

Topic 1: Data models and query languages

Unit 2: Logic & relational calculus

Lecture 6

Wolfgang Gatterbauer

CS7240 Principles of scalable data management (sp23)

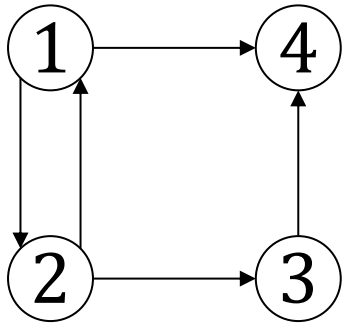
<https://northeastern-datalab.github.io/cs7240/sp23/>

1/27/2023

Pre-class conversations

- Last class recapitulation
 - with more details and intuition
- today:
 - a bit more on logic (I maybe skimming)
 - the relational algebra (RA)

Example: Querying a Graph



What do these queries return ?

$$\{ x \mid \exists y. A(x,y) \}$$

?

$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$

?

$$\{ (x,y) \mid \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

?

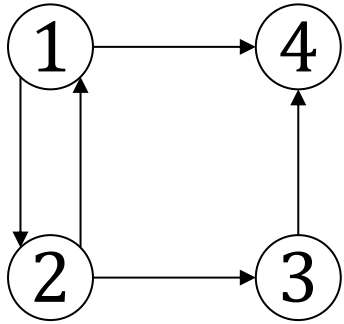
A:

1	2
2	1
2	3
1	4
3	4

A encodes the directed edges of a graph ("arcs")



Example: Querying a Graph



What do these queries return ?

$$\{ x \mid \exists y. A(x,y) \}$$

Nodes that have at least one child:



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1	2
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2	3
1	4
3	4

$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$



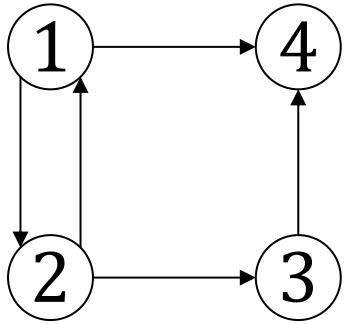
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Nodes that have at least one child: {1,2,3}

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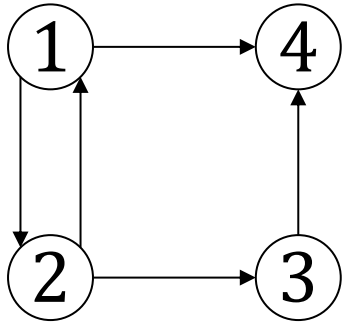
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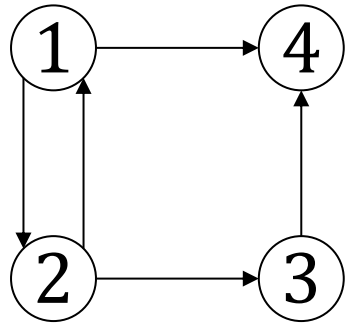
Nodes that have a great-grand-child: ?

$$\{ (x,y) \mid \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

A encodes the directed edges of a graph ("arcs")

?

Example: Querying a Graph



What do these queries return ?

$$\{ x \mid \exists y. A(x,y) \}$$

Nodes that have at least one child: {1,2,3}

$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$

Nodes that have a great-grand-child: {1,2}

*y ≠ u not necessary!
Contrast homomorphism
vs. isomorphism
("Hamiltonian Path")*

A:

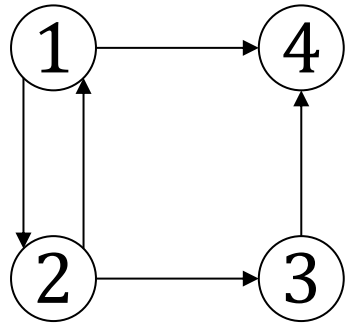
1	2
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$$\{ (x,y) \mid \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

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Example: Querying a Graph



What do these queries return ?

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Nodes that have at least one child: {1,2,3}

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$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$

Nodes that have a great-grand-child: {1,2}

$$\{ (x,y) \mid \exists z. [A(x,z) \wedge \neg A(y,z)] \}$$
$$\{ (x,y) \mid \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

Every child of x is a child of y.

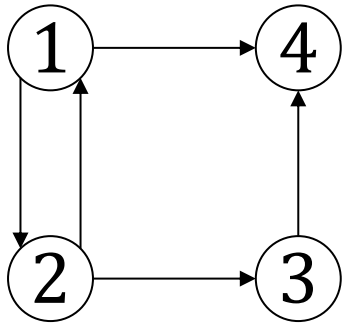
Which of the following tuples fulfill the condition?

(1,3) (3,1)

A encodes the directed edges of a graph ("arcs")

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Example: Querying a Graph



What do these queries return ?

$$\{ x \mid \exists y. A(x,y) \}$$

Nodes that have at least one child: {1,2,3}

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1	2
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$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$

Nodes that have a great-grand-child: {1,2}

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Every child of x is a child of y.

