Updated 6/28/2022

Topic 1: Data models and query languages
Unit 4: Datalog (continued)
Lecture 10

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CS7240 Principles of scalable data management (sp22)

https://northeastern-datalab.github.io/cs7240/sp22/

2/18/2022

Pre-class conversations

- Last class Datalog recapitulation: recursion and stratified negation
- Time for feedback

- today:
  - Datalog with non-stratified negation: playing on the edge of intractability

Outline: T1-4: Datalog

- Datalog
  - Datalog rules
  - Recursion
  - Semantics
  - Datalog$^{\neg}$: Negation, stratification
  - Datalog$^{\pm}$
  - Stable model semantics (Answer set programming)
  - Datalog vs. RA
  - Naive and Semi-naive evaluation (incl. Incremental View Maintenance)

Answer Set Programming (ASP)

- Programming paradigm that can model AI problems (e.g, planning, combinatorics)
- Basic idea
  - Allow non-stratified negation and encode problem (specification & "instance") as logic program rules
  - Solutions are stable models of the program
- Semantics based on Possible Worlds and Stable Models
  - Given an answer set program P, there can be multiple solutions (stable models, answer sets)
  - Each model M: assignment of true/false value to propositions to make all formulas true (combinatorial)
  - Captures default reasoning, non-monotonic reasoning, constrained optimization, exceptions, weak exceptions, preferences, etc., in a natural way
- Finding stable models of answer set programs is not easy
  - Current systems CLASP, DLV, Smodels, etc., extremely sophisticated
  - Work by first grounding the program, suitably transforming it to a propositional theory whose models are stable models of the original program (contrast with "lifted inference" later)
  - These models are found using a SAT solver

Rules with Negation

- Closed world assumption (CWA) as used in standard Datalog:
  - If a fact does not logically follow from a set of Datalog clauses, then we conclude that the negation of this fact is true.
- Problem: CWA can lead to inconsistencies when negation is allowed in rule bodies. Intuition: we can have multiple minimal models ("Herbrand models")

Example 1:

```
boring(chess) :- boring(chess).
```

?

What are all the possible *minimal* models:

- Herbrand universe U_H (set of all constants) ={chess}
- Herbrand base B_H (set of grounded atoms) = {boring(chess)}
- Interpretations (all subsets of B_H) = { {}, {boring(chess)} }
- Model: interpretation that makes each ground instance of each rule true

Rules with Negation

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Example 1:

boring(chess) :- boring(chess).

⇒

$M_1 = \{\}$

What are all the possible *minimal* models:

$M_2 = \{$boring(chess)$\}$ is a model, but not minimal

Rules with Negation

- Closed world assumption (CWA) as used in standard Datalog:
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Example 1:

boring(chess) :- boring(chess).

$\Rightarrow$

M_1 = {}

What are all the possible *minimal* models:

M_2 = {boring(chess)} is a model, but not minimal

Example 2:

boring(chess) :- ¬interesting(chess).

$\Rightarrow$

?

What are all the possible *minimal* models:

Possible interpretations:
{ {},
{b(c)}, {i(c)},
{b(c),i(c)} }

Rules with Negation

- Closed world assumption (CWA) as used in standard Datalog:
  - If a fact does not logically follow from a set of Datalog clauses, then we conclude that the negation of this fact is true.
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Example 1:

boring(chess) :- boring(chess).

M_1 = {}

What are all the possible *minimal* models:

M_2 = {boring(chess)} is a model, but not minimal

Example 2:

boring(chess) :- ¬interesting(chess).

M_1 = {boring(chess)}

M_2 = {interesting(chess)}

What are all the possible *minimal* models:

Semantics: Informally

- Informally, a stable model M of a ground program P is a set of ground atoms such that
  1. Every rule is satisfied:

     i.e., for any rule in P

     $h \text{ :- } a_1, \ldots, a_m, \neg b_1, \ldots, \neg b_n.$

     if each atom a_i is satisfied (a_i's are in M) and no atom b_i is satisfied (i.e. no b_i is in M), then h is in M.
  2. Every $h \in M$ can be derived from a rule by a "non-circular reasoning" (informal for: we are looking for minimal models, or there is some "derivation provenance")

Recall that alternatives to "¬" are "not" and "!". Writing out "not" explicitly is more common in ASP.

Semantics: "non-circular" more formally

Idea: Guess a model M (= a set of atoms). Then verify M is the exact set of atoms that "can be derived" under standard minimal model semantics on P^M on a modified positive program P^M (called "the reduct") derived from P as follows:

1. Create all possible groundings of the rules of program P
2. Delete all grounded rules that contradict M

   $\cancel{h :\text{-}\ a_1, \ldots, a_m, \neg b_1, \ldots, \neg b_n.}$ if some $b_i \in M$

3. In remaining grounded rules, delete all negative literals

   $h :\text{-}\ a_1, \ldots, a_m, \cancel{\neg b_1, \ldots, \neg b_n}.$ if no $b_i \in M$

M is a stable model of P iff M is the least model of P^M

Semantics: "non-circular" more concisely

The reduct of P w.r.t M is:

$$P^M = \left\{ \boxed{h \text{ :- } a_1, \ldots, a_m.} \;\middle|\; \boxed{h \text{ :- } a_1, \ldots, a_m, \neg b_1, \ldots, \neg b_n.} \in \text{grounding of } P \;\wedge\; \text{no } b_i \in M \right\}$$

M is a stable model of P iff M is the least model of P^M

Examples

P1:

```
a :- a.
```

M={a}   Is M a stable model of P1?   ?

Examples

P1: `a :- a.`

M={a} not a stable model (not minimal, derivation of "a" is based on circular reasoning)

? What is a stable model?

Examples

P1: `a :- a.`

M={a} not a stable model (not minimal, derivation of "a" is based on circular reasoning)

M={} stable model

P2: `a :- not b.`

?

{ {a}, {b}, {}, {a,b} }

Examples

P1:

```
a :- a.
```

M={a}   not a stable model (not minimal, derivation of "a" is based on circular reasoning)

M={}   stable model

P2:

```
a :- not b.
```

M={a}   only stable model (contrast with the earlier chess example)

P3:

```
a :- not a.
```

{ {}, {a} }

?

Examples

P1: `a :- a.`

M={a} not a stable model (<u>not minimal</u>, derivation of "a" is based on circular reasoning)

M={} stable model

P2: `a :- not b.`

M={a} only stable model (contrast with the earlier chess example)

P3: `a :- not a.`

~~{a}~~ ~~{}~~

a :- ~~not a~~

{a}

~~a :- not a~~

{}

has no stable model (cp. to earlier "Box(x) :- Item(x), ¬Box(x).")

Examples

P4:

```
a :- not b.
b :- not a.
```

?

Examples

P4:

```
a :- not b.
b :- not a.
```

$M_1=\{a\}$

$M_2=\{b\}$

two stable models

How can you "prove" that M_1 is a stable model?

?

Examples

P4:

a :- not b.

b :- not a.

a :- ~~not b~~.

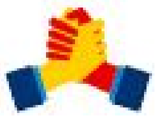
~~b :- not a.~~

M_1={a}

M_2={b}

two stable models

Examples




P4:

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```

$M_1=\{a\}$

$M_2=\{b\}$

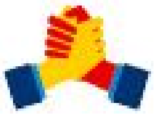
two stable models

P5:

```
a :- not b.
b :- not a.
a :- not a.
```

? { {}, {a}, {b}, {a,b} }

Examples




P4:

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a :- not b.
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```

```
a :- not b.
b :- not a.
```

M_1={a}

M_2={b}

two stable models

P5:

```
a :- not b.
b :- not a.
a :- not a.
```

M={a}

only stable model

How can you "prove" that M is a stable model?

?

Examples

P4:

```
a :- not b.
b :- not a.
```

```
a :- not b.
b :- not a.
```

M_1={a}

M_2={b}

two stable models

P5:

```
a :- not b.
b :- not a.
a :- not a.
```

```
a :- not b.
b :- not a.
a :- not a.
```

M={a}

only stable model

What do empty bodies or heads mean in ASP?

```
a :- b, not c.
```

Think of the head as a disjunction, body as conjunction

$0 \vee a \Leftarrow 1 \wedge b \wedge \neg c$

DLV = "DataLog with Disjunction (=V)"

Empty body:

```
a.
```

?

Empty head:

```
  :- b, not c.
```

?

What do empty bodies or heads mean in ASP?

```
a :- b, not c.
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Think of the head as a disjunction, body as conjunction

$0 \vee a \Leftarrow 1 \wedge b \wedge \neg c$

DLV = "DataLog with Disjunction (=V)"

Empty body:

```
a.
```

$a \Leftarrow 1$

Empty body describes a fact:
"a" needs to be true.
Also in Datalog

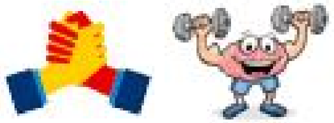
Empty head:

```
:- b, not c.
```

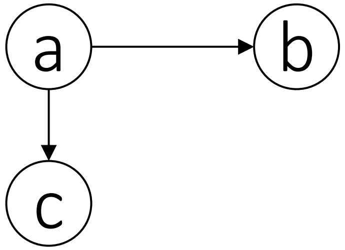
$0 \quad \Leftarrow b \wedge \neg c$

Empty heads describes a constraint: "b and not c" must not be true in any model. Emtpy head describes a condition in the body which leads to contradiction (false)

3-colorability



Q: For a graph (V, E) find an assignment of one of 3 colors to each vertex such that no adjacent vertices share a color.

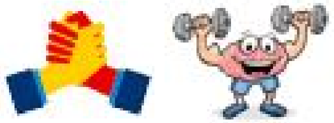


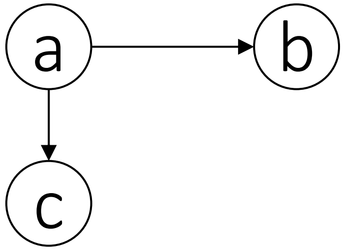
?

Convention in ASP: Capital letters are variables, lower case letters constants

Cp. edge(X,a)
vs. edge(x,'a')

3-colorability





Q: For a graph (V, E) find an assignment of one of 3 colors to each vertex such that no adjacent vertices share a color.

EDB

vertex	A
	a
	b
	c

E	S	T
	a	b
	a	c

```
vertex(a). vertex(b). vertex(c). edge(a,b). edge(a,c).
```

IDB

?

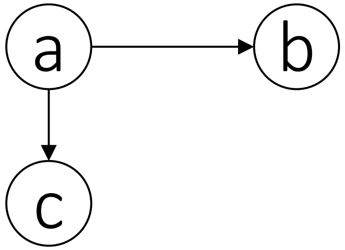
Convention in ASP: Capital letters are variables, lower case letters constants

Cp. edge(X,a)
vs. edge(x,'a')

":- edge(a,X), edge(b,X)" means that "a" and "b" don't share a neighbor

3-colorability

Q: For a graph (V, E) find an assignment of one of 3 colors to each vertex such that no adjacent vertices share a color.



EDB

vertex	X
	a
	b
	c

E	S	T
	a	b
	a	c

```
vertex(a). vertex(b). vertex(c). edge(a,b). edge(a,c).
```

IDB

```
color(V,1) :- not color(V,2), not color(V,3), vertex(V).
color(V,2) :- not color(V,3), not color(V,1), vertex(V).
color(V,3) :- not color(V,1), not color(V,2), vertex(V).
```

?

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Cp. edge(X,a) vs. edge(x,'a')

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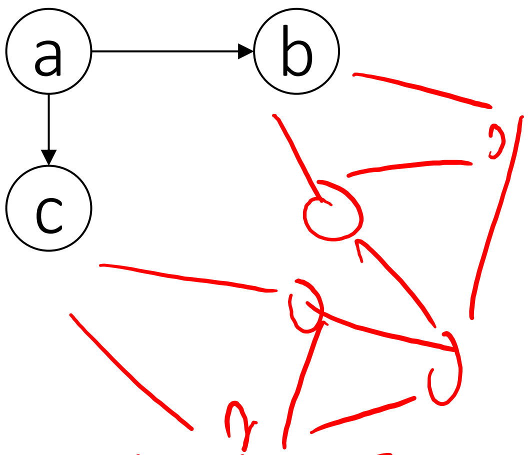

vertex	A
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EDB

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vertex(a). vertex(b). vertex(c). edge(a,b). edge(a,c).
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color(V,1) :- not color(V,2), not color(V,3), vertex(V).
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color(V,3) :- not color(V,1), not color(V,2), vertex(V).

:- edge(V,U), color(V,C), color(U,C).
```

Convention in ASP: Capital letters are variables, lower case letters constants

Cp. edge(X,a) vs. edge(x,'a')

":- edge(a,X), edge(b,X)" means that "a" and "b" don't share a neighbor

Wolfgang Gatterbauer & Dan Suciu

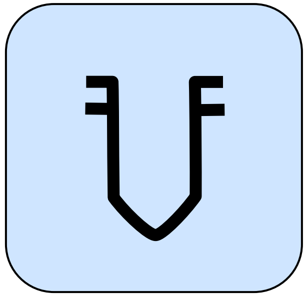
June 8, Sigmod 2010

Paper: https://doi.org/10.1145/1807167.1807193
Full version with proofs: http://arxiv.org/pdf/1012.3320
Old Project web page: https://db.cs.washington.edu/projects/beliefdb/

Problem in social data: often no single ground truth

The Indus Script*

What is the origin
of this glyph?




: cow
Bob


: ship hull
Alice


: jar
Charlie

[*] Current state of knowledge on the Indus Script: Rao et al., Science 324(5931):1165, May 2009
Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Background: Conflicts & Trust in Community DBs

Conflicting beliefs

glyph	*origin*	
	ship hull	Alice
	cow	Bob
	jar	Charlie
	fish	Bob
	knot	Charlie
	arrow	Charlie

"Beliefs": annotated (key,value) pairs

Trust mappings

Alice ← Bob	(100)
Alice ← Charlie	(50)
Bob ← Alice	(80)

Priorities

Bob

100

80

glyph	*origin*
	cow
	fish
	arrow

Alice

50

"Explicit belief"

"Implicit belief"

glyph	*origin*
	ship hull
	fish
	arrow

Charlie

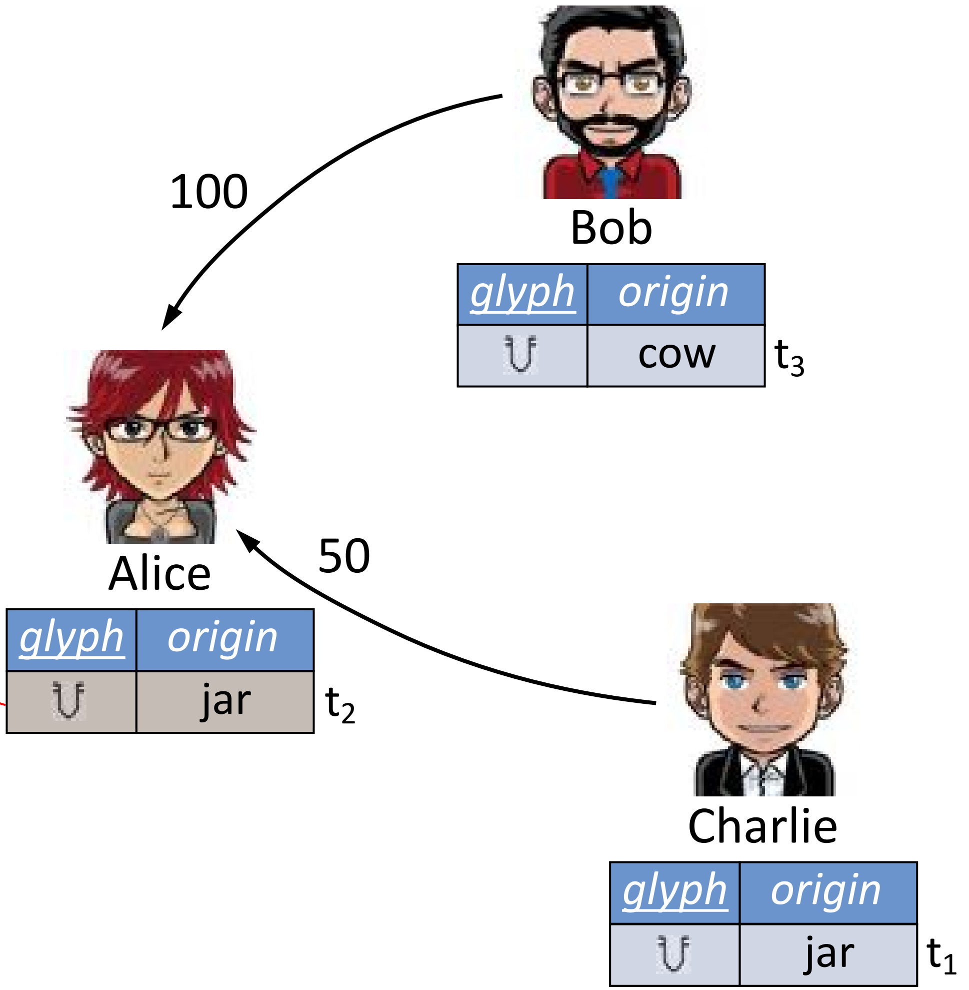
glyph	*origin*
	jar
	knot
	arrow

Recent work on community databases:

- Taylor & Ives [SIGMOD'06] ← Orchestra
- Green et al. [VLDB'07] ← Orchestra
- Kot & Koch [VLDB'09] ← Youtopia
- **GBKS** [VLDB'09] ← BeliefDB

Limitations of previous work: transient effects

1. Incorrect inserts
   – Value depends on order of inserts



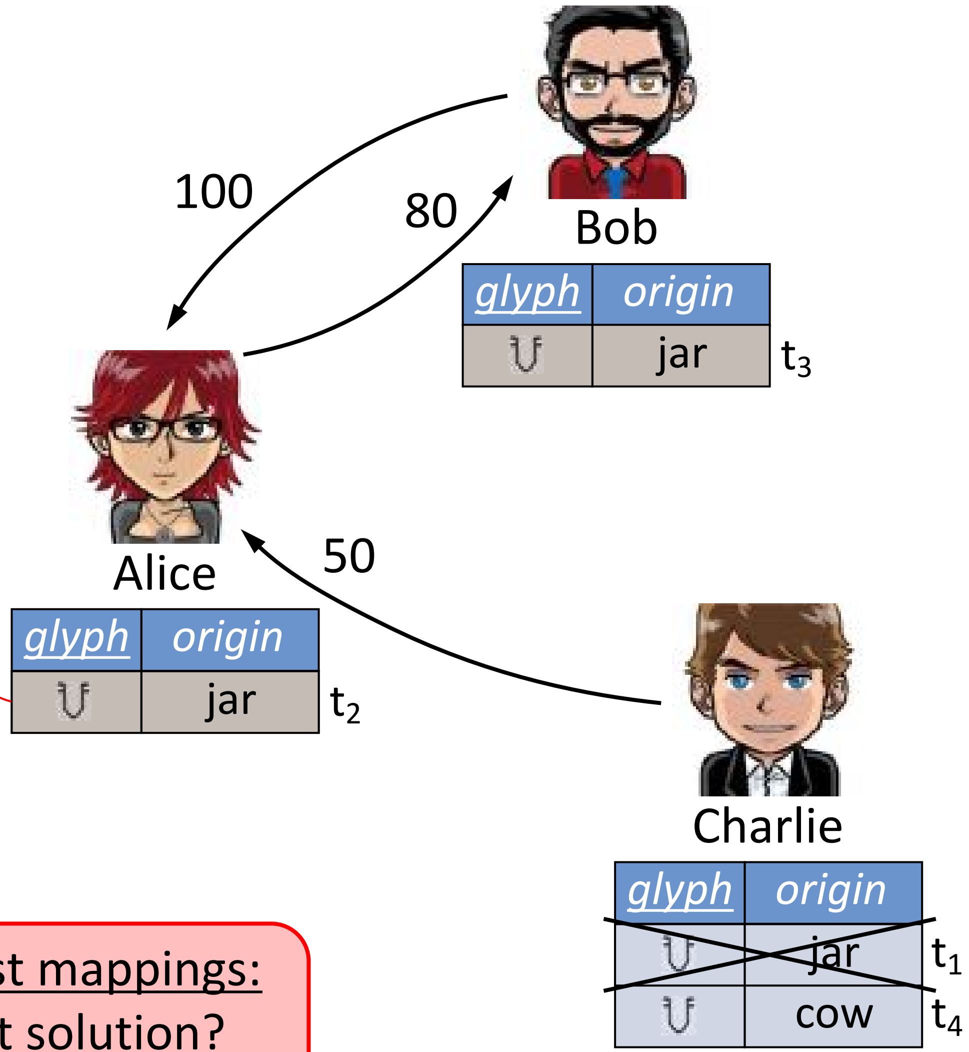
Alice would have
preferred Bob's
value over Charlie's

Limitations of previous work: transient effects

1. Incorrect inserts
   – Value depends on order of inserts

2. Incorrect updates
   – Mis-handling of revokes

Alice and Bob trust each other most, but have lost "justification" for their beliefs



<u>Automatic conflict resolution with trust mappings:</u>
1. How to define a globally consistent solution?
2. How to calculate it efficiently?
(3. Several extensions)

GS [Sigmod'10]

Agenda

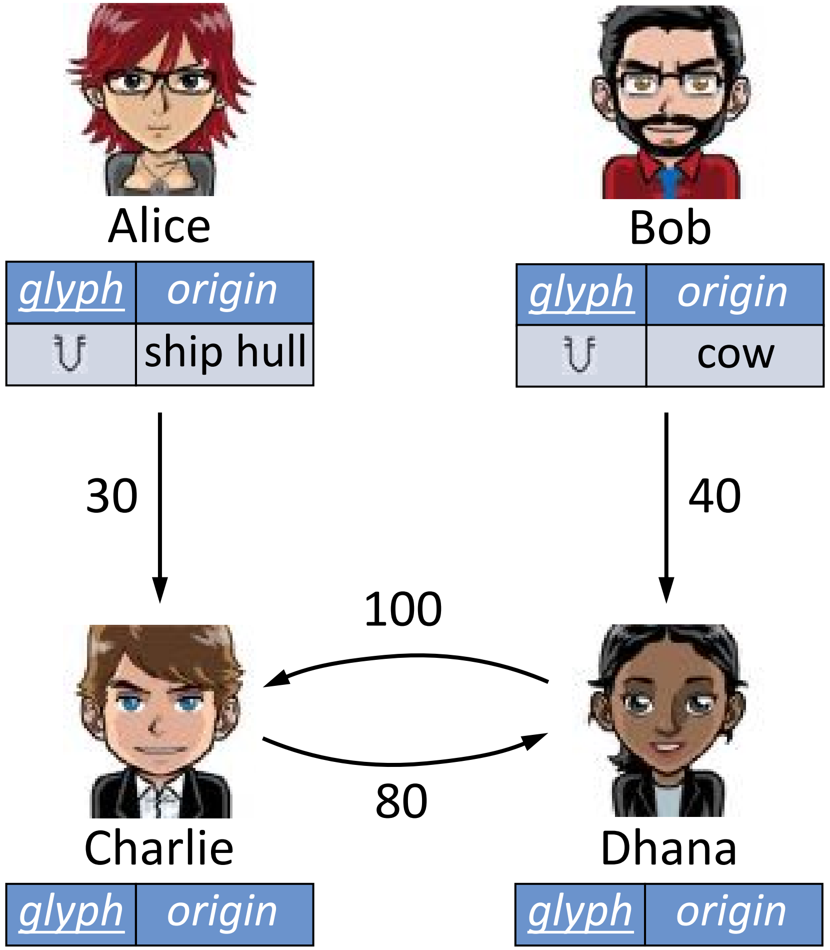
1. Stable solutions
   – how to define a unique and consistent solution?

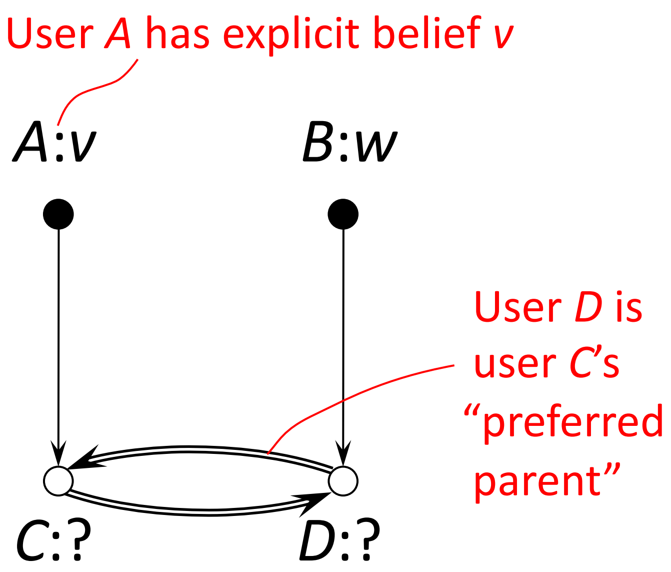
2. Resolution algorithm
   – how to calculate the solution efficiently?

3. Extensions
   – how to deal with “negative beliefs”?

Binary Trust Networks (BTNs)

To simplify presentation: focus on binary TNs



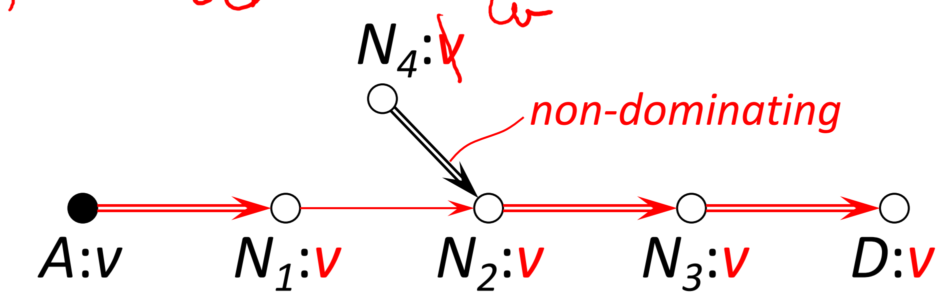


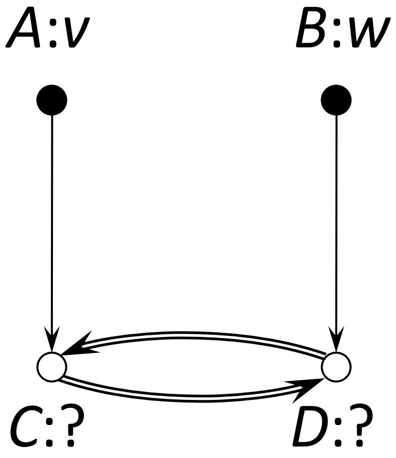
Focus on one single key
(we ignore the glyph)

The definition of a globally consistent solution

- Stable solution
  - assignment of values to each node, s.t. each belief has a "*non-dominated ~~lineage~~*" to an explicit belief

Provenance

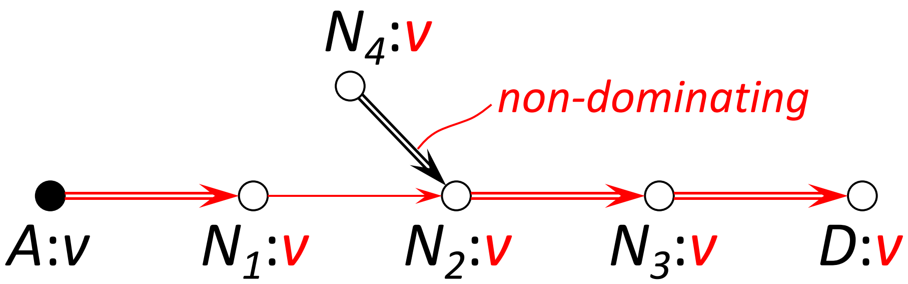


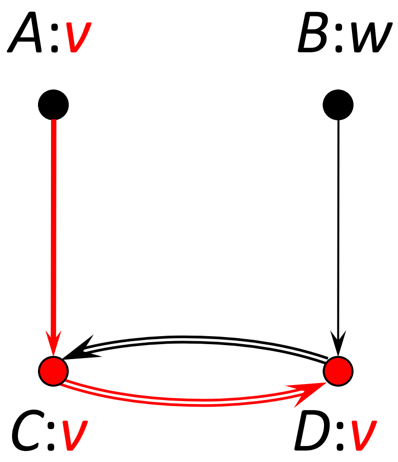


Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

The definition of a globally consistent solution

- Stable solution
  - assignment of values to each node, s.t. each belief has a "*non-dominated lineage*" to an explicit belief

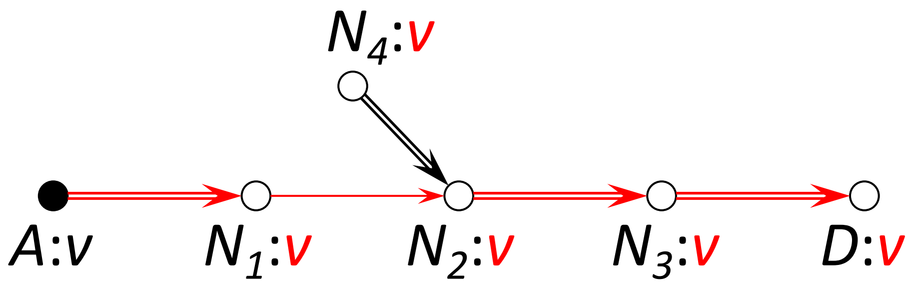


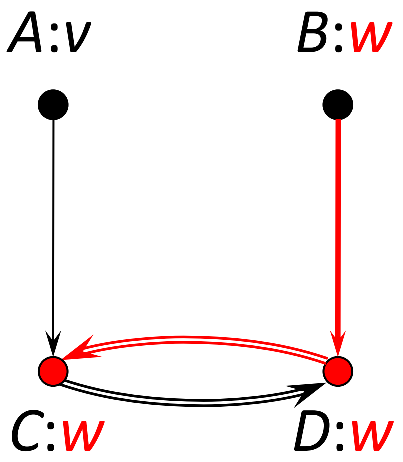


SS_1=(*A*:*v*, *B*:*w*, *C*:*v*, *D*:*v*)

The definition of a globally consistent solution

- Stable solution
  - assignment of values to each node, s.t. each belief has a *"non-dominated lineage"* to an explicit belief

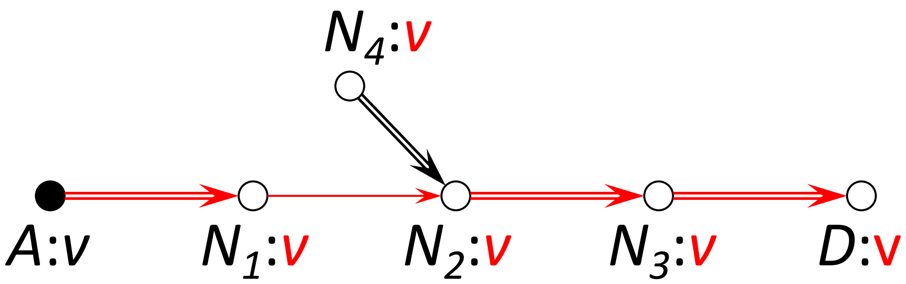


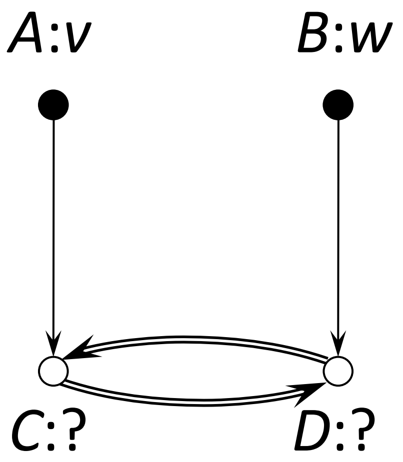


SS_1=(*A*:*v*, *B*:*w*, *C*:*v*, *D*:*v*)
SS_2=(*A*:*v*, *B*:*w*, *C*:*w*, *D*:*w*)

Possible and certain values from all stable solutions

- Stable solution
  - assignment of values to each node, s.t. each belief has a "*non-dominated lineage*" to an explicit belief





SS_1=(*A*:*v*, *B*:*w*, *C*:*v*, *D*:*v*)
SS_2=(*A*:*v*, *B*:*w*, *C*:*w*, *D*:*w*)

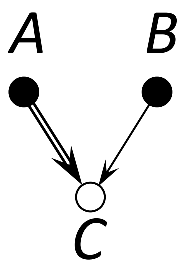
- Possible / Certain semantics
  - a stable solution determines, for each node, a possible value ("**poss**")
  - certain value ("**cert**") = intersection of all stable solutions, per user

X	**poss**(*X*)	**cert**(*X*)
A	{*v*}	{*v*}
B	{*w*}	{*w*}
C	{*v*,*w*}	∅
D	{*v*,*w*}	∅

Logic programs (LP) with stable model semantics

Convention from LP solver DLV: constants and predicates start with lowercase letters, variables with uppercase letters.

