



Frontier-Based RTDP: A New Approach to Solving the Robotic Adversarial Coverage Problem

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The Problem




Area coverage problem

- Example: Roomba

Adversarial coverage

- Example: navigating a minefield

Environment: Grid world



$P(S)=0.2$	$P(S)=0.1$	$P(S)=0.2$	$P(S)=0.2$
$P(S)=0.2$		$P(S)=0.2$	$P(S)=0.2$
start	$P(S)=0$	$P(S)=0.6$	$P(S)=0.5$

- Grid contains the probability of the robot being stopped by a threat at any point.
- **Offline version:** robot knows the map beforehand and can plan ahead

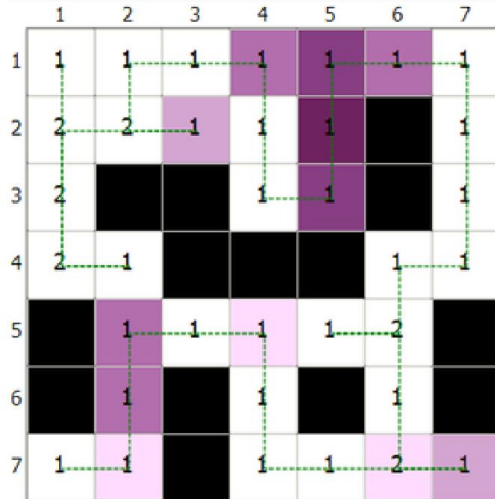
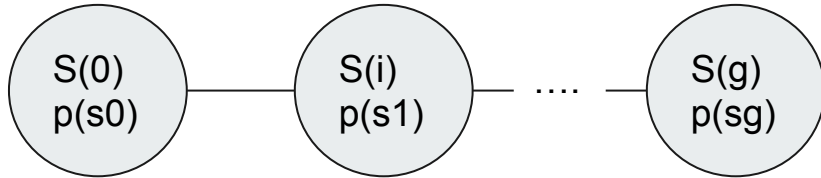
Zhang, Jeremy. "Reinforcement Learning-Implement Grid World From Scratch." *Medium*, Towards Data Science, 24 Aug. 2019, towardsdatascience.com/reinforcement-learning-implement-grid-world-from-scratch-c5963765ebff.

The Approach: Markov Decision Process (MDP)



- Represent costs and uncertainty of actions
- Stochastic Shortest Path Problem
- Possible state space size requires redefining the search algorithm to solve the MDP
- Introduction of the frontier

Adversarial Coverage



Independence: Each (S, S') transition is independent

Given Path A:

P(completion):

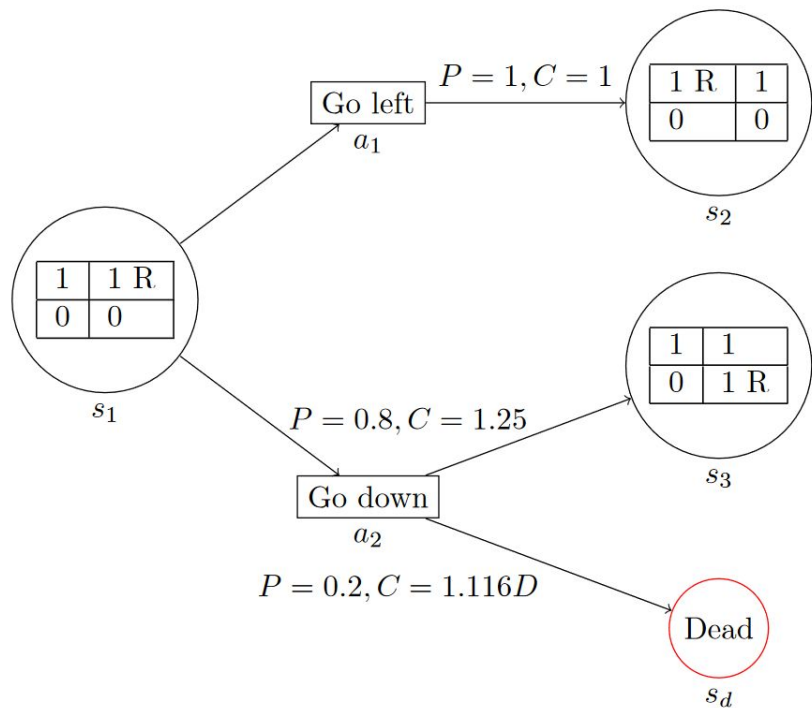
$$P(S_A) = \prod_{i \in (a_1, \dots, a_m)} (1 - p_i)$$

