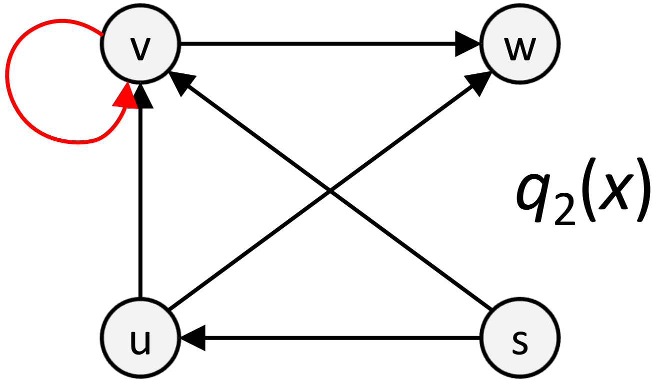
$q_1(x)$:- $R(x,y), R(y,y), R(y,z)$

$q_2(s)$:- $R(s,u), R(u,w), R(s,v), R(u,w), R(u,v)$, $R(v,v)$



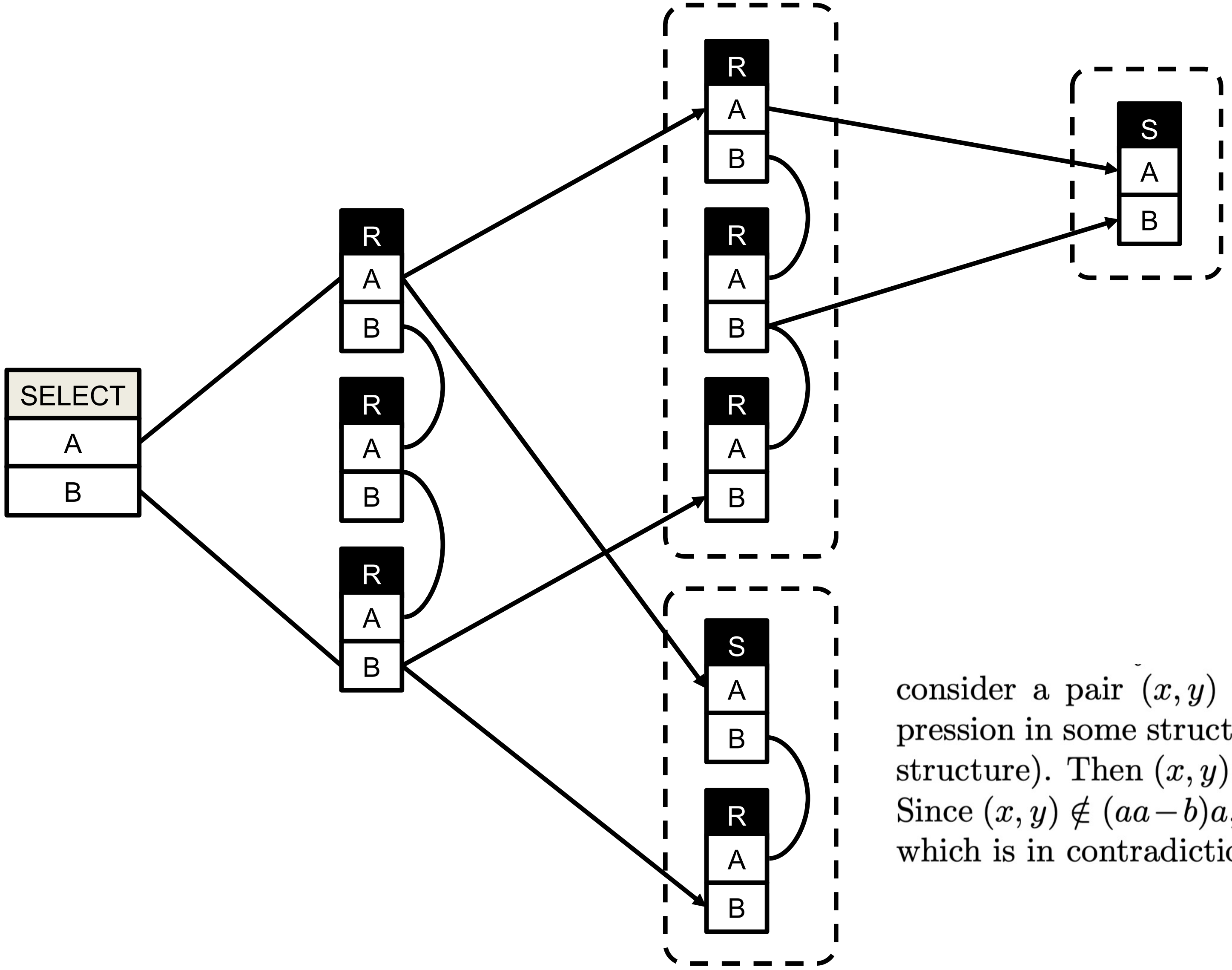
~~$h_{1\to2}$: {(x,s),(y,v),(z,w)}~~ $q_1 \nsubseteq q_2$

$h_{2\to1}$: {(s,x),(u,y),(v,y),(w,z)} $q_1 \subseteq q_2$



Undecidability ☹

- Unfortunately, the following problem is already undecidable
  - Consider the class of nested queries with maximal nesting level 2, no disjunctions, our safety restrictions from earlier, set semantics, arbitrary number of siblings
  - Deciding whether any given query is finitely satisfiable is undecidable.
- This follows non-trivially from from following Arxiv paper:
  - "**Undecidability of satisfiability in the algebra of finite binary relations with union, composition, and difference**" by Tony Tan, Jan Van den Bussche, Xiaowang Zhang, Corr 1406.0349. https://arxiv.org/abs/1406.0349