- LPs can capture this semantics.



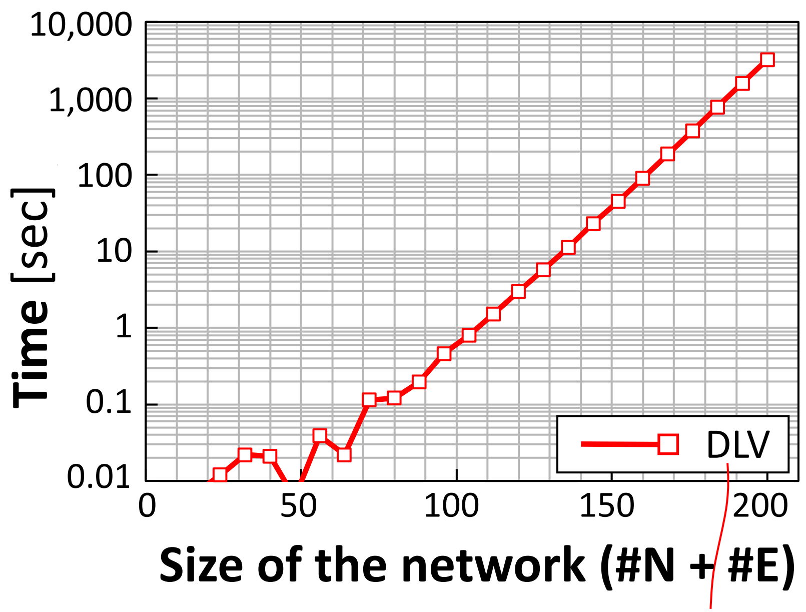

```
   poss(c,X) :- poss(a,X).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
   poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

- There exist powerful and free LP solver available.

- Previous work on peer data exchange suggest using LPs.

Greco et al. [TKDE'03]
Arenas et al. [TLP'03]
Barcelo, Bertossi [PADL'03]
Bertossi, Bravo [LPAR'07]

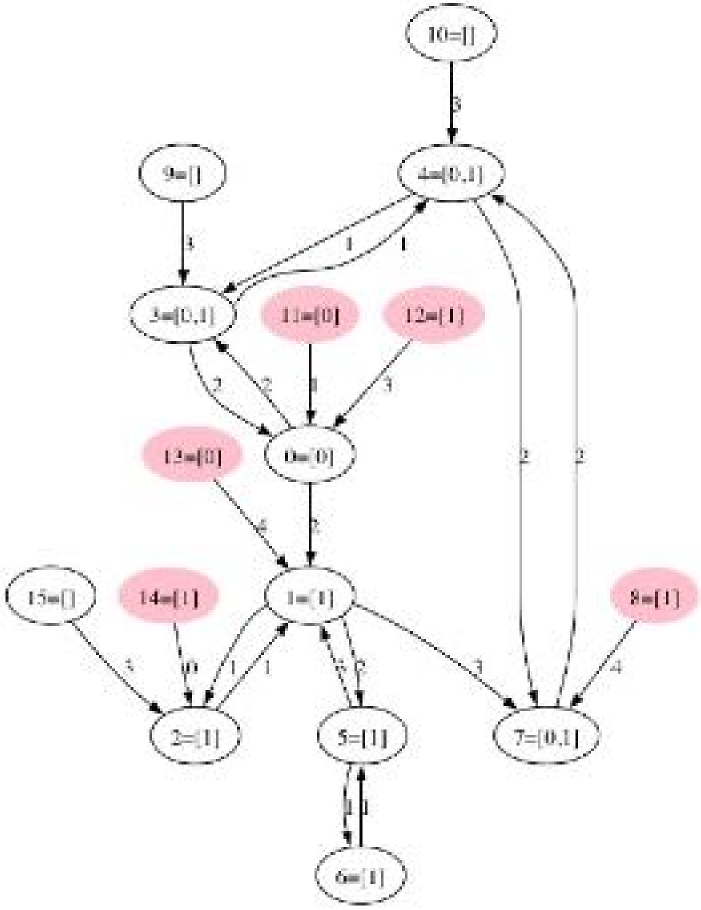
But solving LPs is hard ☹



State-of-the-art LP solver

Yet surprisingly, our problem allows a PTIME solution ☺

DLV example



Size: 38

input.txt

```
% --- Insert explicit beliefs ---
possH(h8_0,1).
possH(h11_0,0).
possH(h12_0,1).
possH(h13_0,0).
possH(h14_0,1).
% --- Node: 0 ---
possH(h0_1,X)       :- possH(h0_0,X).
block(h0_1,11,X)    :- poss(11,X), possH(h0_1,Y), Y!=X.
possH(h0_1,X)       :- poss(11,X), not block(h0_1,11,X).
possH(h0_2,X)       :- possH(h0_1,X).
block(h0_2,3,X)     :- poss(3,X), possH(h0_2,Y), Y!=X.
possH(h0_2,X)       :- poss(3,X), not block(h0_2,3,X).
possH(h0_3,X)       :- possH(h0_2,X).
block(h0_3,12,X)    :- poss(12,X), possH(h0_3,Y), Y!=X.
possH(h0_3,X)       :- poss(12,X), not block(h0_3,12,X).
poss(0,X)           :- possH(h0_3,X).
% --- Node: 1 ---
possH(h1_1,X)       :- possH(h1_0,X).
block(h1_1,2,X)     :- poss(2,X), possH(h1_1,Y), Y!=X.
possH(h1_1,X)       :- poss(2,X), not block(h1_1,2,X).
possH(h1_2,X)       :- possH(h1_1,X).
block(h1_2,0,X)     :- poss(0,X), possH(h1_2,Y), Y!=X.
possH(h1_2,X)       :- poss(0,X), not block(h1_2,0,X).
possH(h1_3,X)       :- possH(h1_2,X).
block(h1_3,5,X)     :- poss(5,X), possH(h1_3,Y), Y!=X.
possH(h1_3,X)       :- poss(5,X), not block(h1_3,5,X).
possH(h1_4,X)       :- possH(h1_3,X).
block(h1_4,13,X)    :- poss(13,X), possH(h1_4,Y), Y!=X.
possH(h1_4,X)       :- poss(13,X), not block(h1_4,13,X).
poss(1,X)           :- possH(h1_4,X).
% --- Node: 2 ---
. . . . . .

% --- Node: 13 ---
poss(13,X)          :- possH(h13_0,X).
% --- Node: 14 ---
poss(14,X)          :- possH(h14_0,X).
% --- Node: 15 ---
poss(15,X)          :- possH(h15_0,X).
```

query.txt