Cost Function:

$$f(A) = -\alpha \cdot P(S_A) + \beta \cdot |A|$$

α : Risk , β : Coverage time

Coverage expressed as an MDP



State: Gridworld coverage and Robot Position

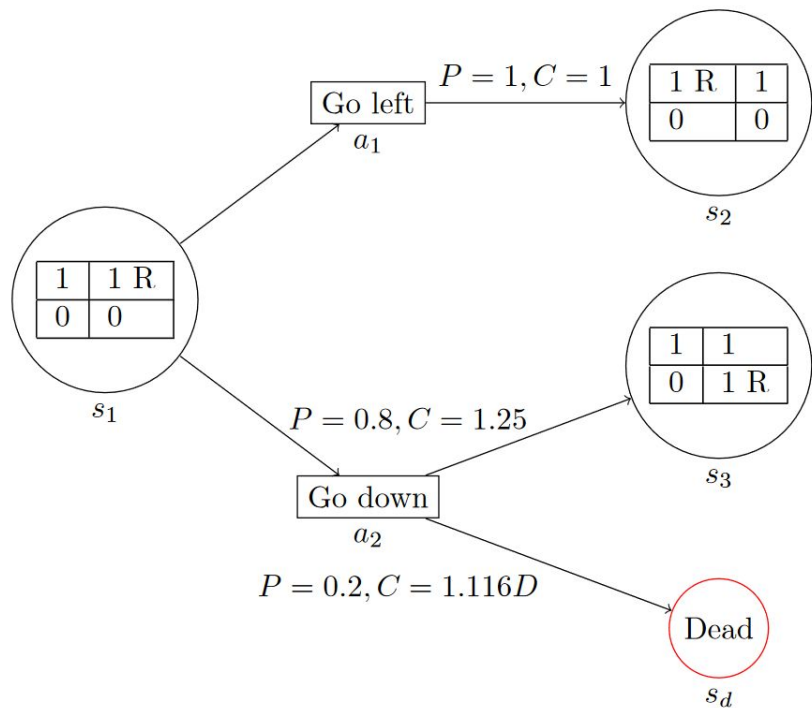
Expansion: State Actions:

- State expansion shows possible transitions
- Action expansion shows possible results

Goal State: All cells in coverage state are 1

Termination State: Goal State or Dead

Transition Function



Transition probability success:

$$P_a(s'|s) = 1$$

Transition probability fail:

$$P_a(s'|s) = 1 - p_j$$

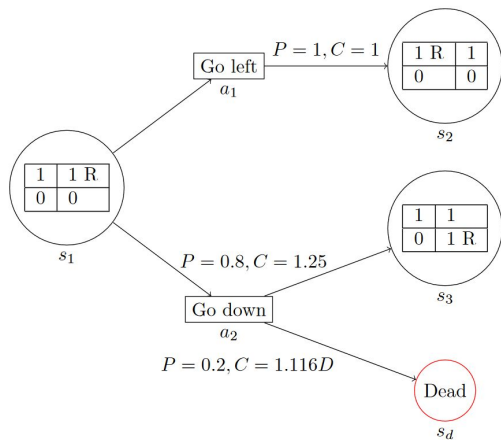
Cost Success:

$$C_a(s, s') = \frac{1}{1 - p_j}$$

Cost Fail:

$$C_a(s, s_d) = -D \cdot \frac{\log(1 - p_j)}{p_j}$$

Transition Function



Discount & Risk Ratio:

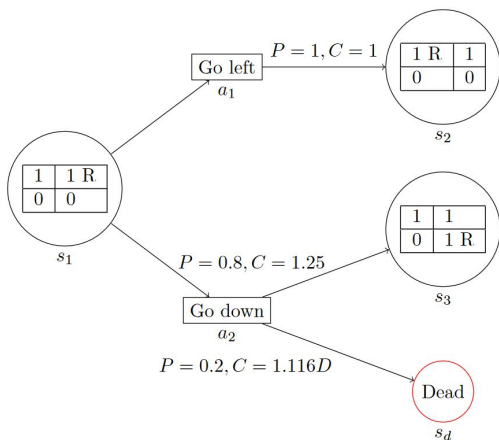
$$D = -\frac{\alpha}{\beta} \cdot \frac{1}{\log(1 - p_{min})}$$