$= aaa - (aa - b)a - ba$

$= aef - (ae - b)f - bf$

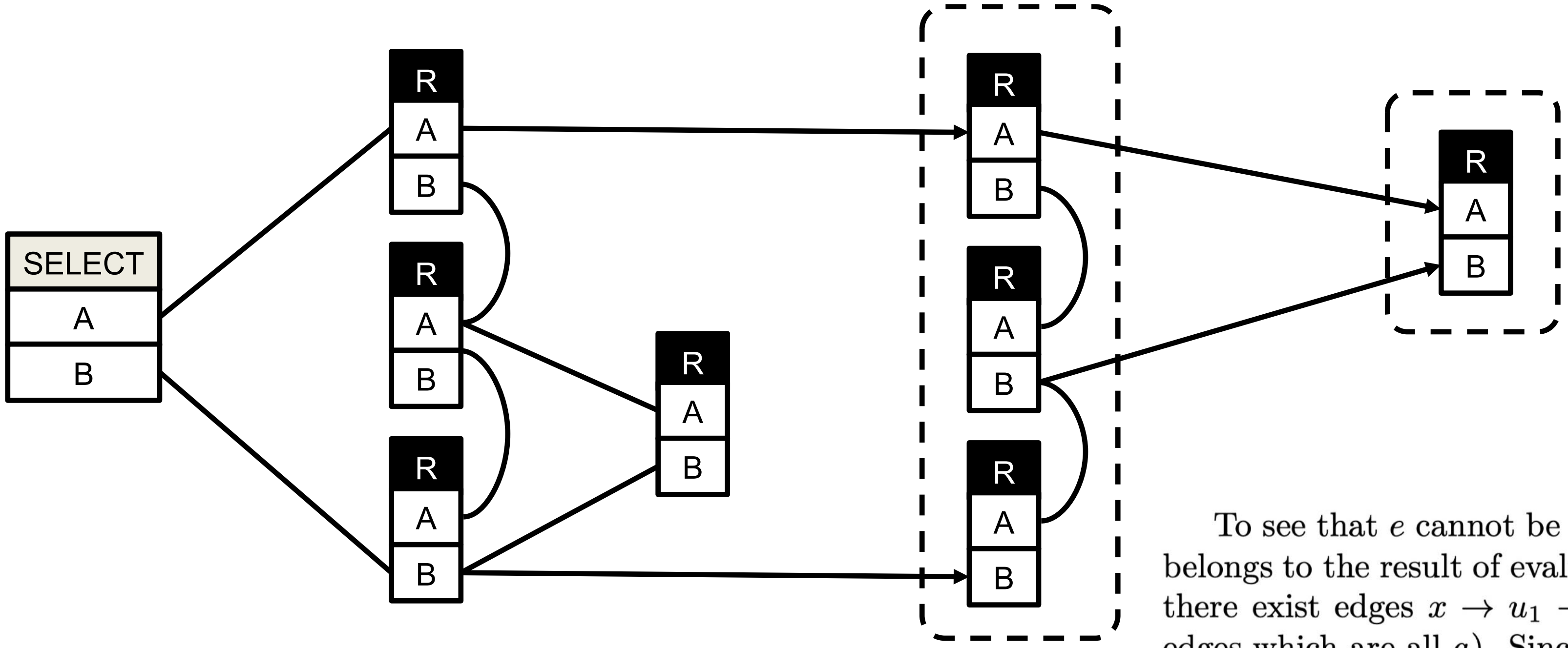
$= aef - aef \cup bf - bf$

consider a pair (x, y) that would belong to the result of evaluating this expression in some structure (for brevity we are omitting explicit reference to this structure). Then $(x, y) \in aaa$ so there exist a-edges (x, x_1), (x_1, x_2), and (x_2, y). Since $(x, y) \notin (aa-b)a$, the b-edge (x, x_2) must be present. But then $(x, y) \in ba$, which is in contradiction with the last part of the expression. □

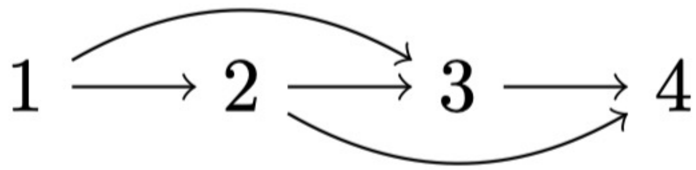
$aaa - ((aa - b)a \cup ba) = aaa - (aa - b)a - ba$

$X - (Y \cup Z) = X - Y - Z$

See "Undecidability of satisfiability in the algebra of finite binary relations with union, composition, and difference" by Tan, Van den Bussche, Zhang. https://arxiv.org/abs/1406.0349



To see that e cannot be satisfied by any series-parallel graph, suppose (x, y) belongs to the result of evaluating e on some structure. Since $(x, y) \in a(a \cap aa)$, there exist edges $x \to u_1 \to u_2 \to y$ and $u_1 \to y$ (we omit the labels on the edges which are all a). Since $(x, y) \notin (aa - a)a$, there must be an edge $x \to u_2$. If at least two of the four elements x, u_1, u_2 and y are identical, the graph contains a cycle and is not series-parallel. If all four elements are distinct, we have a subgraph isomorphic to W above, so the structure is not series-parallel



$$a(aa \cap a) - (aa - a)a$$

See "Undecidability of satisfiability in the algebra of finite binary relations with union, composition, and difference" by Tan, Van den Bussche, Zhang. https://arxiv.org/abs/1406.0349

Open question