```
poss(X,U) ?
```

Executing program

```
./dlv.bin – brave
    input.txt. query-.txt
```

Result

```
Macintosh-2:DLV gat!
8, 1
11, 0
12, 1
13, 0
14, 1
0, 0
1, 1
2, 1
3, 0
3, 1
4, 0
4, 1
5, 1
6, 1
7, 0
7, 1
```

Agenda

1. Stable solutions
   – how to define a unique and consistent solution?

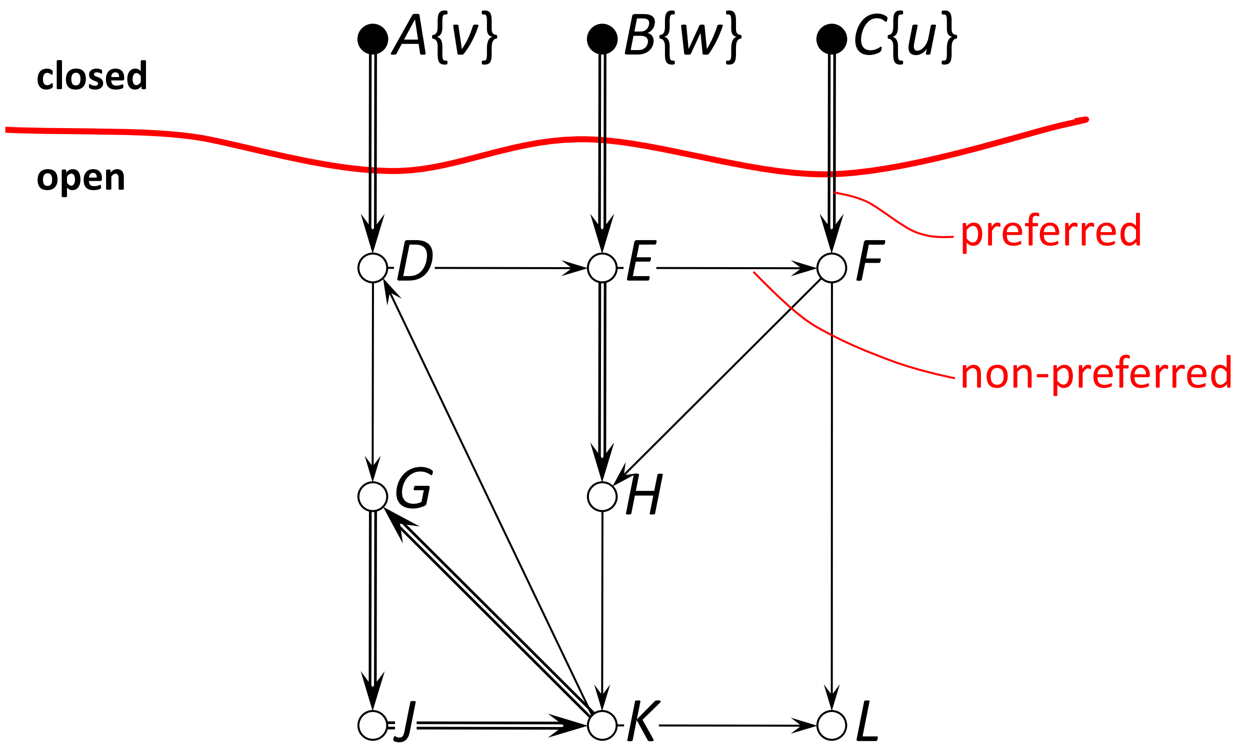
2. Resolution algorithm
   – how to calculate the solution efficiently?

3. Extensions
   – how to deal with “negative beliefs”?

Resolution Algorithm

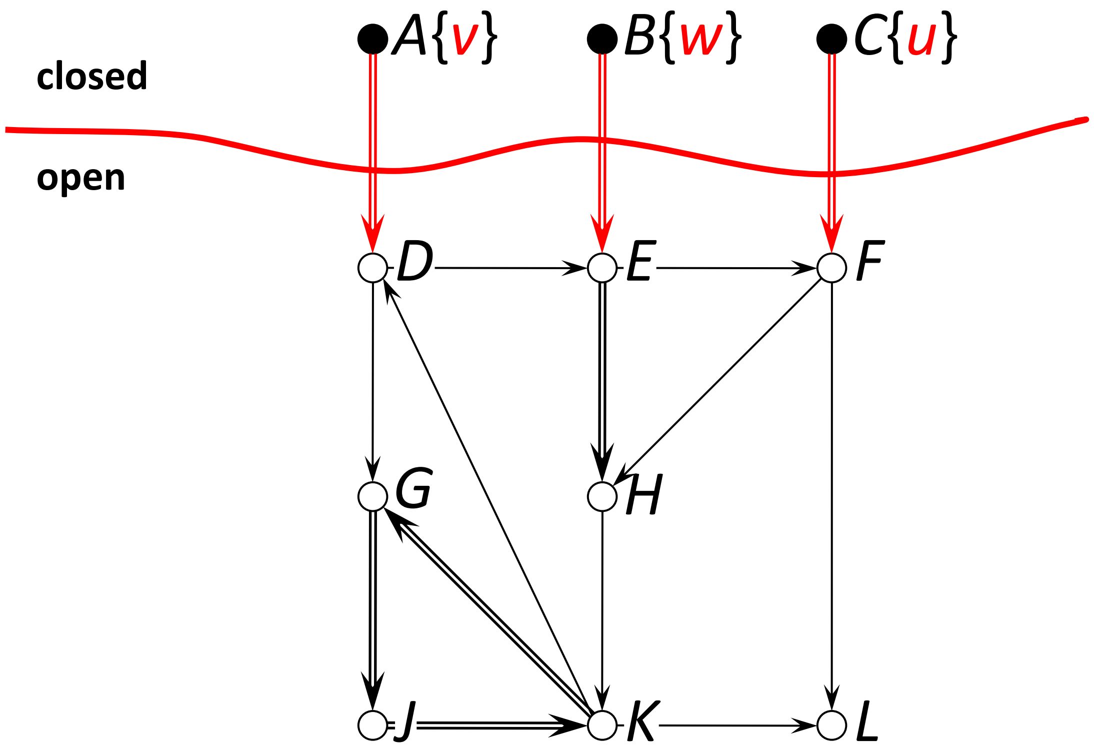
Focus on binary trust network

- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs



X	**poss**(X)	**cert**(X)
A	$\{v\}$	$\{v\}$
B	$\{w\}$	$\{w\}$
C	$\{u\}$	$\{u\}$
D	?	?
E	?	?
F	?	?
G	?	?
H	?	?
J	?	?
K	?	?
L	?	?

Resolution Algorithm



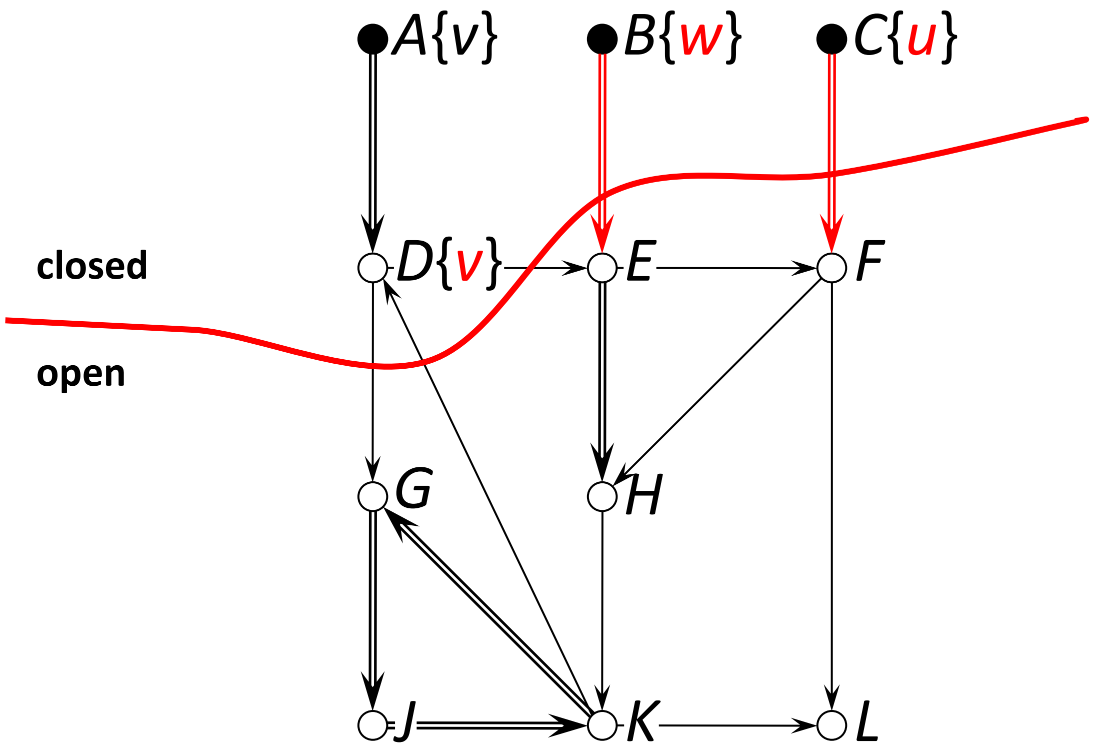
- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from **open** to **closed**
  → follow

X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	?	?
E	?	?
F	?	?
G	?	?
H	?	?
J	?	?
K	?	?
L	?	?

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Resolution Algorithm

- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from
  open to **closed**
  $\rightarrow$ follow

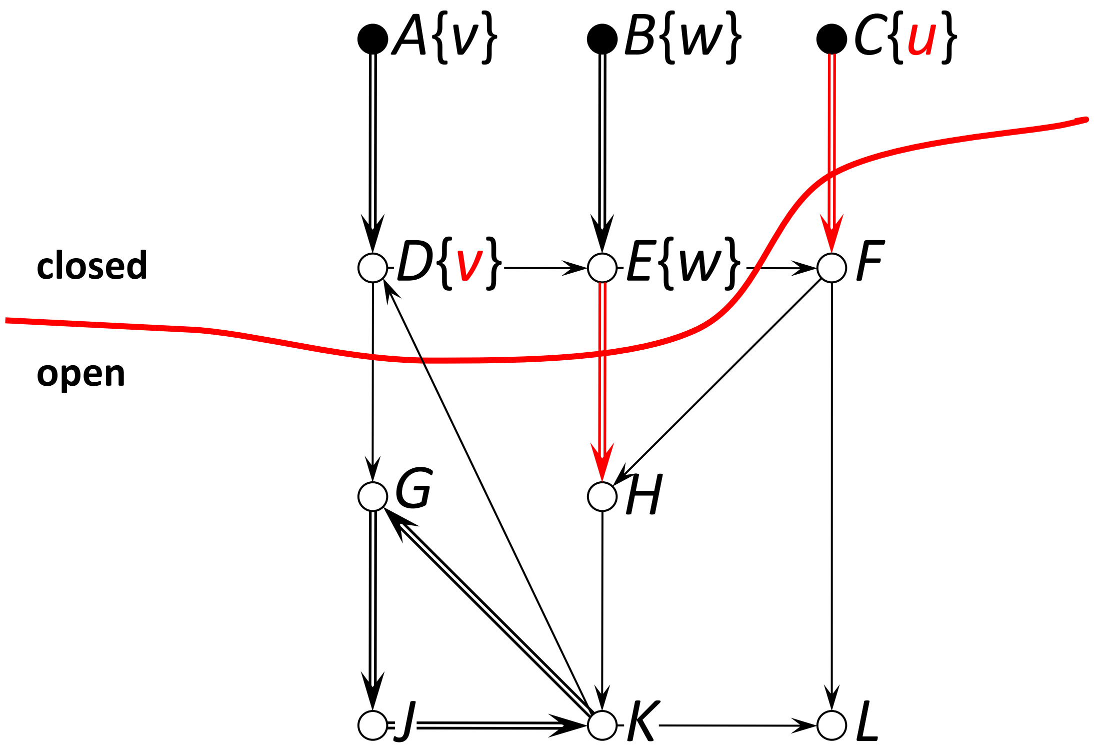


X	**poss**(X)	**cert**(X)
A	$\{v\}$	$\{v\}$
B	$\{w\}$	$\{w\}$
C	$\{u\}$	$\{u\}$
D	$\{v\}$	$\{v\}$
E	?	?
F	?	?
G	?	?
H	?	?
J	?	?
K	?	?
L	?	?

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

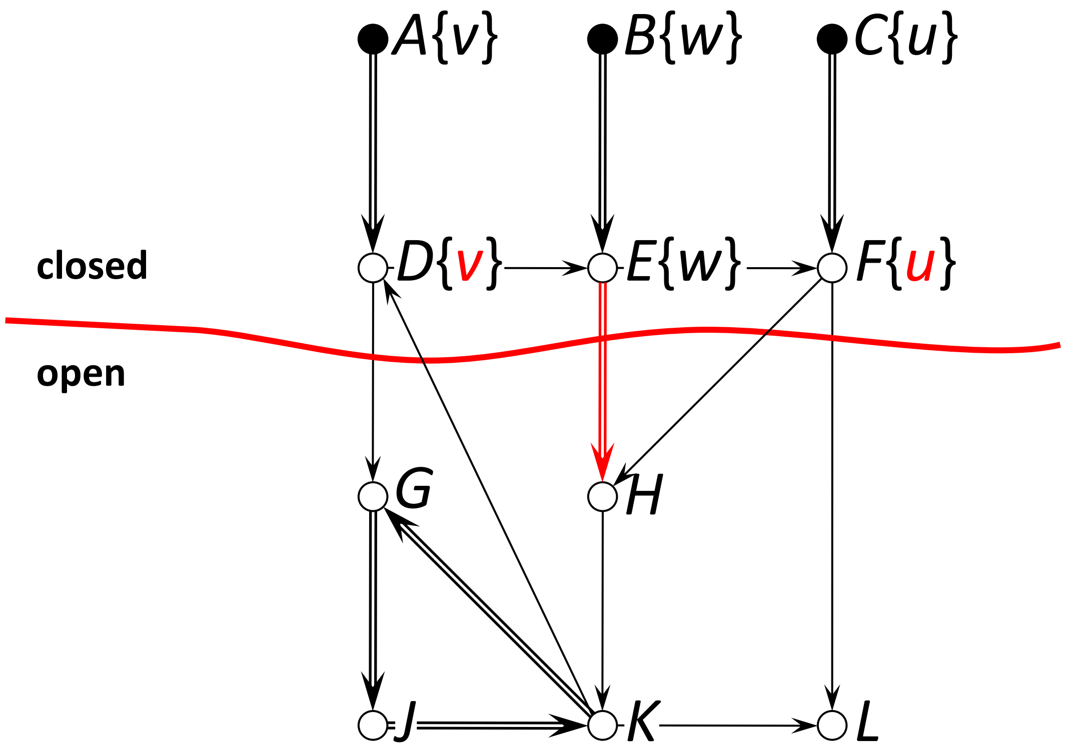
Resolution Algorithm

- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from
  open to **closed**
  → follow



X	**poss**(X)	**cert**(X)
A	$\{v\}$	$\{v\}$
B	$\{w\}$	$\{w\}$
C	$\{u\}$	$\{u\}$
D	$\{v\}$	$\{v\}$
E	$\{w\}$	$\{w\}$
F	?	?
G	?	?
H	?	?
J	?	?
K	?	?
L	?	?

Resolution Algorithm



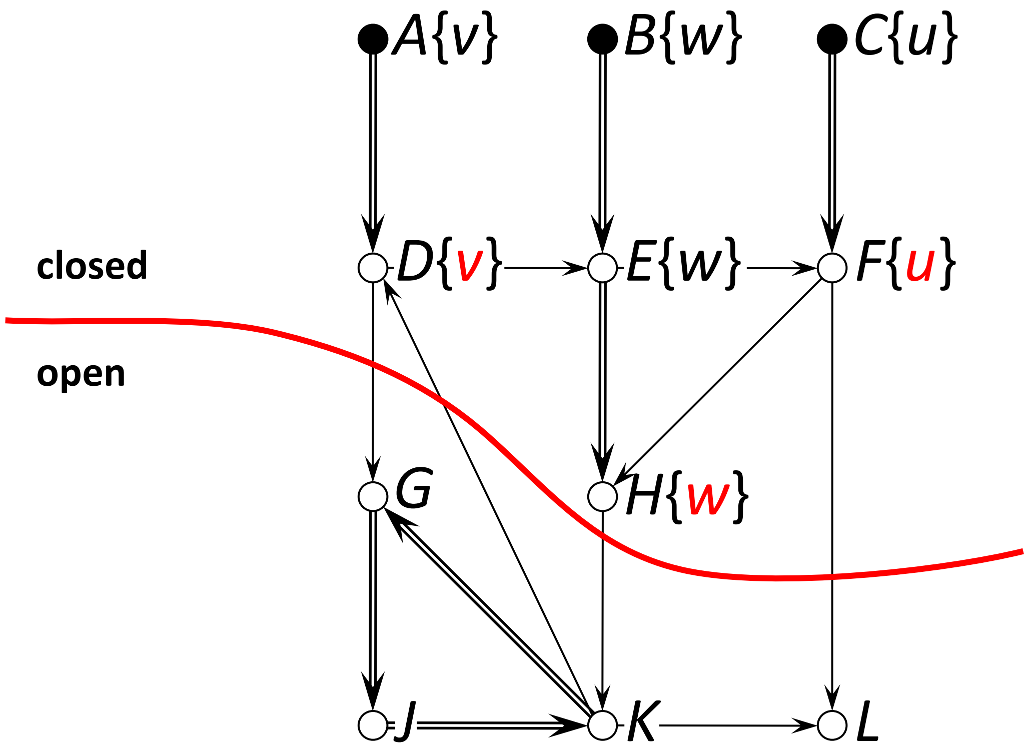
- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  - Step 1: if ∃ preferred edges from **open** to **closed** → follow

X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	?	?
H	?	?
J	?	?
K	?	?
L	?	?

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Resolution Algorithm

- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from
  open to **closed**
  → follow

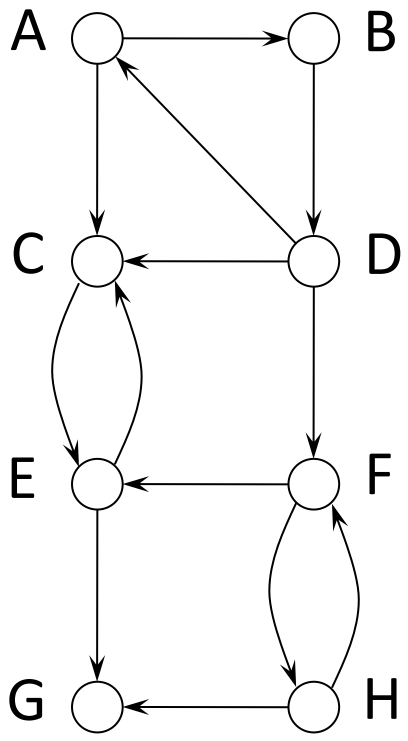


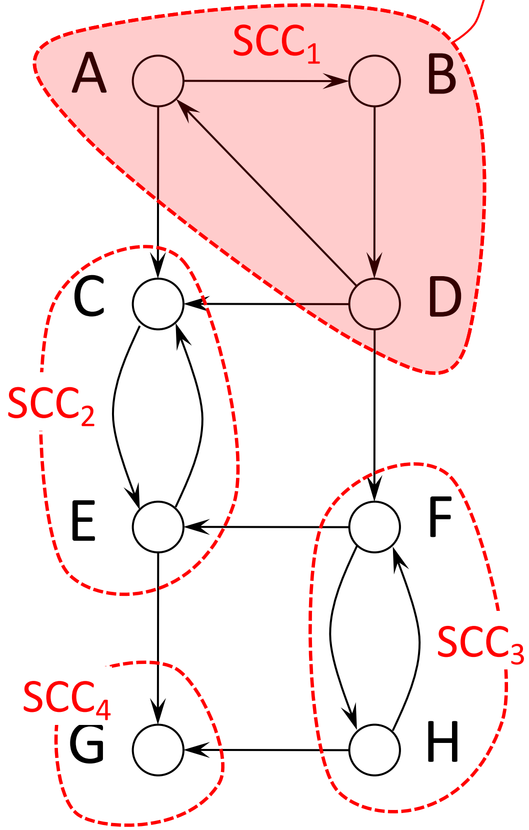
Now we are stuck!

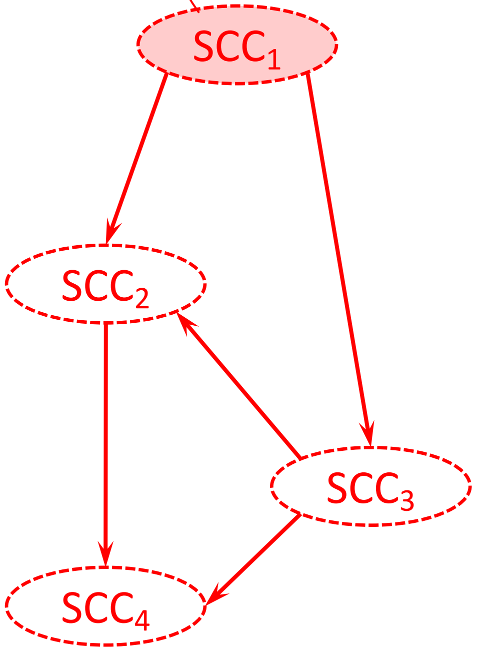
X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	?	?
H	{w}	{w}
J	?	?
K	?	?
L	?	?

Detail: Strongly Connected Components (SCCs)

For every cyclic or acyclic directed graph:
- The Strongly Connected Components graph is a DAG
- can be calculated in **O(*n*)** Tarjan [1972]

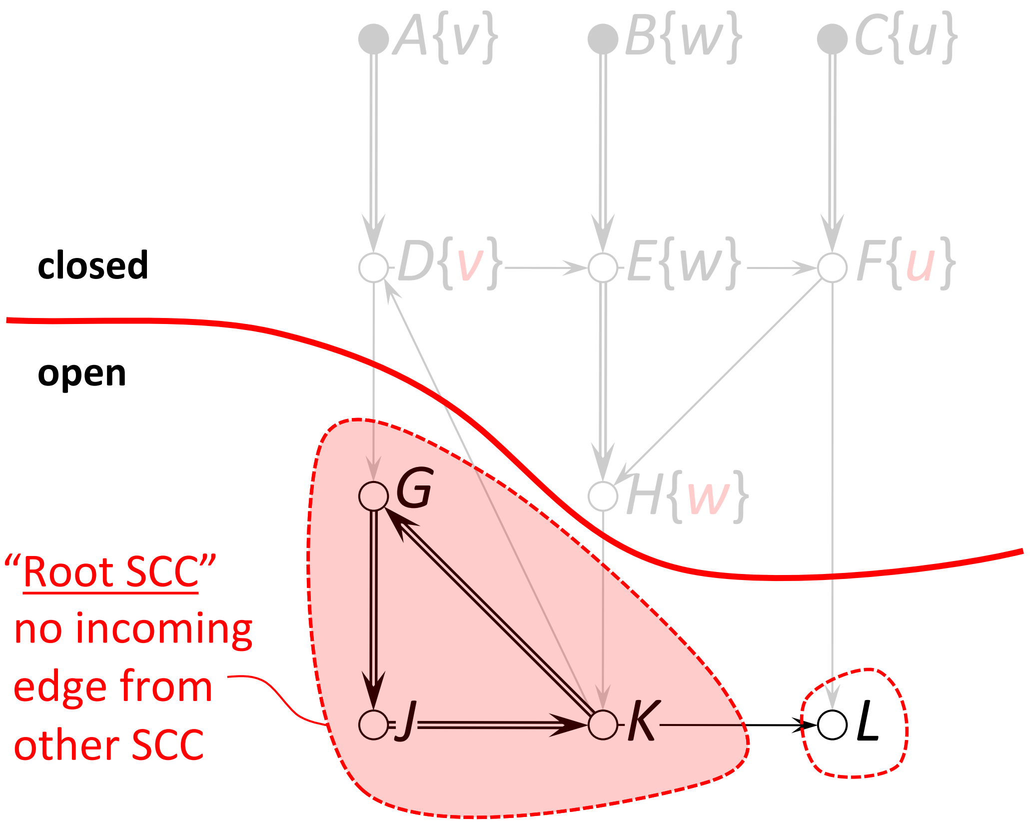






Resolution Algorithm

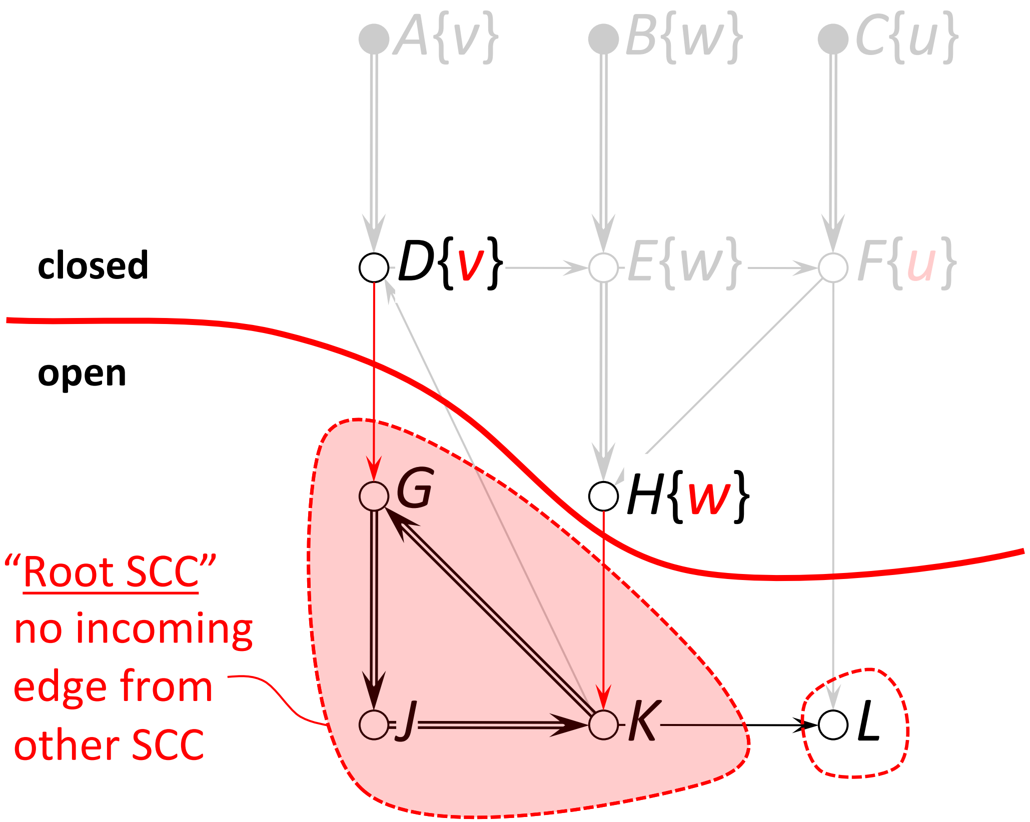
- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  - Step 1: if ∃ preferred edges from **open** to **closed**
    → follow
  - Step 2: else
    → construct SCC graph of **open**



X	**poss**(X)	**cert**(X)
A	$\{v\}$	$\{v\}$
B	$\{w\}$	$\{w\}$
C	$\{u\}$	$\{u\}$
D	$\{v\}$	$\{v\}$
E	$\{w\}$	$\{w\}$
F	$\{u\}$	$\{u\}$
G	?	?
H	$\{w\}$	$\{w\}$
J	?	?
K	?	?
L	?	?

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

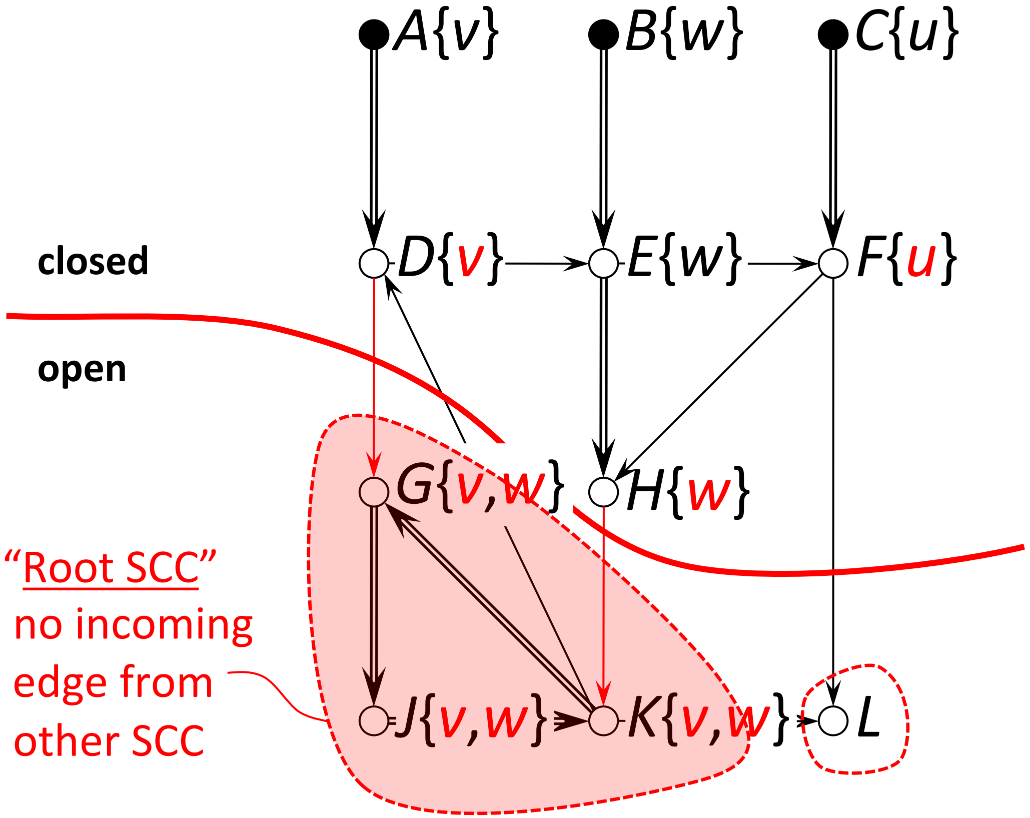
Resolution Algorithm



- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from
  open to **closed**
  → follow
  Step 2: else
  → construct SCC graph of **open**

X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	?	?
H	{w}	{w}
J	?	?
K	?	?
L	?	?

Resolution Algorithm

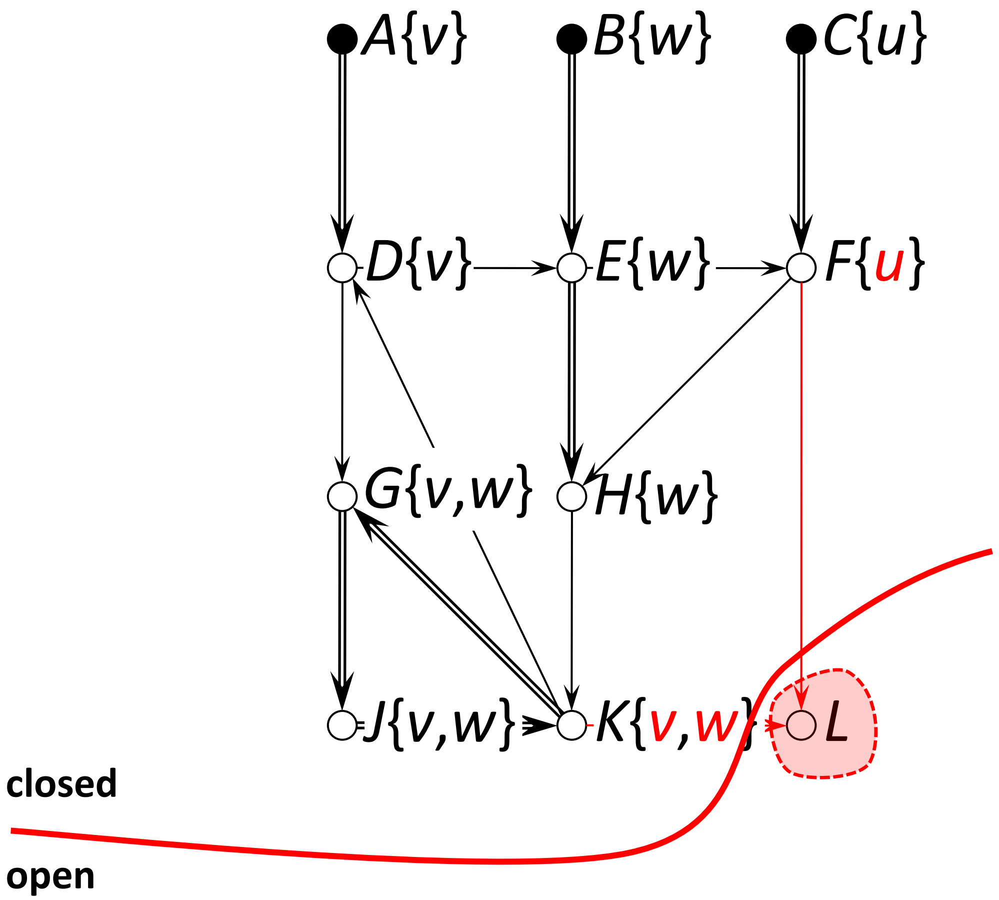


- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  - Step 1: if ∃ preferred edges from **open** to **closed**
    → follow
  - Step 2: else
    → construct SCC graph of **open**
    → resolve minimum SCCs

X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	{v,w}	$\varnothing$