α : Risk , β : Coverage time

Expected cost of state:

$$\begin{aligned} E[C(c_j)] &= (1 - p_j) \cdot \frac{1}{1 - p_j} + p_j \cdot \left[-D \cdot \frac{\log(1 - p_j)}{p_j} \right] \\ &= 1 - D \log(1 - p_j) \end{aligned}$$

Minimum Path Cost and Limitation



Minimum Path Cost:

$$\sum_{t=0}^{n-1} E[C_{a_t}(s_t, s_{t+1})] = n - D \sum_{j=1}^n \log(1 - p_j)$$

Limitation:

- Searching through possible states is a NP-complete problem!
- Processing and memory limitations

Real-Time Dynamic Programming



- Heuristic-search DP algorithm
- Benefits
 - Is focused
 - Has a good anytime behavior
- Repeated trials
 - Initial state s_0
 - Ends in goal state or dead-end state
 - Action selection → greedy
 - Outcome selection → randomly determined
 - Coverage path is built

Frontier Based Real-Time Dynamic Programming



- RTDP wastes time
- Frontier states = separate covered regions from uncovered
- Path with minimum expected cost
- Composite action/outcomes
- Dijkstra's algorithm → shortest path

Algorithm - FBRDTP

Algorithm 2 Frontier-Based-RTDP

Data structures: *frontier* - set of frontier states
visited - set of states already visited by the current trial

```
1: function FBRTDP( $s_0$ ) //  $s_0$  is the initial state
2:   while  $\max_{s \in \text{visited}} \text{RESIDUAL}(s) > \epsilon$  do
3:      $\text{visited} \leftarrow \{s_0\}$ 
4:      $\text{frontier} \leftarrow$  all successors of  $s_0$ 
5:     FBRTDPTRIAL( $s_0$ )

1: function FBRTDPTRIAL( $s$ ) // Execute one trial
2:   while not GOAL( $s$ ) and  $s \neq s_d$  do
3:     // Pick best composite action and update hash
4:      $\hat{a} \leftarrow$  GREEDYCOMPOSITEACTION( $s$ )
5:     UPDATE( $s, \hat{a}$ )
6:     // Stochastically simulate next state
7:      $s \leftarrow$  CHOOSENEXTSTATE( $\hat{a}$ )
8:     if  $s \notin \text{visited}$  then
9:       Add  $s$  to  $\text{visited}$ 
10:    UPDATEFRONTIER( $s$ )
```

```
1: function GREEDYCOMPOSITEACTION( $s$ )
2:   Build a graph  $G$  that consists of the states in  $\text{visited} \cup$ 