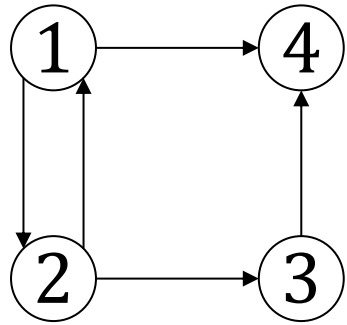
Which of the following tuples fulfill the condition?

~~(1,3)~~ (3,1)

{(1,1),(2,2),(3,1),(3,3),(4,1), (4,2), (4,3), (4,4)} if domain is set of nodes!

A encodes the directed edges of a graph ("arcs")

Example: Querying a Graph



What do these queries return ?

$$\{ x \mid \exists y. A(x,y) \}$$

Nodes that have at least one child: {1,2,3}

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$$\{ x \mid \exists y,z,u. [A(x,y) \wedge A(y,z) \wedge A(z,u)] \}$$

Nodes that have a great-grand-child: {1,2}

$$\{ (x,y) \mid \exists z. [A(x,z) \wedge \neg A(y,z)] \}$$
$$\{ (x,y) \mid \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

Every child of x is a child of y. Which of the following tuples fulfill the condition?

$$\{ (x,y) \mid N(x) \wedge N(y) \wedge \forall z. [A(x,z) \rightarrow A(y,z)] \}$$

{(1,1),(2,2),(3,1),(3,3),(4,1), (4,2), (4,3), (4,4)} if domain is set of nodes!

A encodes the directed edges of a graph ("arcs")

The person/bar/drinks schema

Likes(person, drink)
Frequents(person, bar)
Serves(bar, drink)



What does the following query return?

$$\{ x \mid \forall y. [\text{Frequents}(x, y) \rightarrow \exists z. [\text{Serves}(y, z) \wedge \text{Likes}(x, z)]] \}$$

?

The person/bar/drinks schema

Likes(person, drink)
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What does the following query return?

$$\{ x \mid \forall y. [\text{Frequents}(x, y) \rightarrow \exists z. [\text{Serves}(y, z) \wedge \text{Likes}(x, z)]] \}$$

Find drinkers that frequent only bars
that serve some drink they like.

Is this query domain independent?



The person/bar/drinks schema

Likes(person, drink)
Frequents(person, bar)
Serves(bar, drink)



What does the following query return?

$$\{ x \mid \forall y.[\text{Frequents}(x,y) \rightarrow \exists z.[\text{Serves}(y,z) \wedge \text{Likes}(x,z)]] \}$$

Find drinkers that frequent only bars
that serve some drink they like.

*This query is not domain independent.
How to fix?*

?

*Its output would include all
values from the domain that do
not appear in the Frequents(x,_)*

The person/bar/drinks schema

Likes(person, drink)
Frequents(person, bar)
Serves(bar, drink)



What does the following query return?

Frequents(x,_) \wedge ...
Likes(x,_) \wedge ...

Are those two options to
make it safe identical ?

$$\{ x \mid \forall y. [\text{Frequents}(x,y) \rightarrow \exists z. [\text{Serves}(y,z) \wedge \text{Likes}(x,z)]] \}$$

Find drinkers that frequent only bars
that serve some drink they like.

The person/bar/drinks schema

Likes(person, drink)
Frequents(person, bar)
Serves(bar, drink)



What does the following query return?

$Frequents(x, _) \wedge \dots$

$Likes(x, _) \wedge \dots$

Both safe, but not identical. Tip: Should a drinker who likes a drink but does not frequent any bar be returned?

$\{ x \mid \forall y. [Frequents(x, y) \rightarrow \exists z. [Serves(y, z) \wedge Likes(x, z)]] \}$

Find drinkers that frequent only bars
that serve some drink they like.

Challenge: write this query without the \forall quantifier!
And then in SQL



The person/bar/drinks example



Likes(person, drink)
Frequents(person, bar)
Serves(bar, drink)

Challenge: write these in SQL.

Solutions at: <https://demo.queryvis.com>

Find persons that frequent some bar that serves some drink they like.

?

Find persons that frequent only bars that serve some drink they like.

$$\{ x \mid \exists w. [\text{Likes}(x, w) \wedge \forall y. [\text{Frequents}(x, y) \rightarrow \exists z. [\text{Serves}(y, z) \wedge \text{Likes}(x, z)]]] \}$$

Find persons that frequent some bar that serves only drinks they like.

?

Find persons that frequent only bars that serve only drinks they like.

(= Find persons who like all drinks that are served in all the bars they visit.)

(= Find persons for which there does not exist a bar they frequent that serves a drink they do not like.)

?

SQL example available at: <https://github.com/northeastern-datalab/cs3200-activities/tree/master/sql>

Schema adapted from Jeff Ullman's drinkers/bars/beers example to avoid attributes with same first letters. <https://dl.acm.org/doi/book/10.5555/42790>

Wolfgang Gatterbauer. Principles of scalable data management: <https://northeastern-datalab.github.io/cs7240/>