H	{w}	{w}
J	{v,w}	$\varnothing$
K	{v,w}	$\varnothing$
L	?	?

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Resolution Algorithm

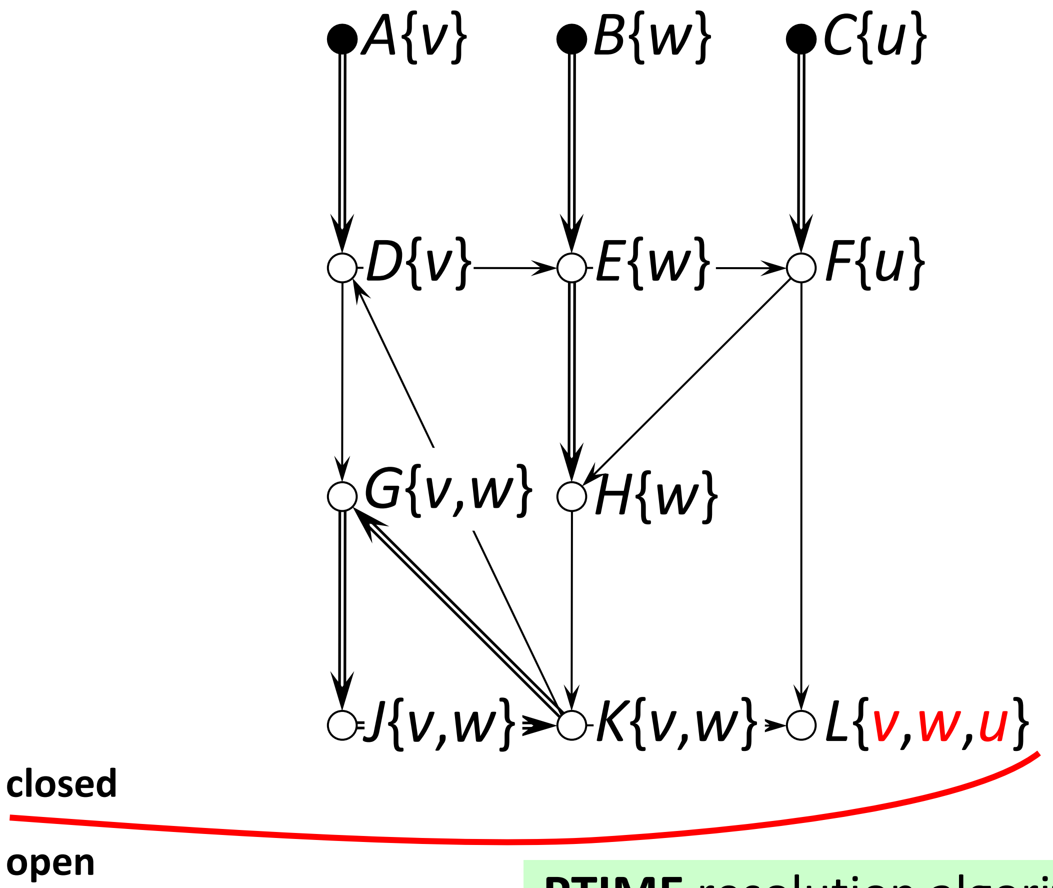


- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  Step 1: if ∃ preferred edges from
    open to **closed**
    → follow
  Step 2: else
    → construct SCC graph of **open**
    → resolve minimum SCCs

X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	{v,w}	∅
H	{w}	{w}
J	{v,w}	∅
K	{v,w}	∅
L	?	?

Resolution Algorithm

- Keep 2 sets: **closed** / **open**
  Initialize **closed** with explicit beliefs
- MAIN
  - Step 1: if ∃ preferred edges from **open** to **closed**
    → follow
  - Step 2: else
    → construct SCC graph of **open**
    → resolve minimum SCCs

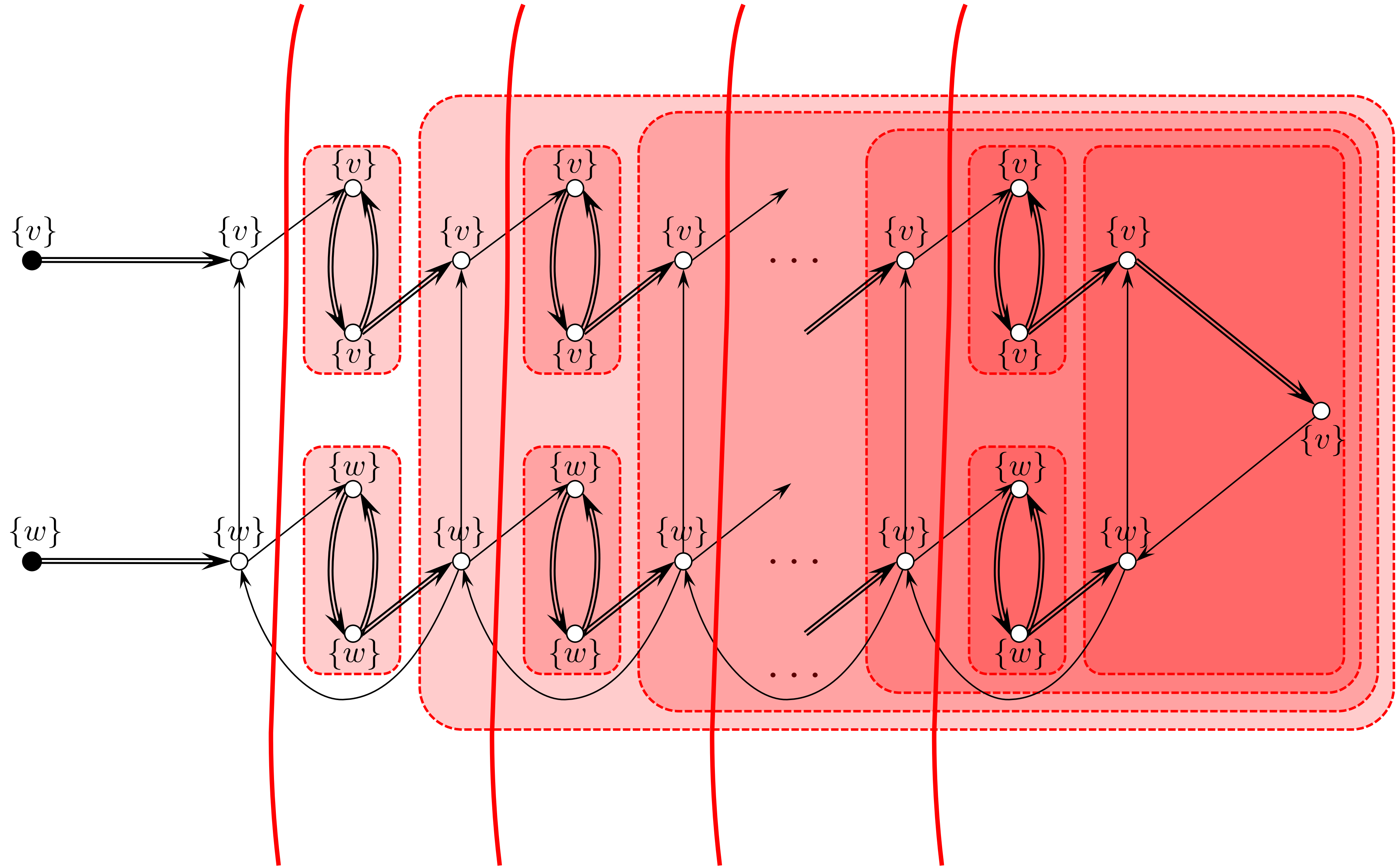


X	**poss**(X)	**cert**(X)
A	{v}	{v}
B	{w}	{w}
C	{u}	{u}
D	{v}	{v}
E	{w}	{w}
F	{u}	{u}
G	{v,w}	∅
H	{w}	{w}
J	{v,w}	∅
K	{v,w}	∅
L	{v,w,u}	∅

Can be implemented in current DBMS with transitive closure ☺

PTIME resolution algorithm
O(*n*²) worst case
O(*n*) on reasonable graphs

O(n2)-worst-case for Resolution Algorithm

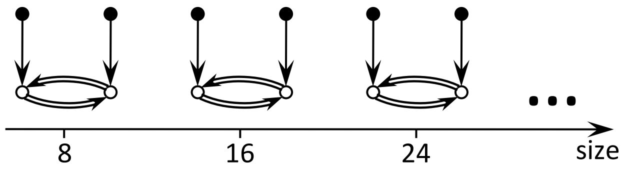


Experiments on large network data

Calculating **poss** / **cert** for fixed key

- **DLV**: State-of-the art logic programming solver
- **RA**: Resolution algorithm

Network 1: “Oscillators”

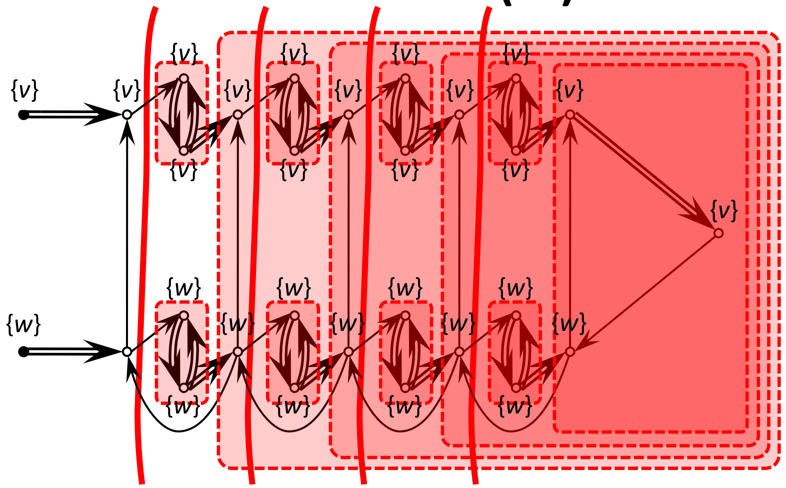


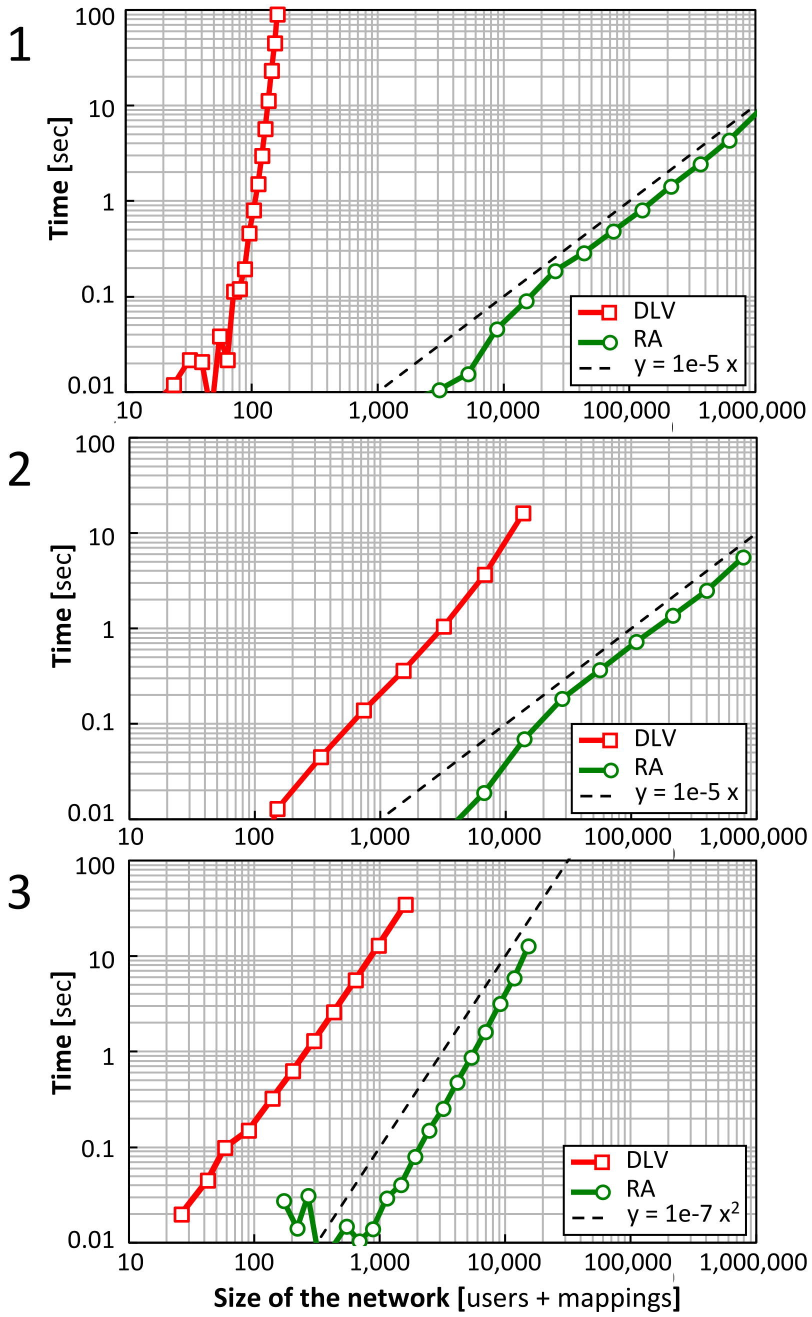
Network 2: “Web link data”

Web data set with 5.4m links between 270k domain names. Approach:

- Sample links with increasing ratio
- Include both nodes in sample
- Assign explicit beliefs randomly

Network 3: “Worst case” **O(n^2)**





Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010

Agenda

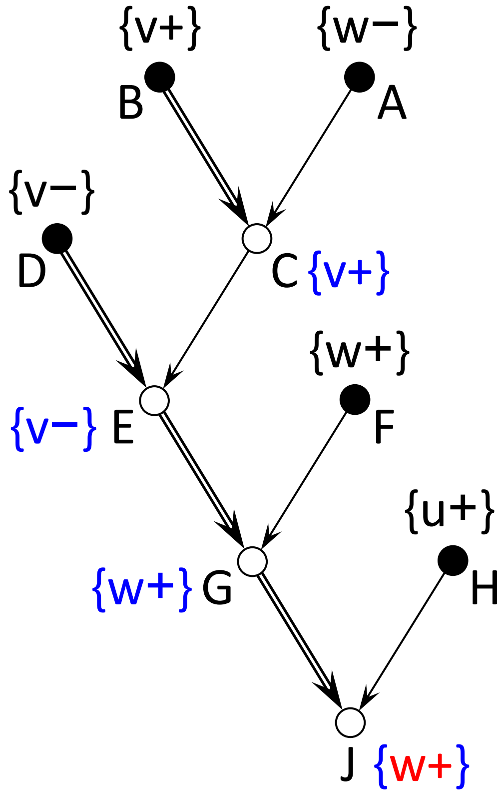
1. Stable solutions
   – how to define a unique and consistent solution?

2. Resolution algorithm
   – how to calculate the solution efficiently?

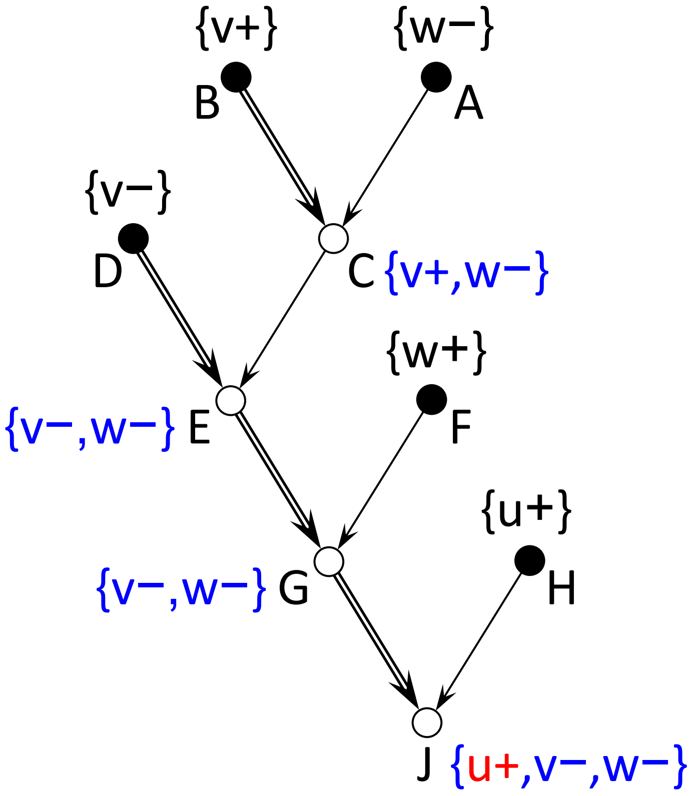
3. Extensions
   – how to deal with “negative beliefs”?

3 semantics for negative beliefs

Agnostic



Eclectic

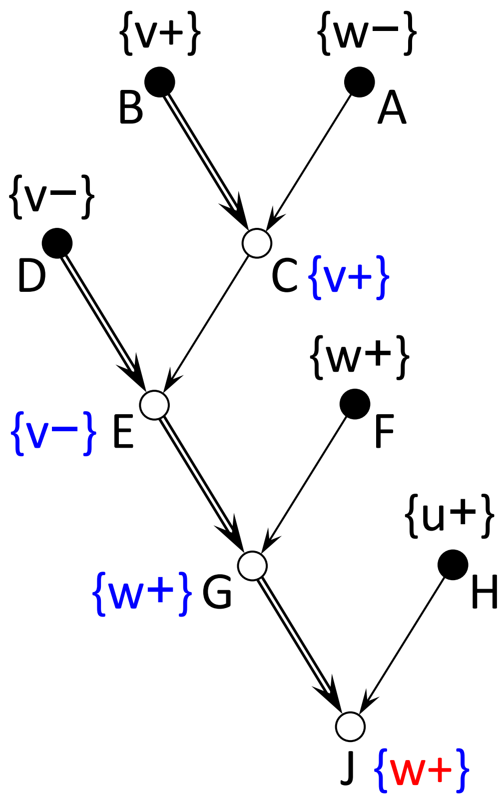
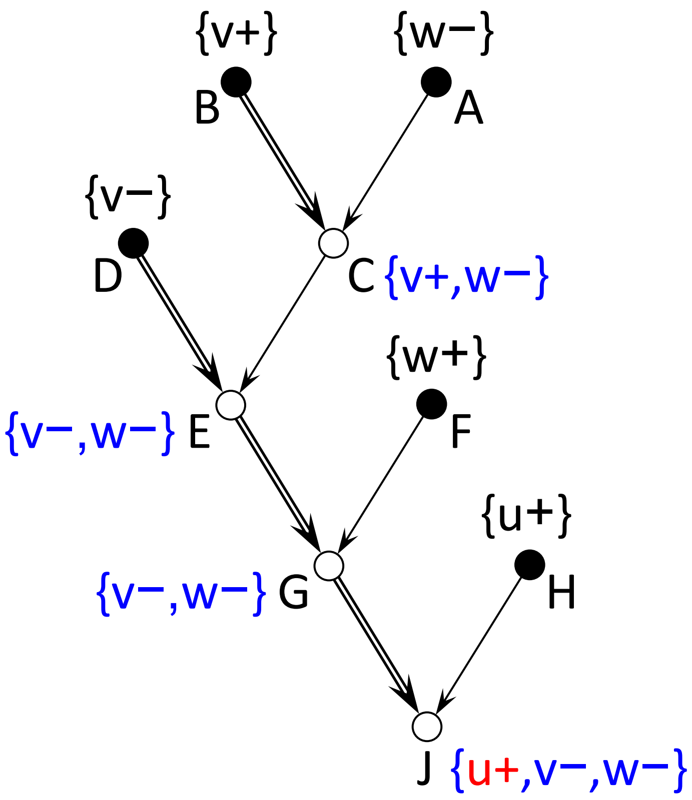
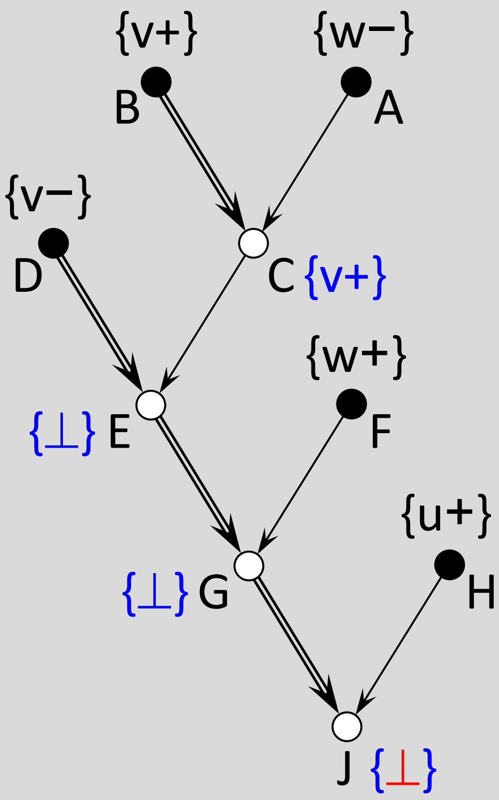


	Agnostic	Eclectic
w/o cycles*	**O(*n*)**	**O(*n*)**
w cycles	**NP-hard**	**NP-hard**

* assuming total order on parents for each node

3 semantics for negative beliefs

Our recommendation

	Agnostic	Eclectic	Skeptic
			
w/o cycles*	$O(n)$	$O(n)$	$O(n)$
w cycles	NP-hard	NP-hard	$O(n^2)$

with a variation of resolution algorithm

* assuming total order on parents for each node

Take-aways automatic conflict resolution

Problem

- Given explicit beliefs & trust mappings, how to assign consistent value assignment to users?

Our solution

- Stable solutions with possible/certain value semantics
- PTIME algorithm [**$O(n^2)$** worst case, **$O(n)$** experiments]
- Several extensions
  - negative beliefs: 3 semantics, two hard, one **$O(n^2)$**
  - bulk inserts
  - agreement checking
  - consensus value
  - lineage computation

in the paper & TR

Please visit us at the poster session Th, 3:30pm

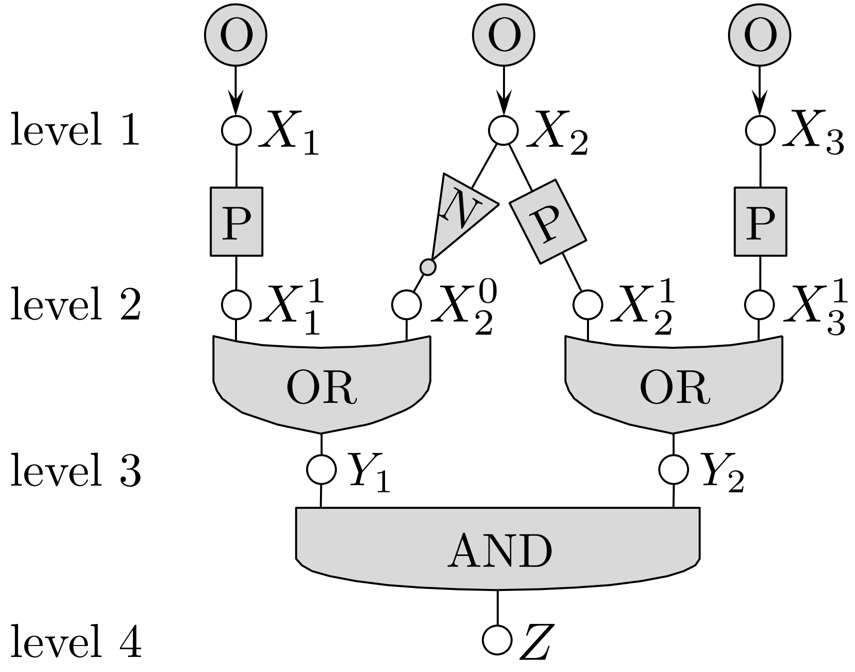
or at: https://db.cs.washington.edu/projects/beliefdb/

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

some details

Fig_ComplexityExampleLong

8-17-2010



Encoding

(0/1) = (a+/b+)

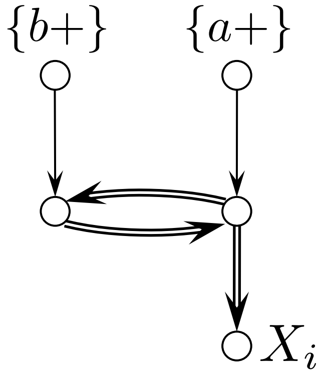
(0/1) = (c+/d+)

(0/1) = (e+/d+)

(0/1) = (e+/f+)

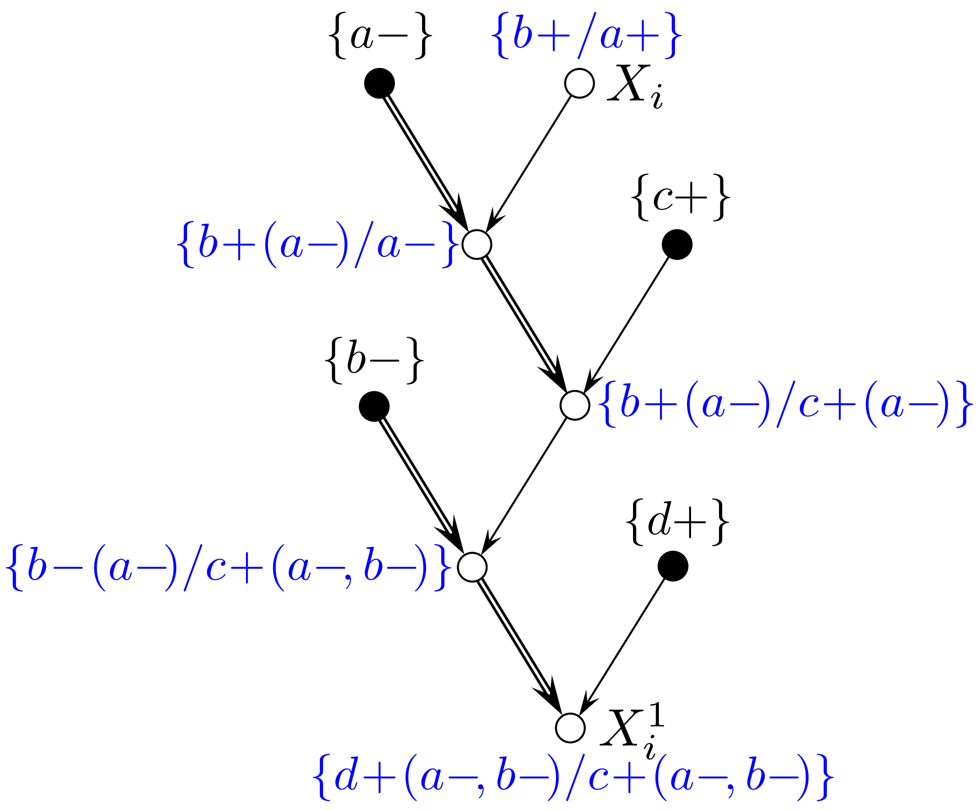
Fig_ComplexityOscillator

8-16-2010



Fig_ComplexityPassLong

8-17-2010



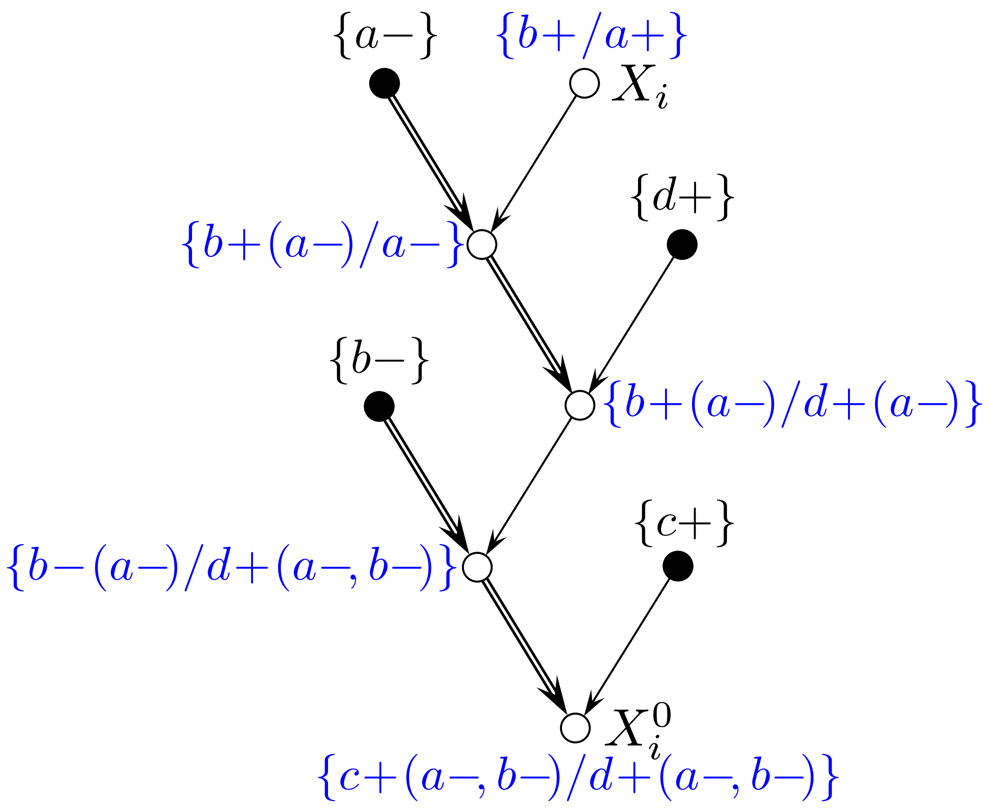
Encoding

(0/1) = (a+/b+)

(0/1) = (c+/d+)

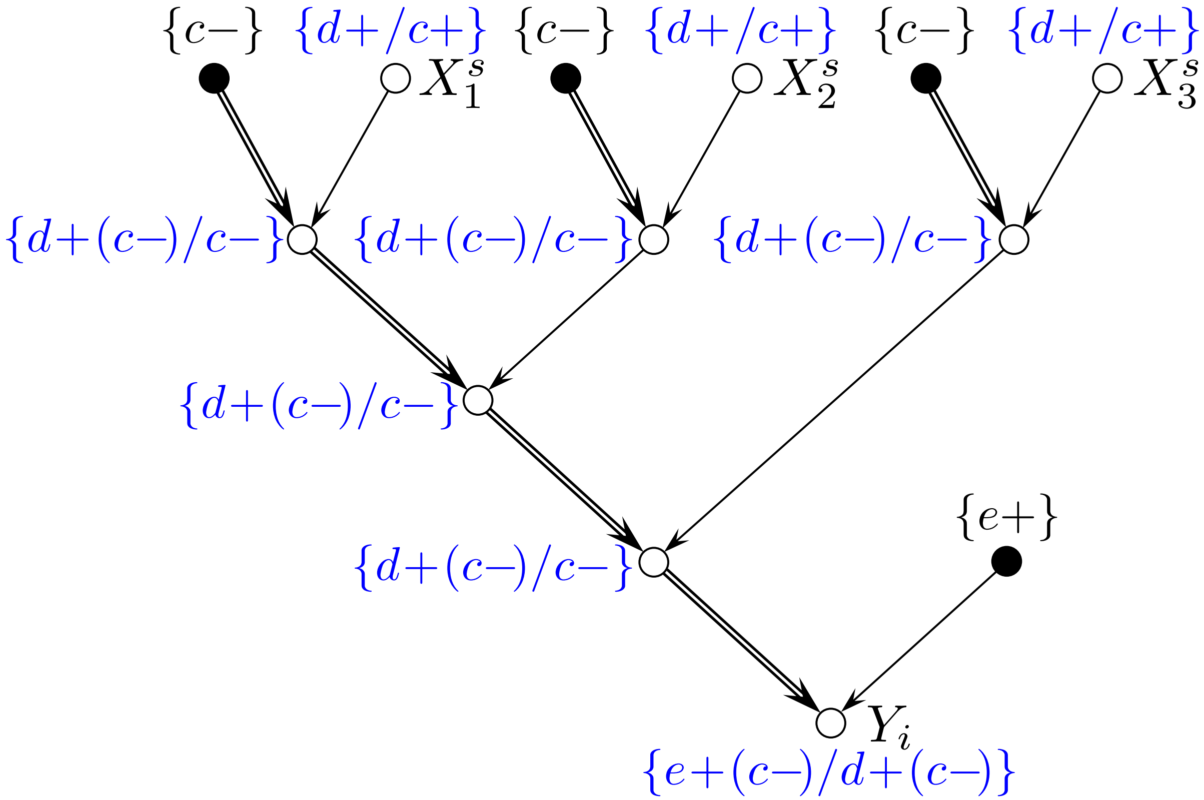
Fig_ComplexityNotLong

8-17-2010