    $\text{frontier}$  and its edge weights defined as the expected costs
   of the state transitions
3:   Run Dijkstra on the graph  $G$  starting from  $s$ 
4:   Find a frontier  $f$  with minimum cost path from  $s$ 
5:   Let  $\hat{a} = (a_1, \dots, a_n)$  be the sequence of actions leading
   from  $s$  to  $f$  on the minimum cost path
6:   return  $\hat{a}$ 

1: function QVALUE( $s, \hat{a}$ )
2:   return  $\sum_{s' \in S} P(s'|s, \hat{a}) [C_{\hat{a}}(s, s') + s'.V]$ 

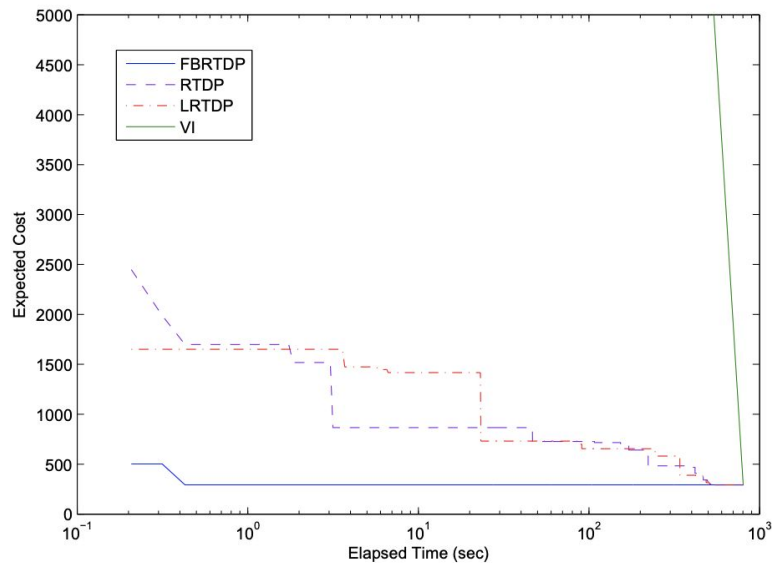
1: function UPDATE( $s, \hat{a}$ )
2:    $s.V \leftarrow$  QVALUE( $s, \hat{a}$ )

1: function CHOOSENEXTSTATE( $s, \hat{a}$ )
2:   Choose  $s'$  with probability  $P(s'|s, \hat{a})$ 
3:   return  $s'$ 

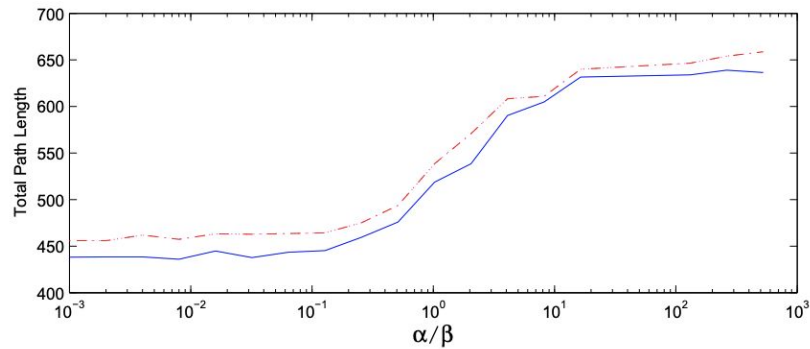
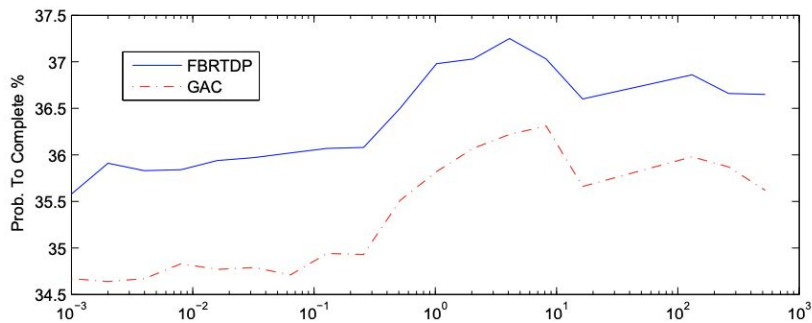
1: function UPDATEFRONTIER( $s$ )
2:   Remove  $s$  from  $\text{frontier}$ 
3:   for every successor state  $s'$  of  $s$  do
4:     if  $s' \notin \text{visited}$  and  $s' \notin \text{frontier}$  then
5:       Add  $s'$  to  $\text{frontier}$ 
```

Empirical Evaluation

- Comparison of FBRTDP to:
 - RTDP
 - Labeled RTDP (LRTDP)
 - Value Iteration (VI)
 - Greedy Adversarial Coverage (GAC)



Larger Maps Comparison



Conclusion



- Model adversarial coverage problem as an MDP for robots
- Smaller maps vs. larger maps
- MDPs
- FBRTDP
 - Provides efficiency while having same optimal convergence as RTDP
 - Allows RTDP to solve the problem on larger maps
- In the future:
 - Handle different variants of the adversarial coverage problem
 - I.e. time delay for robot rather than full stop
 - evaluate FBRTDP on other planning problems and compare its performance to other heuristic algorithms for solving MDPs

Works Cited



[1] Roi Yehoshua, Noa Agmon and Gal A. Kaminka. 2015. Frontier-Based RTDP: A New Approach to Solving the Robotic Adversarial Coverage Problem. Retrieved March 2020 from https://www.researchgate.net/profile/Roi_Yehoshua/publication/281176286_Frontier-Based_RTDP_A_New_Approach_to_Solving_the_Robotic_Adversarial_Coverage_Problem/links/55d9b64208aec156b9ac4ae4.pdf.