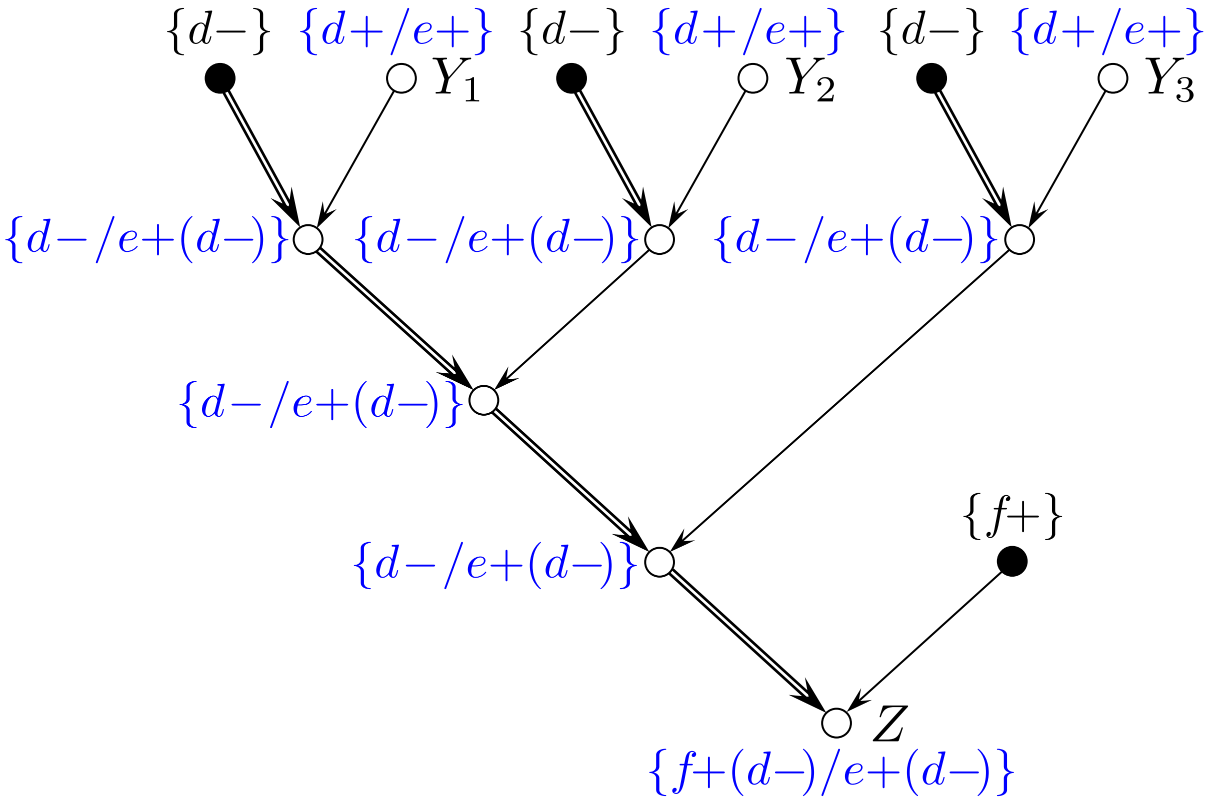
Fig_ComplexityOrLong

8-17-2010



Fig_ComplexityAndLong

8-17-2010



Definition 3.1 (consistency). *Two beliefs* b_1, b_2 *are conflicting* ($b_1 \not\leftrightarrow b_2$) *if they are either distinct positive beliefs* $v+, w+$, *or one is* $v+$ *and the other is* $v-$. *Otherwise,* b_1, b_2 *are consistent* ($b_1 \leftrightarrow b_2$). *A set of beliefs* B *is called consistent if any two beliefs* $b_1, b_2 \in B$ *are consistent.*

Definition 3.2 (preferred union). *Given two consistent sets of beliefs* B_1, B_2, *their preferred union is:*

$$B_1 \vec{\cup} B_2 = B_1 \cup \{b_2 \mid b_2 \in B_2 . \big(\forall b_1 \in B_1 . b_1 \leftrightarrow b_2\big)\}$$

be a consistent set of positive and/or negative beliefs. For each paradigm $\sigma \in \{\texttt{Agnostic}, \texttt{Eclectic}, \texttt{Skeptic}\}$ (abbreviated by $\{\texttt{A}, \texttt{E}, \texttt{S}\}$), the *normal form* $Norm_\sigma(B)$ is:

$$Norm_{\texttt{A}}(B) = \begin{cases} \{v+\} & \text{if } \exists v+ \in B \\ B & \text{otherwise} \end{cases}$$

$$Norm_{\texttt{E}}(B) = B$$

$$Norm_{\texttt{S}}(B) = \begin{cases} \{v+\} \cup (\bot - \{v-\}) & \text{if } \exists v+ \in B \\ B & \text{otherwise} \end{cases}$$

The *preferred union specialized to the paradigm* σ is:

$$B_1 \vec{\cup}_\sigma B_2 = Norm_\sigma \left(Norm_\sigma(B_1) \vec{\cup} Norm_\sigma(B_2) \right) \qquad (1)$$

For example:

$$\{a-\} \vec{\cup}_{\texttt{A}} \{b+\} = \{b+\}$$

$$\{a-\} \vec{\cup}_{\texttt{E}} \{b+\} = \{b+, a-\}$$

$$\{a-\} \vec{\cup}_{\texttt{S}} \{b+\} = \{b+, a-, c-, d-, \ldots\}$$

$$\{b-\} \vec{\cup}_{\texttt{S}} \{b+\} = \bot$$

$a^- \ (a^+ \ \ b^+)$

$(a^- \ \ a^+) \ \ b^+$

A puzzling question is why is the `Skeptic` paradigm in PTIME, while the other two are hard. It is easy to see that the Boolean gates in Fig. 7 no longer work under `Skeptic`, but we do not consider this a satisfactory explanation. While we cannot give an ultimate cause, we point out one interesting difference. The preferred union for `Skeptic` is *associative*, while it is not associative for either `Agnostic` nor `Eclectic`. For example, consider the two expressions $B_1 = \{a-\} \vec{\cup}_\sigma (\{a+\} \vec{\cup}_\sigma \{b+\})$, $B_2 = (\{a-\} \vec{\cup}_\sigma \{a+\}) \vec{\cup}_\sigma \{b+\}$. For `Agnostic`, we have $B_2 = \{b+\}$, for `Eclectic` $B_2 = \{a-, b+\}$, while for both $B_1 = \{a-\}$. By contrast, one can show that $\vec{\cup}_\mathrm{S}$ is associative. Associativity as a desirable property during data merging was pointed out in [14].

The issue of associativity

null appears in a join column. No matter what choice is taken, ⟕ is not associative. Consider the relations

q(A	B)
1	2

r(B	C)
2	3

s(A	C)
1	4

Computing (q ⟕ r) ⟕ s we get

q'(A	B	C)
1	2	3
1	⊥	4

while q ⟕ (r ⟕ s) gives

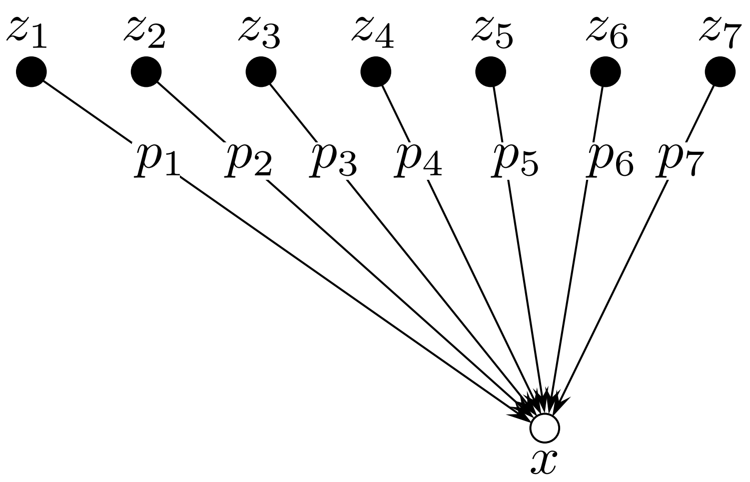
q''(A	B	C)
1	2	4
⊥	2	3

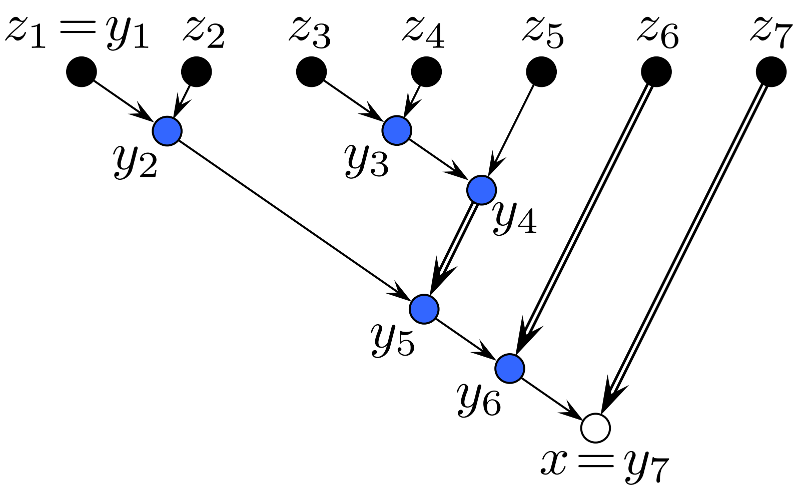
$$\{a^-\} \;\vec{\cup}_a\; (\{a\} \;\vec{\cup}_a\; \{b\}) \qquad = \{a^-\}$$

$$(\{a^-\} \;\vec{\cup}_a\; \{a\}) \;\vec{\cup}_a\; \{b\} \qquad = \{b\}$$

Source: left outer join example from p392 in "Maier. The theory of relational databases, 1983." https://web.cecs.pdx.edu/~maier/TheoryBook/TRD.html
Source: right preferred union example from "Gatterbauer, Suciu. Conflict resolution using trust mapping. SIGMOD 2010. https://doi.org/10.1145/1807167.1807193

Binarization example



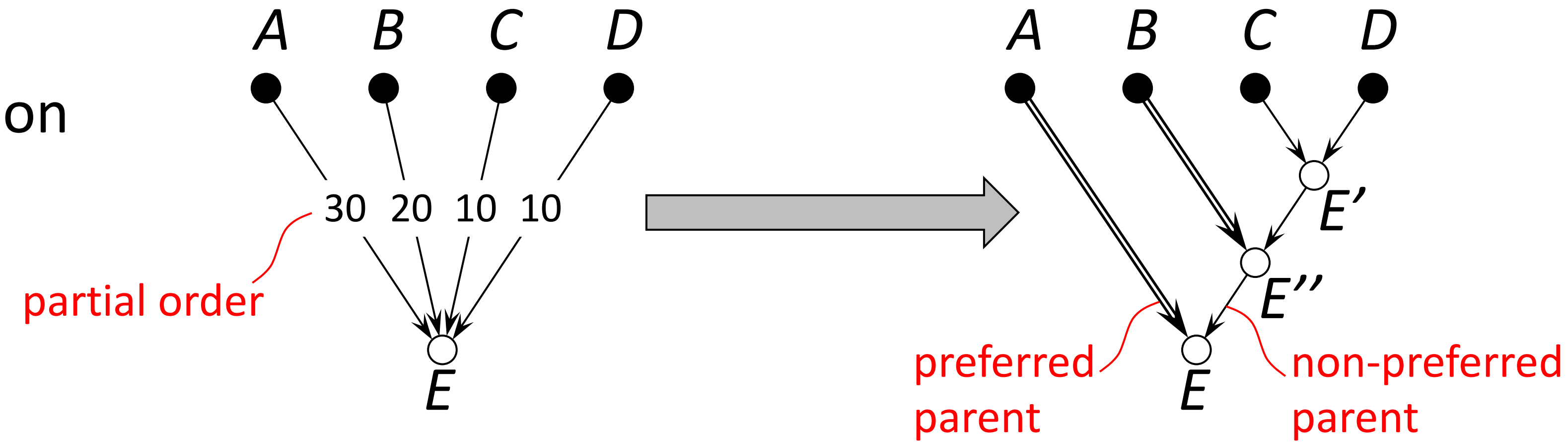


$p_1 = p_2 < p_3 = p_4 = p_5 < p_6 < p_7$

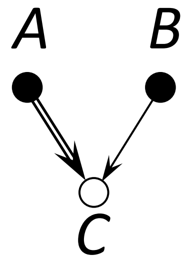
Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

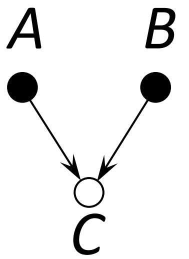
Logic programs with stable model semantics

Step 1:
Binarization



Step 2:
Logic program





1: accept all **poss** of preferred parent

```
poss(c,X) :- poss(a,X).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

```
block(c,a,Y) :- poss(a,Y), poss(c,X), X!=Y.
poss(c,Y) :- poss(a,Y), not block(c,a,Y).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

2: accept **poss** from non-preferred parent, that are not conflicting with an existing value

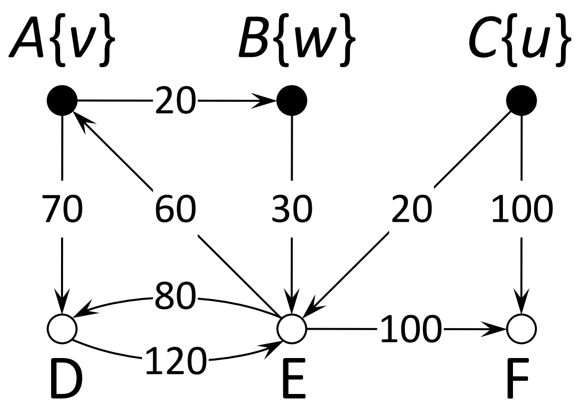
Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

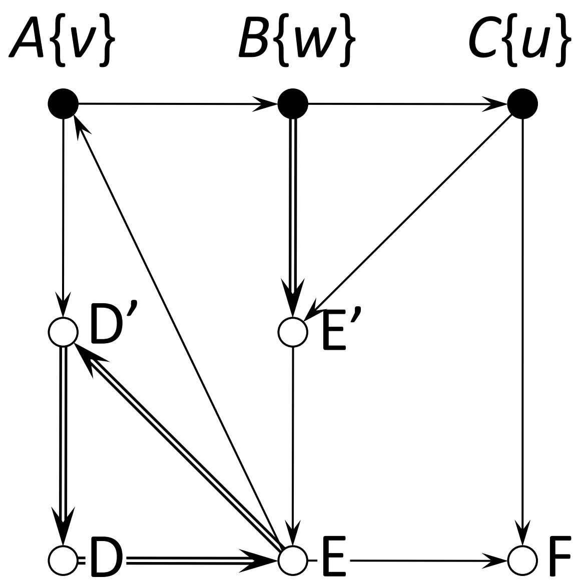
Binarization for Resolution Algorithm*

Example Trust Network (TN)
6 nodes, 9 arcs (size 15)
3 explicit beliefs: A:v, B:w, C:u

Corresponding Binary TN (BTN)
8 nodes, 12 arcs (size 20)

Size increase (N+E): ≤ 3



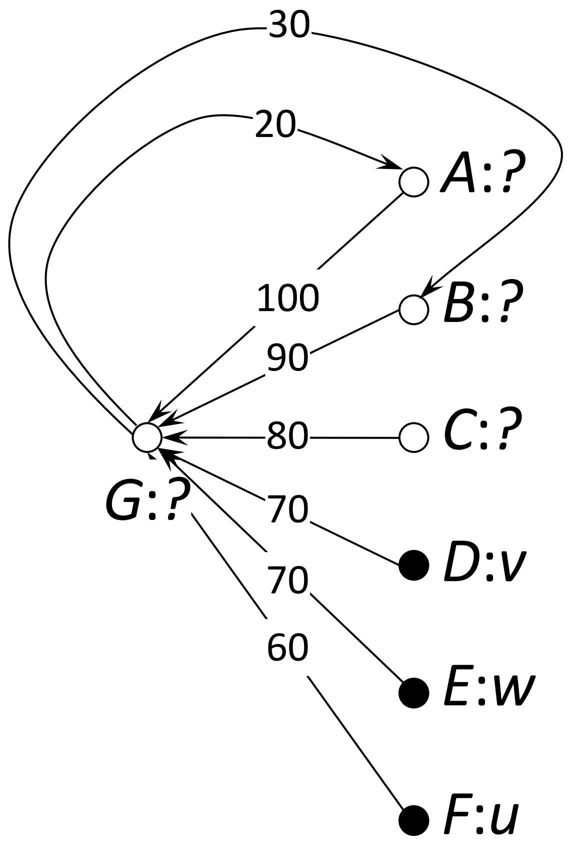


* Note that binarization is not necessary, but greatly simplifies the presentation

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Stable solutions: example 2

- Priority trust network (TN)
  - assume a fixed key
  - users (nodes): *A*, *B*, *C*
  - values (beliefs): *v*, *w*, *u*
  - trust mappings (arcs) from "parents"
- Stable solution
  - assignment of values to each node*, s.t. each belief has a "*non-dominated lineage*" to an explicit belief
- Certain values
  - all stable solution determine, for each node, a possible value ("poss")
  - certain value ("cert") = intersection of all stable solutions



* each node with at least one ancestor with explicit belief

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

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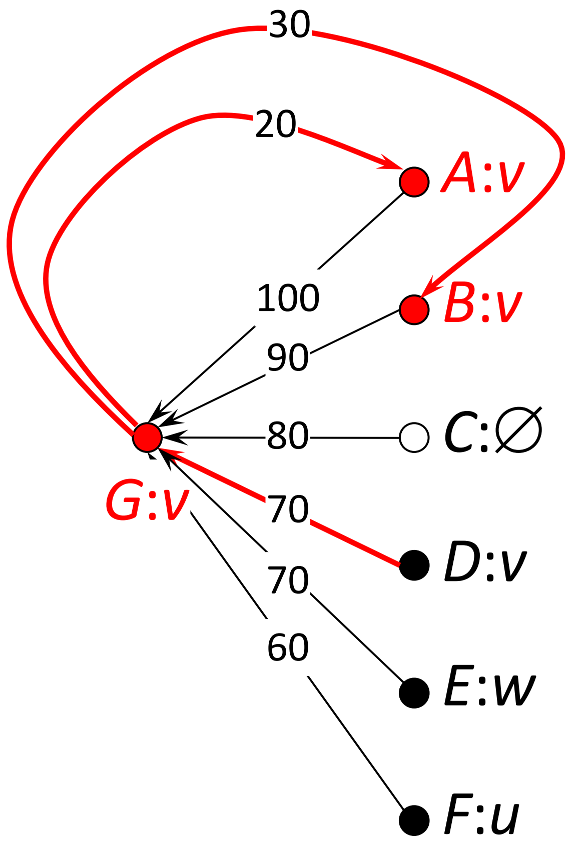

poss(*G*) = {*v*,...}

* each node with at least one ancestor with explicit belief

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Stable solutions: example 2

- Priority trust network (TN)
  - assume a fixed key
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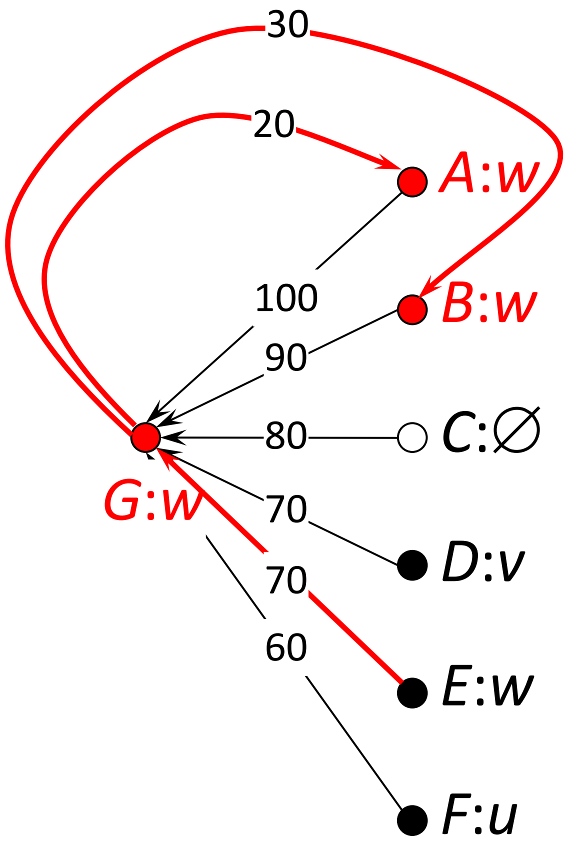


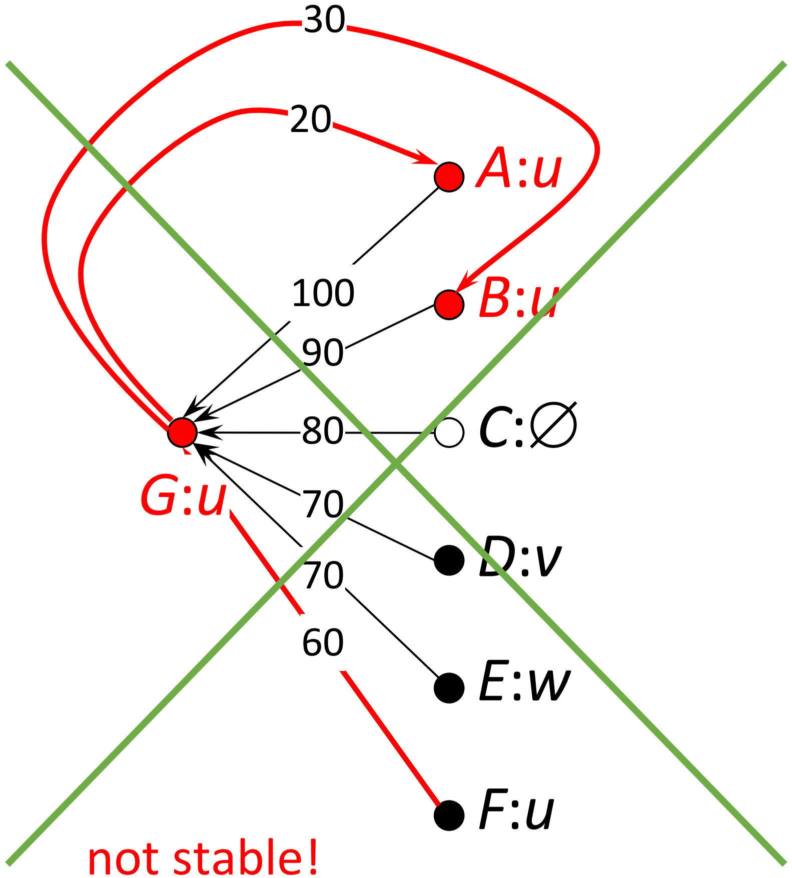
poss(*G*) = {*v*,*w*,...}

* each node with at least one ancestor with explicit belief

Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

Stable solutions: example 2

- Priority trust network (TN)
  - assume a fixed key
  - users (nodes): *A*, *B*, *C*
  - values (beliefs): *v*, *w*, *u*
  - trust mappings (arcs) from "parents"
- Stable solution
  - assignment of values to each node*, s.t. each belief has a "*non-dominated lineage*" to an explicit belief
- Certain values
  - all stable solution determine, for each node, a possible value ("poss")
  - certain value ("cert") = intersection of all stable solutions



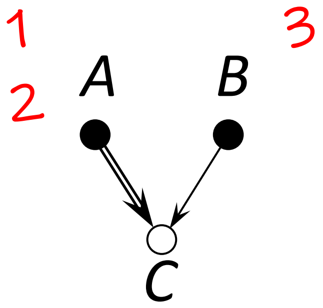
not stable!
F→G dominated by E→G

poss(*G*) = {*v*,*w*}
cert(*G*) = ∅

* each node with at least one ancestor with explicit belief
Gatterbauer, Suciu. Data Conflict Resolution Using Trust Mappings, SIGMOD 2010, https://doi.org/10.1145/1807167.1807193

exercise

Logic programs with stable model semantics




```
   poss(c,X) :- poss(a,X).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
   poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

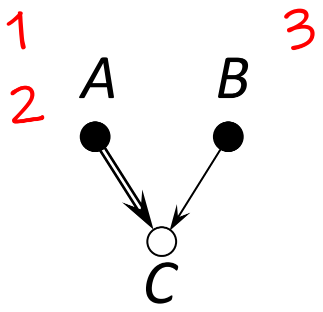
```
poss(a,1).
poss(a,2).
poss(b,3).
```

⇨

poss(c,X) ?

?

Logic programs with stable model semantics




```
poss(c,1) :- poss(a,1)
poss(c,2) :- poss(a,2)
poss(c,3) :- poss(a,3)
block(c,b,3) :- poss(b,3), poss(c,1), X!=Y
block(c,b,3) :- poss(b,3), poss(c,2), X!=Y
block(c,b,3) :- poss(b,1), poss(c,3), X!=Y
...
poss(c,3) :- poss(b,3), not block(c,b,3)
poss(c,2) :- poss(b,2), not block(c,b,2)
...
```

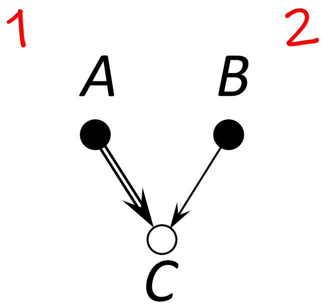
```
poss(c,X) :- poss(a,X).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

```
poss(a,1).
poss(a,2).
poss(b,3).
```

poss(c,X) ?

M={ poss(a,1), poss(a,2), poss(b,3),
poss(c,1), poss(c,2) }

Logic programs with stable model semantics




```
block(c,a,Y) :- poss(a,Y), poss(c,X), X!=Y.
   poss(c,Y) :- poss(a,Y), not block(c,a,Y).
block(c,b,Y) :- poss(b,Y), poss(c,X), X!=Y.
   poss(c,Y) :- poss(b,Y), not block(c,b,Y).
```

```
poss(a,1).
poss(a,2).
```

⇨ poss(c,X) ?

?